# Mode Participation Factor and Effective Mass

Modal Analysis – Lesson 4

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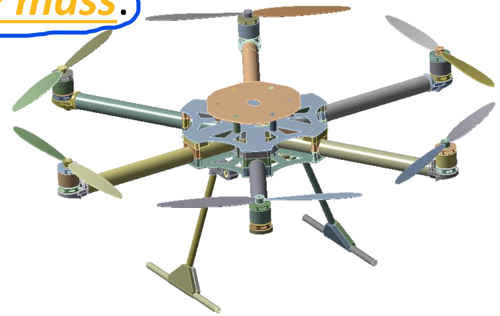
# Mode Participation Factor

Is every natural frequency equally important?

- For a realistic model, we might have thousands or millions of DOFs, which means that we can find that many natural frequencies.
- For example, take this recreational drone problem. There are one million DOFs, that's one million natural frequencies and modes. Is it necessary to find them all? Are they important on the same level? The answer is no.
- In most cases, high frequency modes can be  neglected. And not every mode participates at the same level to the deformation of the structure under dynamic load.
- How do we determine what number of modes is sufficient to extract and how do we  find the most important natural frequencies or modes? To quantify these, we use two simple scalars: the *mode participation factor and the effective mass*.



| Mode | Natural Frequency |
|---|---|
| 1 | 16.625 |
| 2 | 25.834 |
| 3 | 41.947 |
| 4 | 51.015 |
| 5 | 53.128 |
| 6 | 53.188 |
| 7 | 53.208 |
| 8 | 53.311 |
| 9 | 53.499 |
| 10 | 53.523 |
| 11 | 53.548 |
| 12 | 53.725 |
| 13 | 53.764 |
| 14 | 53.785 |
| 15 | 59.337 |
| 16 | 63.139 |
| 17 | 93.975 |
| 18 | 111.41 |
| 19 | 111.75 |
| 20 | 112.29 |
| 21 | 114.51 |
| 22 | 116.1 |
| 23 | 116.95 |
| 24 | 120.2 |
| 25 | 127.15 |
| 26 | 142.37 |
| 27 | 291.23 |
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# Mode Participation Factor (cont.)

Let's see how the participation factor and effective mass are calculated.

- Participation factor

Mass Matrix

$$\gamma_i = \{\phi\}_i^T [\boldsymbol{M}] \{\boldsymbol{D}\}$$

Mode shapes

Excitation Direction Vector

where {*D*} is an assumed unit displacement vector and depends on the direction of excitation in each of the global Cartesian directions and rotation about each of these axes.

- Effective mass

$$M_{eff,i} = \gamma_i^2$$

The square of the participation factor is the effective mass.

# Mode Participation Factor (cont.)

$$\gamma_i = \{\phi\}_i^T [M] \{D\}$$

$$M_{eff,i} = \gamma_i^2$$

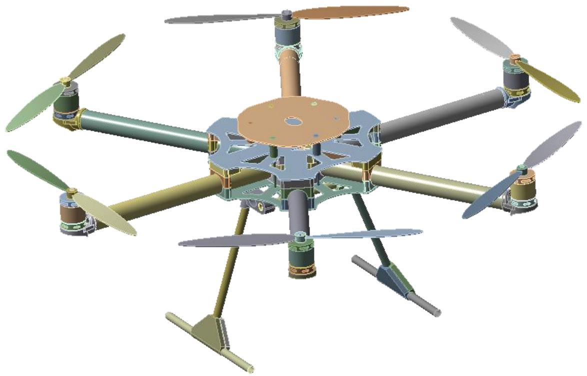
- The mode participation factor and the effective mass measures the amount of mass moving in each direction for each mode.
- Vector {D} represents the direction the participation factor is calculated in.
- A high value in a direction indicates that the mode will be excited by forces in that direction.

In some textbooks and articles, the effective mass here is called the participation factor. The participation factor and effective mass serve similar roles in modal analysis.

# Mode Participation Factor (cont.)

Let's have a look at the modes of a recreational drone structure.

- 12 modes are extracted for this structure.
- The data shows the participation factor and the effective mass calculated in the z direction.
- Modes 1, 5 and 11 contribute significantly to deformation in the z direction. These modes are most easily excited in vibration.

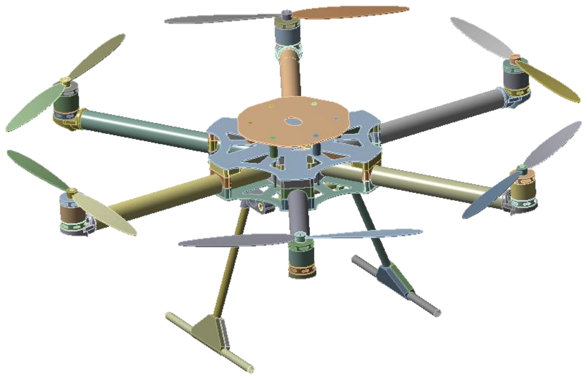


***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

| MODE | FREQUENCY | PERIOD | PARTIC.FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF.MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 436.755 | 0.22896E-02 | 3.9458 | 1.000000 | 15.5693 | 0.832972 | 0.734572 |
| 2 | 742.299 | 0.13472E-02 | -0.21103E-11 | 0.000000 | 0.445330E-23 | 0.832972 | 0.210111E-24 |
| 3 | 773.960 | 0.12921E-02 | -0.19217E-10 | 0.000000 | 0.369277E-21 | 0.832972 | 0.174228E-22 |
| 4 | 867.928 | 0.11522E-02 | -0.39713E-12 | 0.000000 | 0.157711E-24 | 0.832972 | 0.744094E-26 |
| 5 | 1349.07 | 0.74125E-03 | -1.6909 | 0.428524 | 2.85903 | 0.985933 | 0.134892 |
| 6 | 1452.63 | 0.68841E-03 | 0.51860E-10 | 0.000000 | 0.268943E-20 | 0.985933 | 0.126890E-21 |
| 7 | 1851.94 | 0.53997E-03 | -0.83610E-13 | 0.000000 | 0.699063E-26 | 0.985933 | 0.329824E-27 |
| 8 | 2960.43 | 0.33779E-03 | 0.52013E-10 | 0.000000 | 0.270534E-20 | 0.985933 | 0.127640E-21 |
| 9 | 2992.35 | 0.33419E-03 | -0.78978E-10 | 0.000000 | 0.623746E-20 | 0.985933 | 0.294289E-21 |
| 10 | 3094.57 | 0.32315E-03 | -0.57469E-09 | 0.000000 | 0.330264E-18 | 0.985933 | 0.155822E-19 |
| 11 | 3099.14 | 0.32267E-03 | 0.51276 | 0.129951 | 0.262921 | 1.00000 | 0.124049E-01 |
| 12 | 3494.04 | 0.28620E-03 | 0.20305E-09 | 0.000000 | 0.412285E-19 | 1.00000 | 0.194520E-20 |
| sum | | | | | 18.6912 | | 0.881869 |

# Mode Participation Factor (cont.)

- In doing modal analysis, we should include most of the significant modes, meaning that we should extract a sufficient number of modes to evaluate. This can be judged by the ratio between the effective mas and the total mass.
- If the ratio of effective mass to total mass is close to 1, it means most of the significant modes have been extracted.
- In this case, 12 modes were extracted. The ratio of effective mass to total mass reached 0.88.



***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

| MODE | FREQUENCY | PERIOD | PARTIC.FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF.MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 436.755 | 0.22896E-02 | 3.9458 | 1.000000 | 15.5693 | 0.832972 | 0.734572 |
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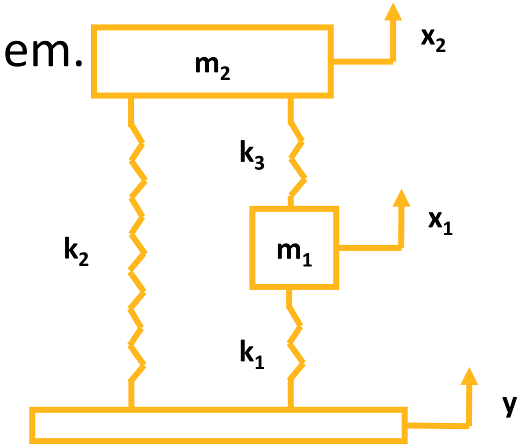
| sum | | | | | 18.6912 | | 0.881869 |

# Mode Participation Factor: Solve a Simple Spring-Mass System

Calculate the participation factor and effective mass of a 2-DOF spring-mass problem.




$$\gamma = \phi^T MD = \begin{bmatrix} 0.6280 & 0.4597 \\ -0.3251 & 0.8881 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.7157 \\ -0.2379 \end{bmatrix}$$

$$M_{eff,i} = \frac{\gamma_i^2}{\{\phi\}_i^T[M]\{\phi\}_i} = \gamma_i^2 = \begin{bmatrix} 2.9436 \\ 0.0566 \end{bmatrix}$$

| Parameters | |
|---|---|
| Variable | Value |
| $m_1$ | 2.0 kg |
| $m_2$ | 1.0 kg |
| $k_1$ | 1000 N/m |
| $k_2$ | 2000 N/m |
| $k_3$ | 3000 N/m |

Since there are only two modes for this 2-DOF problem, the sum of the effective mass is 3 kg, the total mass of the system.

$$\text{1.757^2+(-0.2375)^2} = \boxed{3\ \text{kg}}$$

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