How to report the integral of Far Field values such as Directivity over part of the infinite sphere (for example, upper hemisphere)

Frequently Asked Question:
In HFSS designs, Far Field characteristics such as rE, Directivity etc. are reported vs. spherical coordinates Theta and Phi. Also, total radiated power is reported through antenna parameters. Sometimes it is necessary to report Far Field variables integrated over some part of the infinite sphere. For example, for calculating noise temperature of antenna, it is necessary to report the part of radiated power that goes into the upper and lower hemisphere.

Solution:

Direcitivity as a function of Theta and Phi is derived from Far Field using the formula:

DirTotal=2*pi*(rETotal)^2/(377Ohm*Radiated Power)

The following definitions are used:

- **Direcitivity=U/U_isotropic,**
- **U_isotropic = Radiated Power/(4*pi)**
- **U(Theta,Phi)=(rETotal)^2/(2*377Ohm)**

The part of the radiated power through the particular surface of the infinite sphere can be calculated through the integral:

$$\int d\text{Phi} \int d\text{Theta} * \text{Dir} * \sin(\text{Theta}).$$

If the integration is over the whole area, Phi=-pi to +pi (-180deg to +180deg), and Theta =0 to pi (or 0deg to 180 deg).

In HFSS reporting, there are Trace Characteristics dedicated to integration. Imagine that we plot sin(Theta) vs. Theta as shown in Fig.1. Righclick on the graph to add Trace Characteristics, choose from Math category. Use 'integ' (or "integabs" if the integration covers negative Theta values) operation, you may manually control the range or use full and control the range in the Infinite Sphere setup (see Fig.2).
We expect to see $\int d\text{Theta} * \sin(\text{Theta}) = 2$ while integrating from Theta=0 to pi, but we see different value of ~114.58. Note that HFSS reports Theta and Phi in degrees, but the power calculations should be done in radians. Every time we integrate, it is necessary to multiply value to pi/180 to get the proper integral. It is easier to include normalization in the plotted value as in Fig.3.

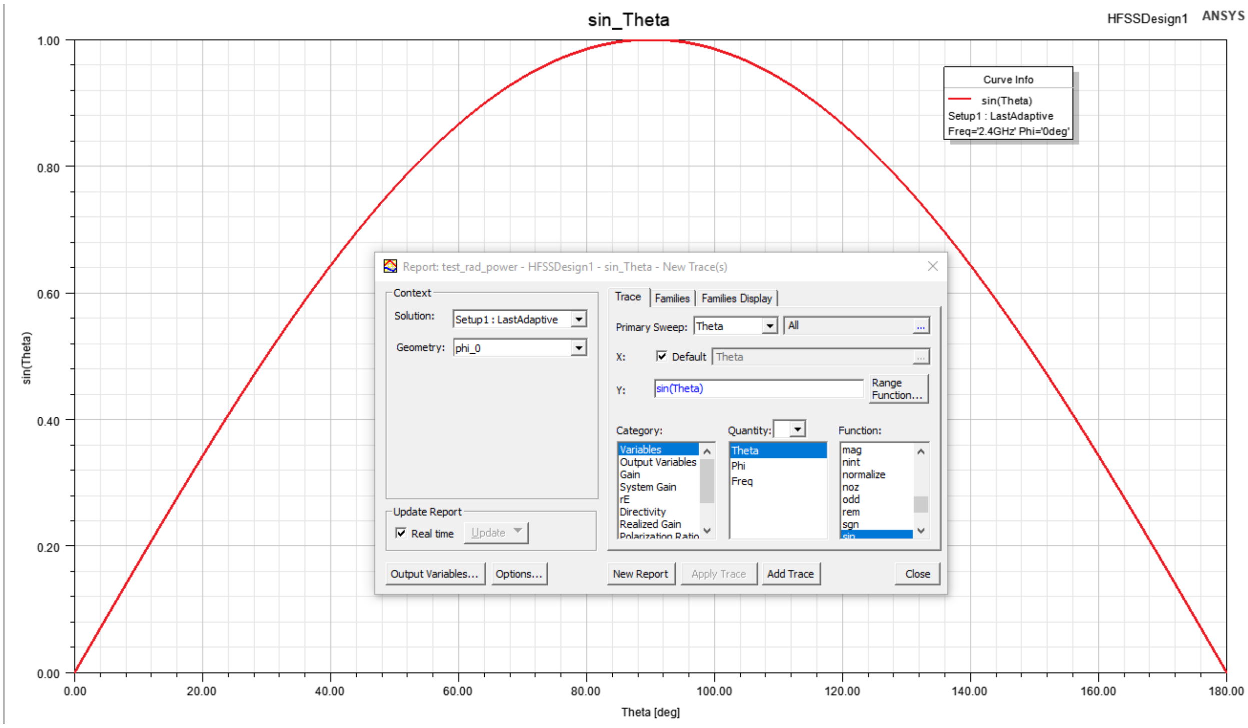

Fig.1 Plot sin(Theta) vs. Theta at phi=0 deg

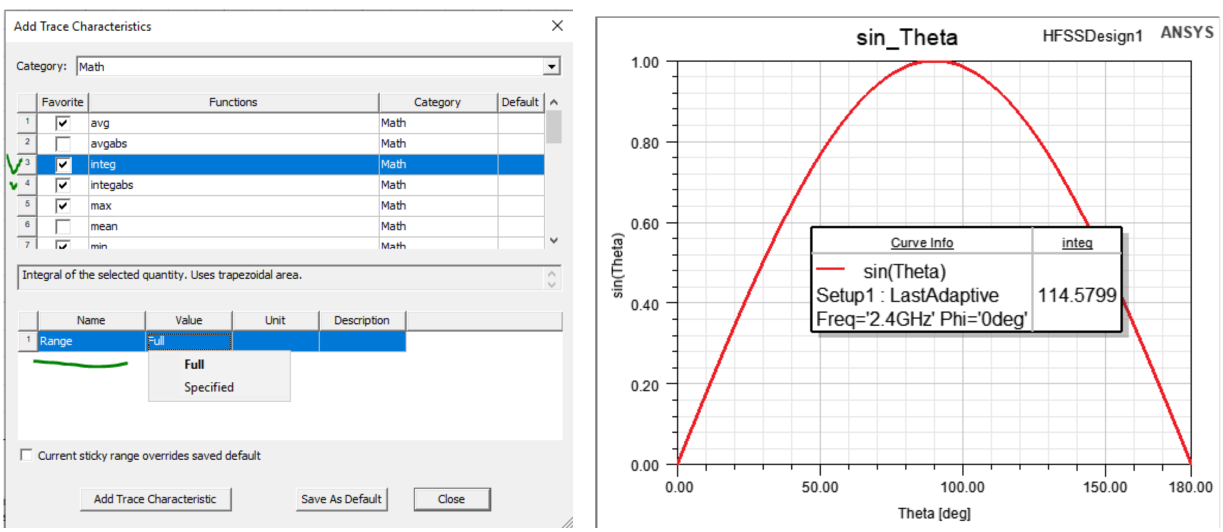

Fig.2 Trace Characteristic window and resulted integral value

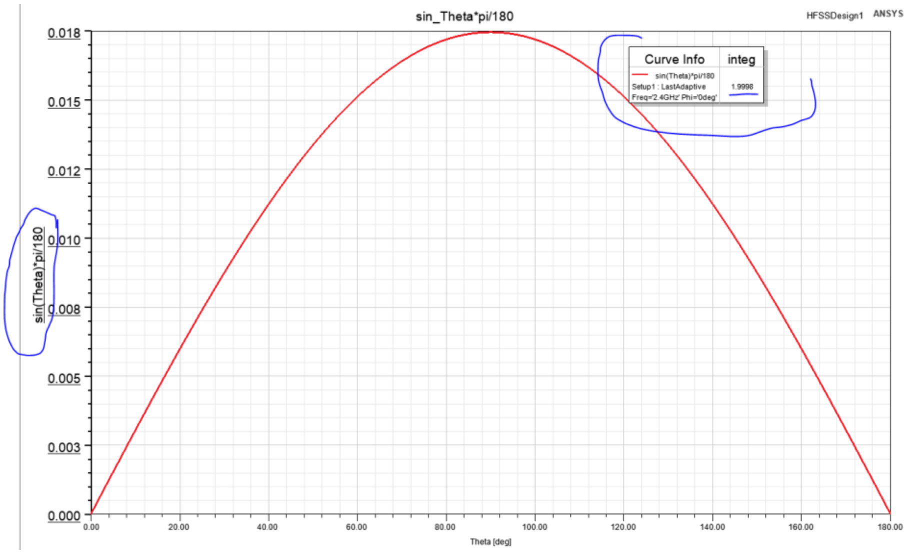

Fig.3 Normalized plot of sin(Theta) and expected integral value

Part of Total Radiated power : renormalization of integration using degrees

Note that if we integrate over the whole infinite sphere, we get 4*pi.

∫*d*Phi∫*d*Theta*sin(Theta)=4*pi

By the definition of Directivity (here **Dir** means Total Directivity)
∫*d*Phi∫*d*Theta*Dir*sin(Theta)=Total RadiatedPower=T

The power into the part of infinite sphere, P1, can be calculated as:
P1/T= [∫*d*Phi∫*d*Theta*Dir*sin(Theta)]/(4*pi),
Where integrations should be done in radians over the desirable part of the sphere. Since we know that integration in HFSS are sone in degrees, and we integrate over both phi and theta, we can normalize the value by (pi/180)^2.

Finally, the normalized Directivity value should be (pi/180)^2/(4*pi)=pi/(360^2).

Part of Total Radiated power that goes into upper hemisphere

Follow the steps to report the fraction of radiated power into the upper hemisphere:

Step1: create the Infinite sphere that covers upper Hemisphere (Fig.4)
Phi=-180deg to 180deg
Theta = 0deg to 90deg
Step size does not change the results if the Directivity is smooth enough.

Step2: plot integral of Dir*sin(Theta) over Theta vs. the value of Phi angle.
Check every option on Fig.4 that underscored or circled by red.

Step3: Use Trace Characteristics for the plot from the previous step as shown in Fig.5

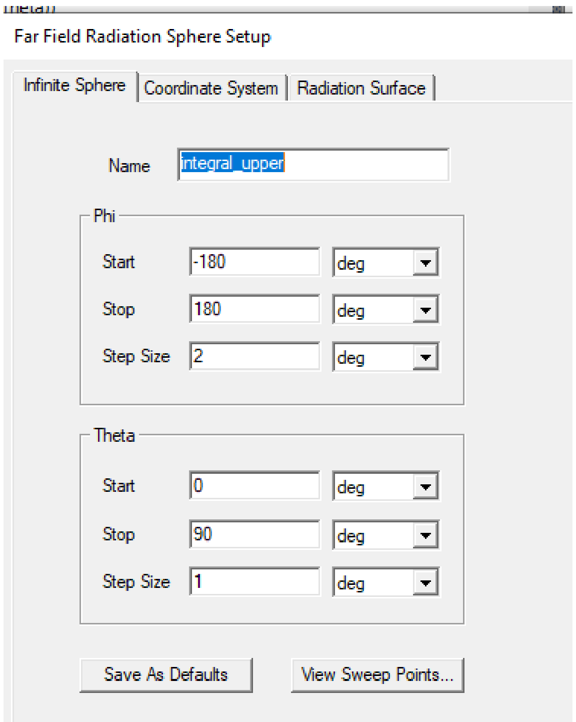

Fig.3 Infinite sphere setup that covers only upper hemisphere

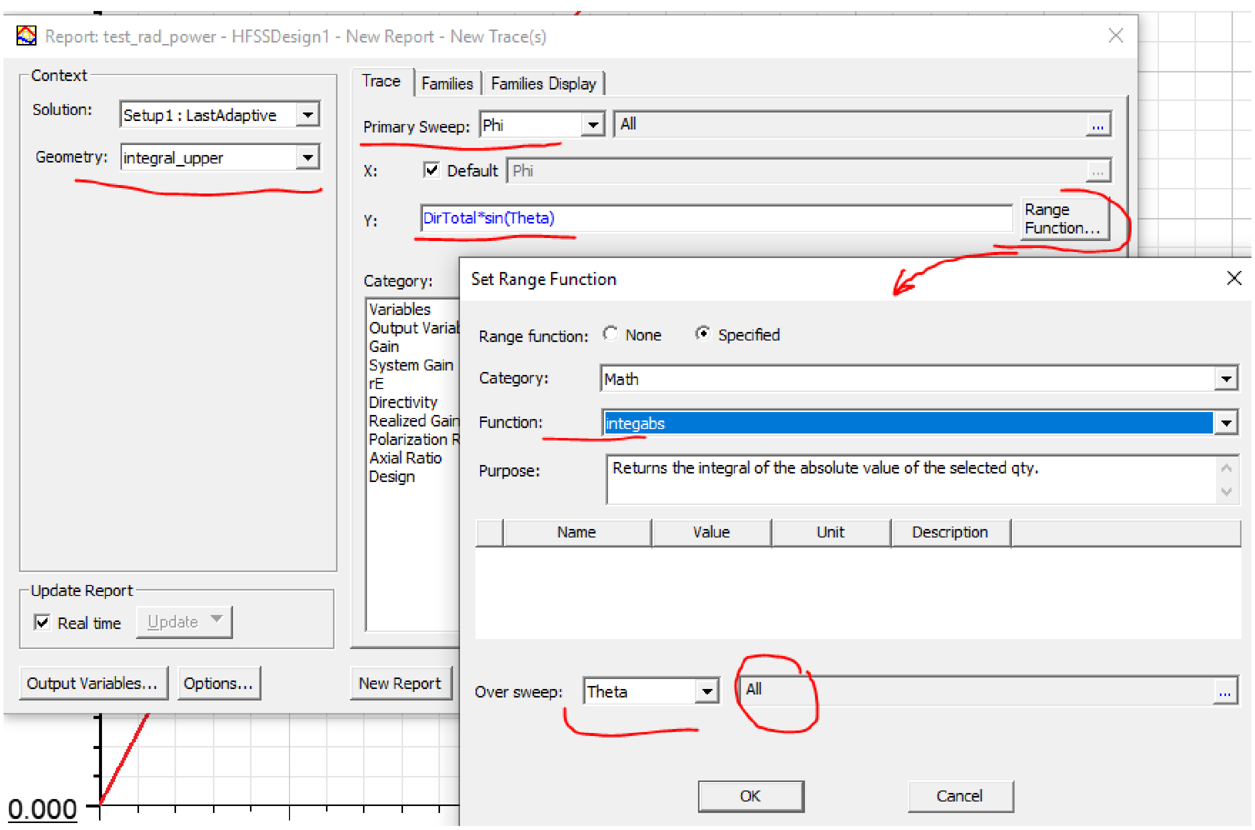

Fig.4.Plot Integral over Theta vs. phi

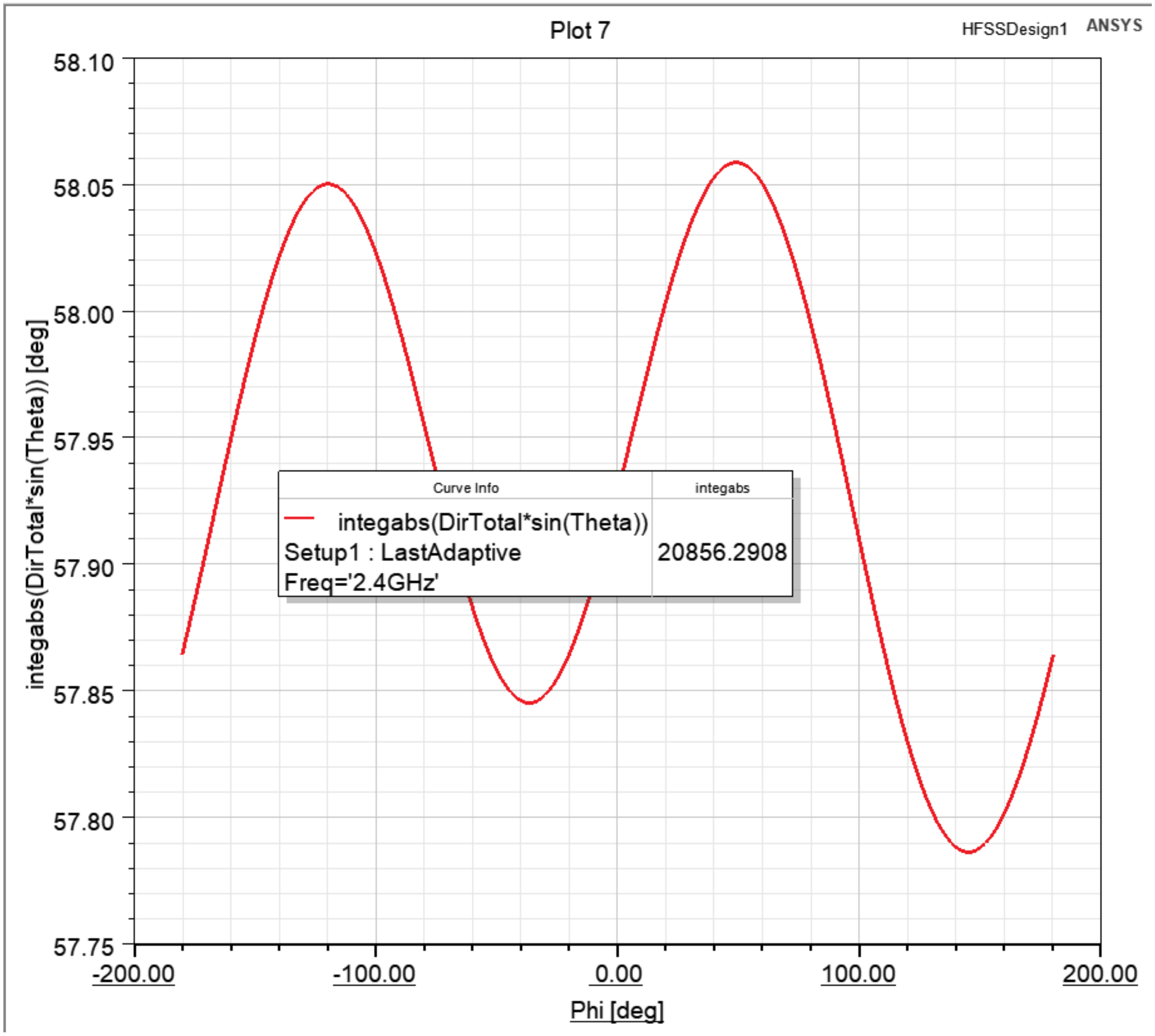

Fig.5.Plot from Fig.4 and Integral over Phi using Trace Characteristics option

Step4. Normalize the resulting value by pi/(360^2)
We get expected portion of radiated power into upper hemisphere for vertical dipole:
P1/T= 20856.2908*pi/360/360=0.505

Alternatively, one can include the normalization into the value plotted as shown in Fig.6

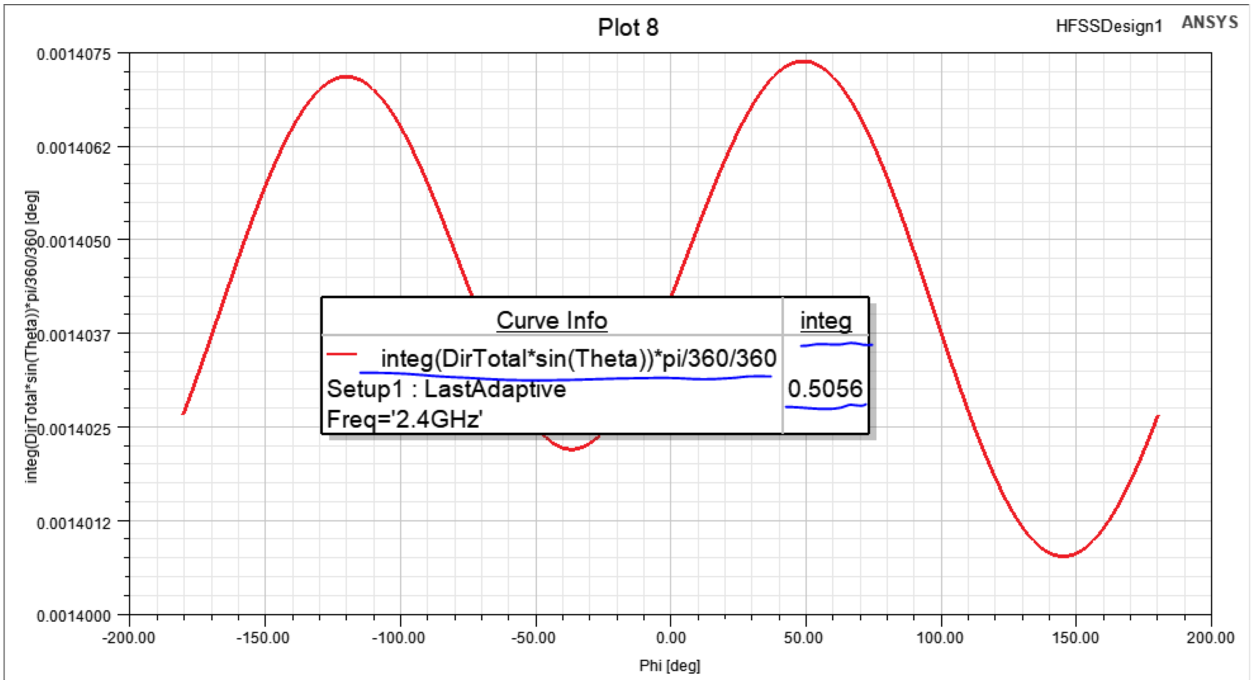

Fig.6.Plot of normalized value and Integral over Phi using Trace Characteristics option