Understanding Modal Expansion in Harmonic Analysis

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Background

- Modal expansion is carried out in harmonic analysis using modal super position method.
- The document explains about the modal expansion method available in ANSYS Mechanical.

What is expansion of modes?

- Expansion of modes is based on the ***Expansion Theorem***. According to ***Expansion Theorem***, any response of the linear structural system can be uniquely represented by linear combination of eigen modes of the system. Mathematically it is represented as follows,

Expansion of first *n* modes to predict the response {u}

$$\{u\} = \sum_{i=1}^{n} y_i \{\emptyset_i\}$$

Where y_i is called a modal coordinate and $\{\phi_i\}$ is the i^{th} mode shape vector.

- This becomes the fundamental theoretical basis for modal superposition method.

Modal Superposition Method

- Harmonic analysis using modal superposition method is carried out by going through the following steps,

Harmonic analysis with following governing equation

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = [F]$$

Modal Analysis

Step 1: Find out mode shapes ϕ_i and its corresponding natural frequency ω_i for set of *n* modes to be included in the harmonic analysis.

Harmonic Solution

Step 2: Solve *n* set of uncoupled equations for *n* set of modal coordinates y_i

$$\ddot{y}_i + 2\omega_i\zeta\dot{y}_i + \omega_i^2 y_i = f_i$$

Expansion of Results

Step 3: Expansion of all modes to predict the response {u} from mode shapes calculated in Step 1 and modal coordinate y_i calculated in Step 2.

$$\{u\} = \sum_{i=1}^{n} y_i\{\emptyset_i\}$$

Expansion of modes

- Specific ANSYS Mechanical (and Mechanical APDL), expansion refers to following two

1. **MADPL Command – mxpand:** This is used in the modal analysis. When the *mxpand* command is issued, the mode shapes corresponding to the natural frequencies of the structure are calculated and written in the modal analysis results file (*.rst). *Note that this expansion has no relation with the **Expansion Theorem** discussed in the 3rd slide.*

    - In ANSYS Mechanical APDL scripts, *mxpand* command is required to be given to get the mode shape results in the results file.
    - In ANSYS Workbench Mechanical, the number of modes selected are expanded by default and there is no need to separately specify mxpand command

2. **MAPDL Command – numexp:** This is used in the harmonic analysis. *This command calculates the frequency response of the structure at the frequencies of interest using the **Expansion Theorem** explained in the 3rd slide.*

    - It is important to specify the frequencies for which the previously calculated modes should be expanded using Expansion theorem in ANSYS Mechanical as well as ANSYS Mechanical APDL.

Selection of Appropriate Modes for Expansion

- It is possible to select only specific modes for calculation of mode shape and harmonic response in ANSYS Mechanical APDL. In mathematical terms, it can be represented as follows,

Expansion of all modes to predict the response {u}

$$\{u\} = \sum_{i=1}^{n} y_i \{\emptyset_i\}$$

Expansion of only certain selected nodes to predict the response {u}

$$\{u\} = y_1\{\emptyset_1\} + y_3\{\emptyset_3\} + y_6\{\emptyset_6\} + \cdots$$

- It is also possible to select the specific modes for expansion based on ratio of effective mass to the total mass in a given direction.

Selection of modes based on modal participation

- As mentioned in the previous slide, it is possible to select only specific modes for expansion based on the ratio of effective mass to the total mass of the structure. This modal selection is done in ***Modal Analysis*** using following commands

  - **MADPL Command – MODSLOPTION:** This command specifies the criteria for selecting modes based on the value of ratio of effective mass to the total mass in any given direction.

Ratio of effective mass to total mass

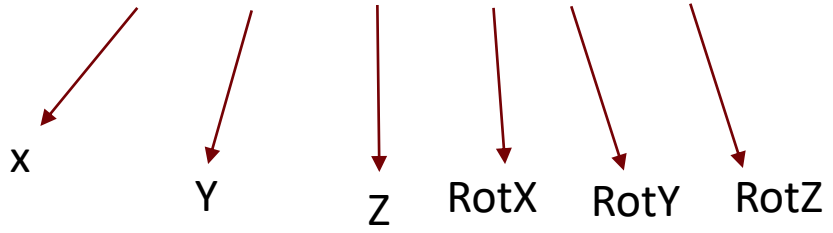
***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	1297.39	0.77078E-03	0.0000	0.000000	0.00000	0.00000	0.00000
2	3441.14	0.29060E-03	0.18218E-01	1.000000	0.331895E-03	1.00000	0.704661
3	3502.43	0.28552E-03	0.0000	0.000000	0.00000	1.00000	0.00000
4	4669.38	0.21416E-03	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.331895E-03		0.704661

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	1297.39	0.77078E-03	0.18007E-01	1.000000	0.324261E-03	1.00000	0.688453
2	3441.14	0.29060E-03	0.0000	0.000000	0.00000	1.00000	0.00000
3	3502.43	0.28552E-03	0.0000	0.000000	0.00000	1.00000	0.00000
4	4669.38	0.21416E-03	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.324261E-03		0.688453

MODSELOPTION, *dir1*, *dir2*, *dir3*, *dir4*, *dir5*, *dir6*

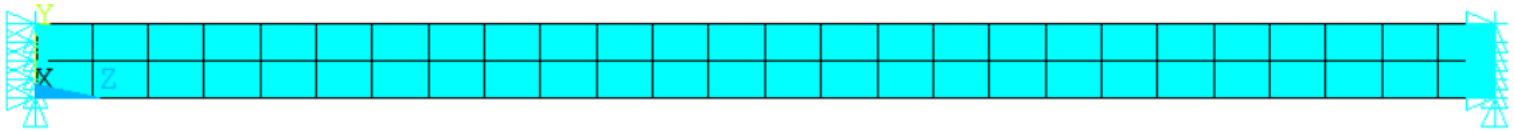


- If any decimal number is specified in any of these directions, all modes are considered till the summation of ratio of effective mass to total mass is less than the specified decimal. (starting from largest to smallest)
- If a 'no' is specified at any of the directions in the above commands, selection criteria is not considered for that particular direction.
- If 'yes' is specified at any directions in the above command, criteria from significance level from MXPAND command is considered as the criteria for selecting modes.

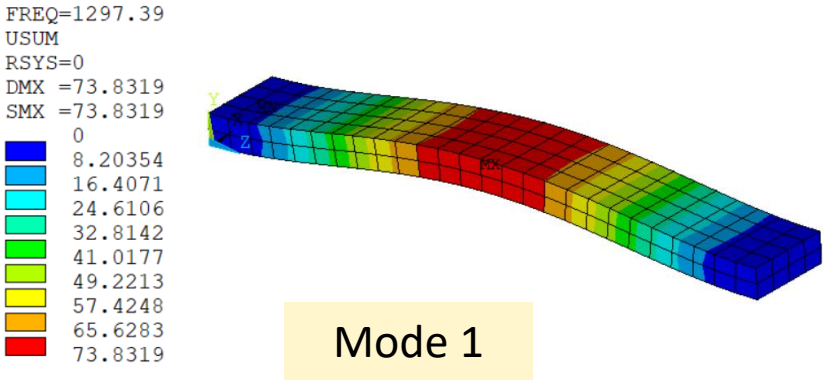
MXPAND, *NMODE*, *FREQB*, *FREQE*, *Elcalc*, ***SIGNIF***, *MSUPkey*, *ModeSelMethod*, *EngCalc*

Demo Example: Selecting only certain modes for expansion in harmonic analysis

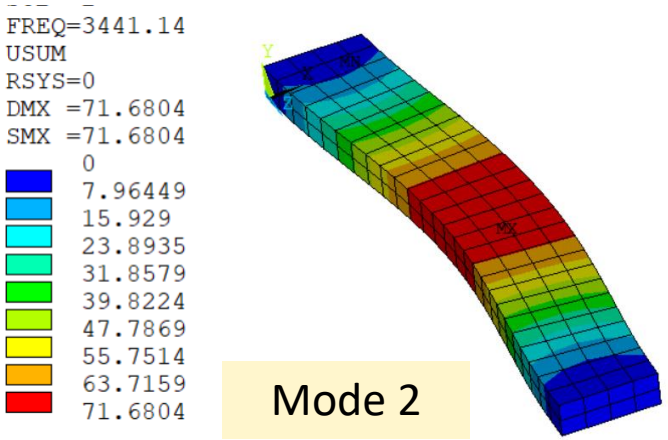
- Demo example of a beam fixed at both the ends is shown



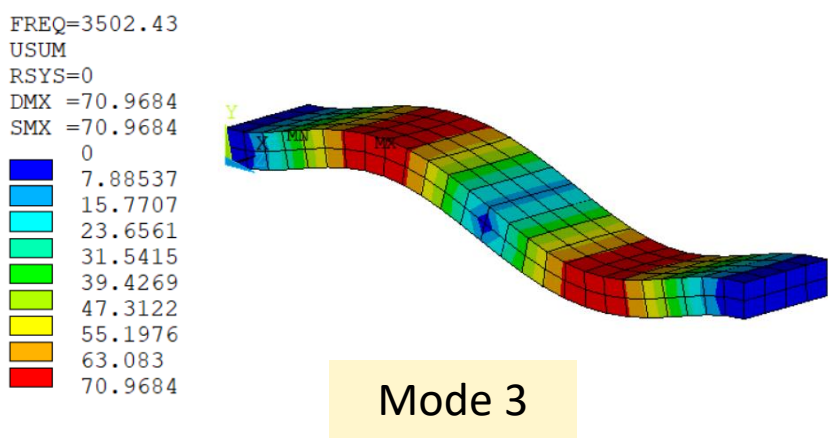
*Mode dominant in X direction



Mode 1

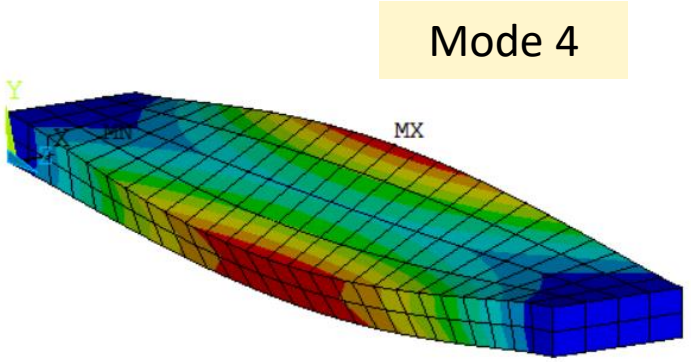


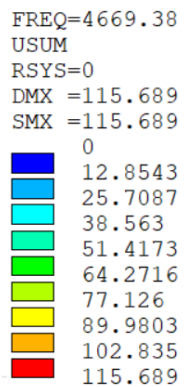
Mode 2



Mode 3

Mode 4





Demo Example: Selecting only certain modes for expansion in harmonic analysis

Expansion of all modes in Modal Analysis

```
modopt,lanb,10,0,5000,,,,,
mxpand,all,,,,,,EFFM
```

***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

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4	4669.38	0.21416E-03	0.0000	0.000000	0.00000	1.00000	0.00000
sum					0.324261E-03		0.688453

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	1297.39	0.77078E-03	0.0000	0.000000	0.00000	0.00000	0.00000
2	3441.14	0.29060E-03	0.0000	0.000000	0.00000	0.00000	0.00000
3	3502.43	0.28552E-03	0.0000	0.000000	0.00000	0.00000	0.00000
4	4669.38	0.21416E-03	0.0000	0.000000	0.00000	0.00000	0.00000

Expansion of modes only dominant in X direction Modal Analysis

```
modopt,lanb,10,0,5000,,,,,
mxpand,all,,,,,,EFFM
modseloption,,no,no,no,no,no,
```

***** PARTICIPATION FACTOR CALCULATION ***** X DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	3441.14	0.29060E-03	0.18218E-01	1.000000	0.331895E-03	1.00000	0.704661
sum					0.331895E-03		0.704661

***** PARTICIPATION FACTOR CALCULATION ***** Y DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	3441.14	0.29060E-03	0.0000	0.000000	0.00000	0.00000	0.00000

***** PARTICIPATION FACTOR CALCULATION ***** Z DIRECTION

MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	3441.14	0.29060E-03	0.0000	0.000000	0.00000	0.00000	0.00000

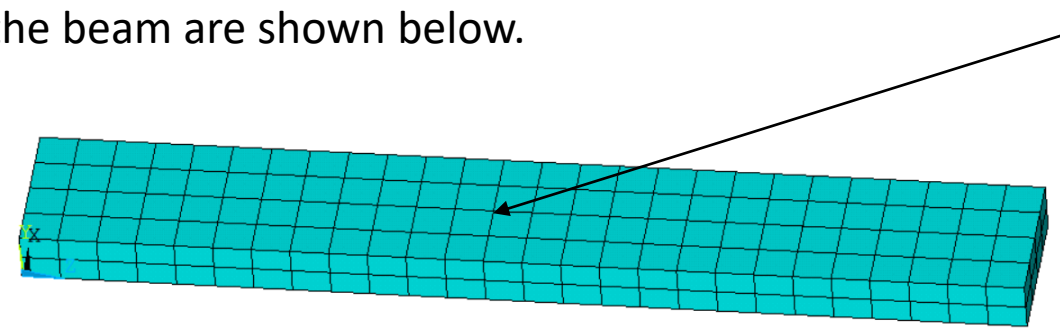
***** PARTICIPATION FACTOR CALCULATION *****ROTX DIRECTION

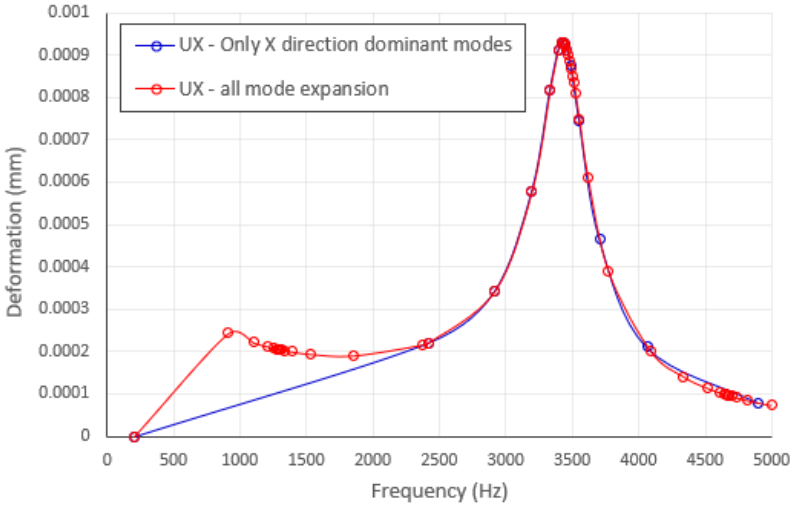
MODE	FREQUENCY	PERIOD	PARTIC.FACTOR	RATIO	EFFECTIVE MASS	CUMULATIVE MASS FRACTION	RATIO EFF.MASS TO TOTAL MASS
1	3441.14	0.29060E-03	0.0000	0.000000	0.00000	0.00000	0.00000

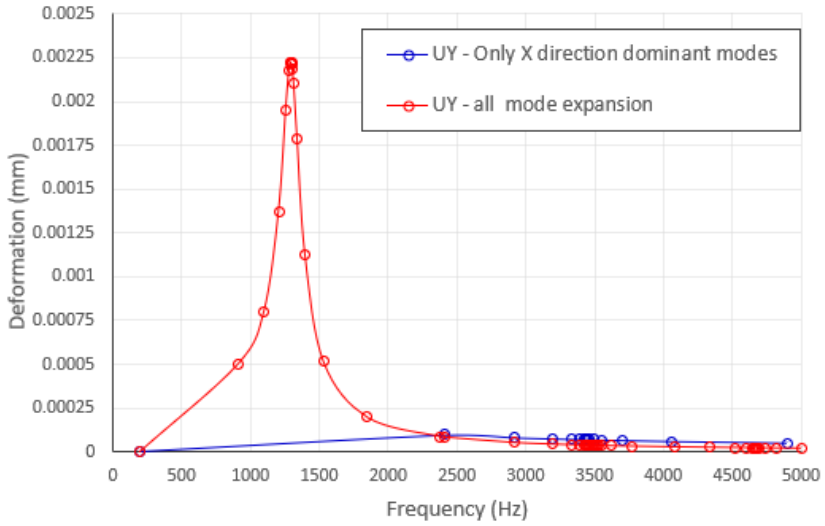
Only the mode with large effective mass ratio in X direction is included in the modal analysis results.

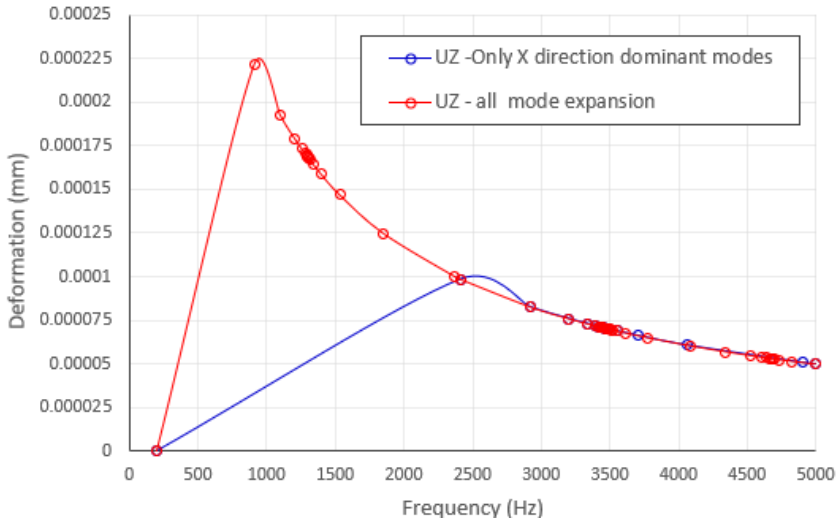
Demo Example: Harmonic Analysis

- Harmonic analysis is carried out on both the FE models (all modes included and only X direction dominant modes included)
- Base excitation of 10G applied in X,Y and Z direction each.
- The results at the center of the beam are shown below.









- Refer the cdb and script attached with this solution for the results shown above.