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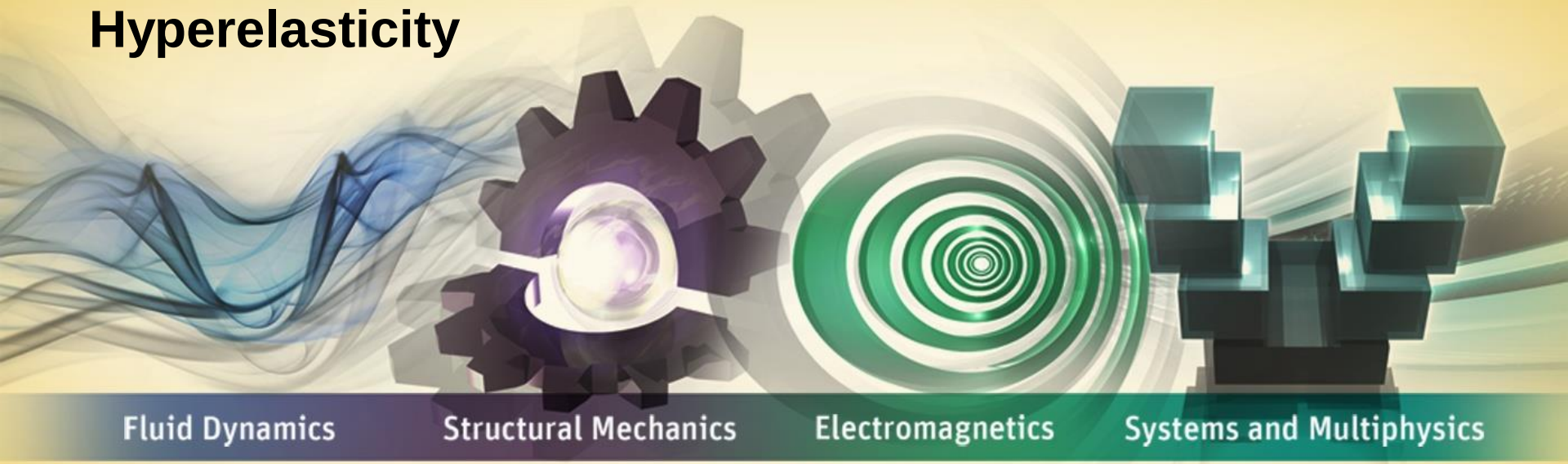
Appendix 4A

Hyperelasticity



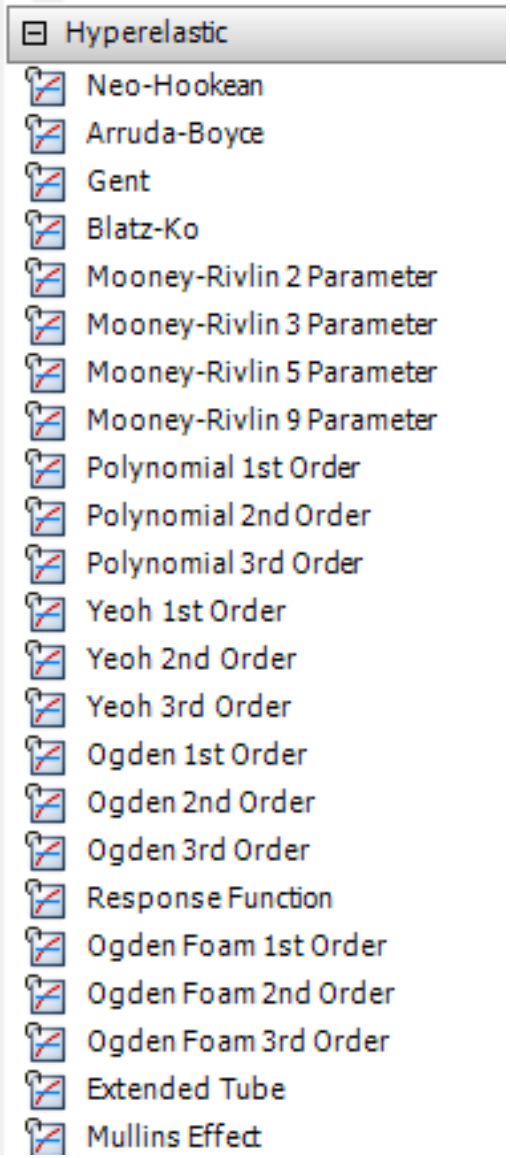
ANSYS Mechanical
Advanced Nonlinear Materials

Hyperelasticity Chapter Overview

This appendix is an optional supplement to Lecture 4, offering a more rigorous explanation of each particular form of the strain energy density function (W) and the differences between them.

Some *guidelines* will also be presented to aid the user in the selection the best strain energy density function.

Prerequisite is Chapter 4



... Particular Forms of W

The strain energy potential (*W*), introduced in Chapter 4, will require certain types of parameters input as material constants.

- **The number of material constants will differ, depending on the strain energy function *W* chosen.**
- **The choice of *W* will depend on the type of elastomer analyzed, the loading conditions, and the amount of data available.**
- **Some *very general guidelines* will be presented to aid the user in the selection of W. Keep in mind that, because of the above-mentioned factors, no guidelines can cover 100% of situations.**
- **From the selection of W and material constants which are input, stress and strain behavior are calculated by the solver.**
- **The next slides discuss the different forms of strain energy potential W available in ANSYS with some comments on the selection and of their use.**

... Particular Forms of W

The *polynomial form* is based on the first and second strain invariants. It is a phenomenological model of the form

$$W = \sum_{i+j=1}^{N} c_{ij}\left(\overline{I}_1 - 3\right)^i \left(\overline{I}_2 - 3\right)^j + \sum_{k=1}^{N} \frac{1}{d_k}\left(J-1\right)^{2k}$$

where the initial bulk modulus and initial shear modulus are

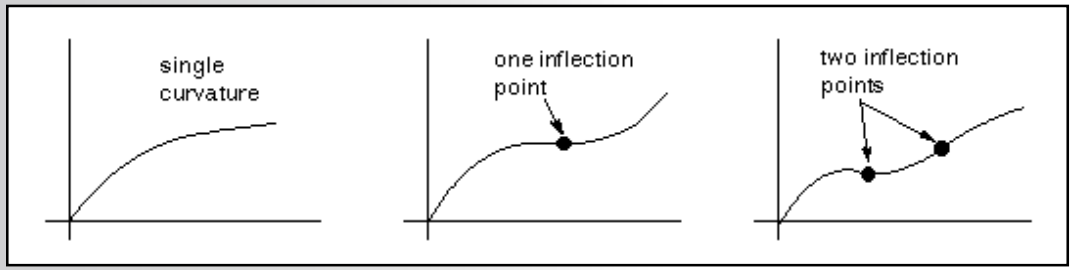
$$\mu_o = 2\left(c_{10} + c_{01}\right)$$

$$\kappa_o = \frac{2}{d_1}$$

... Polynomial Form Guidelines

Comments on the General Polynomial Form (PF):

- **As noted in the figure below, more terms will be required to capture any inflection points in the engineering stress-strain curve. *The user must ensure that enough data is supplied with inclusion of higher-order terms.* Polynomial form with N=2 or N=3 may be used up to 100-300% strains (general guideline). Usually, values of N greater than 3 are rarely used.**
- **PF is a very general form, so it can produce very good curve fits. As with all models, *data of expected modes of deformation is required when curve-fitting*. If limited (e.g., uniaxial) test data exists, consider use of Yeoh model (see Yeoh section later).**



PF with N=1 **PF with N=2** **PF with N=3**

... Mooney-Rivlin Model

There are two-, three-, five-, and nine-term *Mooney-Rivlin models* available in ANSYS. *These can also be thought of as particular cases of the polynomial form.*

- ***The Mooney-Rivlin model is commonly considered with N=1 (two-term M-R),* although ANSYS allows for any value of N.**
- **The two-term Mooney-Rivlin model is equivalent to the polynomial form when N=1:**

$$W = c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + \frac{1}{d}\left(J - 1\right)^2$$

- **The three-term Mooney-Rivlin model is similar to the polynomial form when N=2 and $c_{20}=c_{02}=0$:**

$$W = c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + c_{11}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right) + \frac{1}{d}\left(J - 1\right)^2$$

... Mooney-Rivlin Model

The five-term Mooney-Rivlin model is equivalent to the polynomial form when N=2:

$$W = c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + c_{20}\left(\overline{I}_1 - 3\right)^2 + c_{11}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right) + c_{02}\left(\overline{I}_2 - 3\right)^2 + \frac{1}{d}(J-1)^2$$

The nine-term Mooney-Rivlin model can also be thought of as the polynomial form when N=3:

$$W = c_{10}\left(\overline{I}_1 - 3\right) + c_{01}\left(\overline{I}_2 - 3\right) + c_{20}\left(\overline{I}_1 - 3\right)^2 + c_{11}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right) + c_{02}\left(\overline{I}_2 - 3\right)^2 + c_{30}\left(\overline{I}_1 - 3\right)^3 + c_{21}\left(\overline{I}_1 - 3\right)^2\left(\overline{I}_2 - 3\right) + c_{12}\left(\overline{I}_1 - 3\right)\left(\overline{I}_2 - 3\right)^2 + c_{03}\left(\overline{I}_2 - 3\right)^3 + \frac{1}{d}(J-1)^2$$

... Mooney-Rivlin Model

For all of the preceding Mooney-Rivlin forms, the initial shear and initial bulk moduli are defined as:

$$\mu_o = 2\left(c_{10} + c_{01}\right)$$

$$\kappa_o = \frac{2}{d}$$

... Mooney-Rivlin Guidelines

Comments on the Mooney-Rivlin (M-R) model:

- **Because of its equivalence to polynomial forms (N=1, 2, 3), as discussed earlier, *the same PF guidelines apply to M-R models.***
- **The 2-term Mooney-Rivlin model is most commonly used. Some *very broad rules-of-thumb* are presented below.**
  - **The 2-term M-R may be valid up to 90-100% tensile strains, although it will not account for stiffening effects of the material, usually present at larger strains.**
  - **Pure shear behavior may be characterized up to 70-90%. This is because the 2-term M-R model exhibits a constant shear modulus.**
  - **Although moderate compression behavior can be characterized well (up to 30%), significant compression response may not be captured with only 2-term MR.**

... Yeoh Model

The *Yeoh model* (a.k.a. reduced polynomial form) is similar to the polynomial form but is based on first strain invariant only.

$$W = \sum_{i=1}^{N} c_{i0}\left(\overline{I}_1 - 3\right)^i + \sum_{i=1}^{N} \frac{1}{d_i}\left(J-1\right)^{2i}$$

***The Yeoh model is commonly considered with N=3 (a.k.a. "cubic" form),* although solver allows for any value of N.**

The initial shear and bulk moduli are defined similar to other invariant-based models:

$$\mu_o = 2c_{10}$$

$$\kappa_o = \frac{2}{d_1}$$

... Yeoh Model Guidelines

One may note from the previous slides that the Yeoh model is dependent on the first invariant I_1 only.*

- Yeoh proposed omitting the second invariant term. The justification of this comes from the observation that changes in the strain energy potential is less sensitive to changes in the second invariant than the first (i.e., $\partial W/\partial I_1 >> \partial W/\partial I_2$). This is especially true for larger strains.
- Also, if only limited test data is available (e.g., uniaxial test), it has been shown that ignoring the second invariant leads to better prediction of general deformation states.
- As strain increases, the shear modulus (slope) decreases slightly then increases. To reflect this, use a *cubic form* (N=3):
  - c_{10} is positive, equal to half of initial shear modulus value
  - c_{20} is negative (softening at small strains), ~ 0.1 to 0.01 * c_{10}
  - c_{30} is positive (stiffening at larger strains), ~ 1e-2 to 1e-4 * c_{10}

* For a detailed discussion on this topic, please refer to the papers by O.H. Yeoh, "Characterization of Elastic Properties of Carbon-Black-Filled Rubber Vulcanizates," Rubber Chem. Tech. 63, 1990 and "Some Forms of the Strain Energy Function for Rubber," Rubber Chem. Tech. 66, 1993

... Neo-Hookean Form

The *neo-Hookean form* can be thought of as a subset of the polynomial form for N=1, c_{01}=0, and c_{10}=μ/2:

$$W = \frac{\mu}{2}\left(\overline{I}_1 - 3\right) + \frac{1}{d}\left(J - 1\right)^2$$

where the initial bulk modulus is defined as

$$\kappa_o = \frac{2}{d}$$

... *Neo-Hookean* Guidelines

The neo-Hookean model is the simplest hyperelastic model, and it may be a good way to start.

- **Although it will probably not predict moderate/large strains well, for small strain applications, it may be suitable.**

Similar considerations apply to the neo-Hookean model as to the 2-term Mooney-Rivlin model (discussed earlier):

- **The neo-Hookean form may be valid up to 30-40% tensile strains, and it will not account for stiffening effects of the material, usually present at larger strains.**
- **Pure shear behavior may be characterized up to 70-90%. This is because the neo-Hookean model exhibits a constant shear modulus.**
- **Although moderate compression behavior can be characterized well (up to 30%), *significant* compression response may not be captured with neo-Hookean.**

... Arruda-Boyce Model

The *Arruda-Boyce form* (a.k.a. eight-chain model) is a statistical mechanics-based model. This means that the form was developed as a statistical treatment of non-Gaussian chains emanating from the center of the element to its corners (eight-chain network).

$$W = \mu \sum_{i=1}^{5} \frac{C_i}{\lambda_L^{2i-2}} \left(\overline{I_1^i} - 3^i \right) + \frac{1}{d} \left(\frac{J^2\text{-}1}{2} - \ln J \right)$$

- **where the constants C_i are defined as**

$$C_1 = \frac{1}{2}, C_2 = \frac{1}{20}, C_3 = \frac{11}{1050}, C_4 = \frac{19}{7050}, C_5 = \frac{519}{673750}$$

... Arruda-Boyce Model

- The initial shear modulus is μ.
  - In the Arruda-Boyce paper,* the rubbery modulus (shear modulus) is defined as $nk\Theta$, which is a function of chain density (n), Boltzmann's constant (k), and temperature (Θ). In ANSYS, $\mu = nk\Theta$.
- The limiting network stretch λ_L is the chain stretch at which stress starts to increase without limit.
  - Note that as λ_L becomes infinite, the Arruda-Boyce form becomes the Neo-Hookean form.
  - Also in the paper,* the equation references the locking stretch (limiting network stretch) as $\sqrt{N}$. In ANSYS, $\lambda_L = \sqrt{N}$.
- The initial bulk modulus κ is defined as usual by 2/d.

* For a detailed discussion on this topic, please refer to the paper by M.C. Boyce and E.M. Arruda, "A Three-Dimensional Constitutive Model for the Large Stretch Behavior of Rubber Elastic Materials," J. Mech. Phys. Solids, Vol 41 No 2, 1993.

... Arruda-Boyce Guidelines

A few comments on the Arruda-Boyce (A-B) model:

- **It is apparent that the A-B model has some similarities to the Yeoh model, although *the coefficients are fixed, predefined functions of the limiting network stretch* λ_L.**
  - **This means that discussion of the Yeoh model and consideration of I_1-dependency are applicable here as well.**
  - **From a physical standpoint, the use of I_1 only means that the eight chains are equally stretched under any deformation state, i.e., $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$ represents this chain elongation.**
  - **Additional usefulness of the Arruda-Boyce model stem from the fact that the material behavior can be characterized well even with limited test data (uniaxial test), and fewer material parameters are required. *However, this is a fixed formulation, which may limit its applicability for any material.***
  - **Generally speaking, suited for large strain ranges.**
  - **No stress softening but only stress stiffening with increasing strain.**

... Arruda-Boyce Guidelines

More comments on the Arruda-Boyce (A-B) model:

- **The A-B equation used here is actually the first five terms of the strain energy function. The original equation is a stress relationship which contains an inverse Langevin function. This equation needs to be converted to a series expansion and numerically integrated to get W (i.e., integration is not exact).**
  - **Because only the first five terms of W are commonly used, this may cause the limiting network stretch to be slightly less pronounced.**
  - **This does not invalidate the model but is simply mentioned in case one does an academic exercise to stretch a model near the limiting stretch value. The stress will rise dramatically but will not experience any 'limiting' stretch value.**
  - **It is important to note that, in this discussion, one is referring to the *chain stretch*, which is defined as:**

$$\lambda_{chain} = \frac{1}{\sqrt{3}}\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} = \sqrt{\frac{I_1}{3}}$$

... Gent Model

The *Gent model* is a micromechanical model, similar to Arruda-Boyce, which also utilizes the concept of limiting network stretch:

$$W = -\frac{\mu J_m}{2}\ln\left(1-\frac{\bar{I}_1-3}{J_m}\right)+\frac{1}{d}\left(\frac{J^2\text{-}1}{2}-\ln J\right)$$

where the constants μ, J_m, and d are input. μ is the initial shear modulus. J_m is the limiting value of (I_1-3), analogous to λ_L for Arruda-Boyce.

- **The initial shear modulus is μ**
  - **In Gent's paper,* the tensile modulus E is defined instead, with μ = E/3.**
- **J_m is the limiting value of (I_1-3) where stresses become infinitely large.**
  - **Note that as J_m becomes infinite, the Gent model approaches the Neo-Hookean model.**
- **The initial bulk modulus κ is defined as usual by 2/d.**

*** For a detailed discussion on the model, please refer to A. N. Gent, "A New Constitutive Relation for Rubber," Rubber Chem. Tech. 69, 1996.**

... Gent Guidelines

Some comments on the Gent model:

- **If a series expansion of the natural logarithm is performed, the resulting expression will be similar to the Yeoh model. The coefficients, however, are *predefined functions of J_m*.**
- **It is quite clear that there are many similarities between the Gent and A-B models.***
  - **J_m is the limiting value of (I_1-3) in Gent, analogous to λ_L being the limiting value of chain stretch λ_{chain} for A-B.**
  - **As stated in Gent's paper, the value of J_m should be on the order of 100.**
- **Because of the fact that Gent's strain energy function is used exactly, J_m is the limiting value of (I_1-3) where stresses will increase without bounds.**
- **Like the Yeoh and Arruda-Boyce models, the Gent model is applicable for large strain cases.**

*** For a detailed discussion on the comparison of the two models, see M.C. Boyce, "Direct Comparison of the Gent and the Arruda-Boyce Constitutive Models of Rubber Elasticity," Rubber Chem. Tech. 69, 1996**

... Ogden Model

The *Ogden form*, another phenomenological model, is directly based on the principal stretch ratios rather than the strain invariants:

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}\left(\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3\right) + \sum_{i=1}^{N} \frac{1}{d_i}(J-1)^{2i}$$

where the initial bulk and shear moduli are defined as

$$\mu_o = \frac{\sum_{i=1}^{N} \mu_i \alpha_i}{2} \qquad \kappa_o = \frac{2}{d_1}$$

- **The model is equivalent to the (two-term) Mooney-Rivlin form if**
  N=2 $\mu_1=2c_{10}$ $\alpha_1=2$ $\mu_2=-2c_{01}$ $\alpha_2=-2$
- **The model degenerates to the Neo-Hookean form when**
  N=1 $\mu_1=\mu$ $\alpha_1=2$

... Ogden Guidelines

Some comments on the Ogden model:

- Since Ogden is based on principal stretch ratios directly, it may be more accurate and often provides better curve fitting of data. However, it may also be a little more computationally expensive.
- Note that if limited test data exists and multiple modes of deformation are expected, curve-fitting uniaxial data only may not yield realistic behavior in other modes.
- Ogden noted that a minimum of three terms should be used.*
  - The first term represents small strain values:
    $1.0 < \alpha_1 < 2.0$ and $\mu_1 > 0$
  - The second term represents stiffening at larger strains:
    $\alpha_2 > 2.0$ and $\mu_2 > 0$ with $\mu_2 << \mu_1$
  - The third term represents behavior in compression:
    $\alpha_3 < -0.5$ and $\mu_3 < 0$ with $\mu_3 << \mu_1$
  - The product $\mu_i\alpha_i$ should always be positive (shear modulus)

* For a detailed discussion, see R.W. Ogden, "Large Deformation Isotropic Elasticity - On the Correlation of Theory and Experiment for Incompressible Rubberlike Solids," Rubber Chem. and Tech. 46, 1973 and "Recent Advances in the Phenomenological Theory of Rubber Elasticity," Rubber Chem. and Tech. 59, 1986

... Ogden Guidelines

- Yeoh added some additional insights on the Ogden constants by examining the Ogden equation:*
  - In shear, when $|\alpha_i| > 2.0$, material stiffens with increasing strain. Conversely, when $|\alpha_i| < 2.0$, material softens with increasing strain. (Shear behavior is insensitive to sign of α_i)
  - When α_i is negative, it has a large contribution to compressive behavior but small contribution to tensile behavior. For positive, small values of α_i, the compression behavior is insensitive to α_i and behaves like neo-Hookean material ($\alpha_1=2$).
  - Yeoh proposed the following guideline for 2-term Ogden:
    $1.2 < \alpha_1 < 1.6$ and $\mu_1 > 0$ (small-strain behavior)
    $\alpha_2 \sim 6.0$ and $\mu_2 > 0$ with $\mu_2 << \mu_1$ (large-strain tensile behavior)
- Generally speaking, the Ogden model can be used to characterize small or large strain behavior.

* For a detailed discussion, see O.H. Yeoh, "On the Ogden Strain-Energy Function," Rubber Chem. and Tech. 70, 1997

... Ogden Compressible Foam Model

The *Ogden compressible foam model* (a.k.a. Hyperfoam model) is similar to the Ogden incompressible model:

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}\left(J^{\alpha_i/3}\left(\overline{\lambda}_1^{\alpha_i} + \overline{\lambda}_2^{\alpha_i} + \overline{\lambda}_3^{\alpha_i}\right) - 3\right) + \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i \beta_i}\left(J^{-\alpha_i \beta_i} - 1\right)$$

where the initial bulk and shear moduli are

$$\mu_o = \frac{\sum_{i=1}^{N} \mu_i \alpha_i}{2} \qquad \kappa_o = \sum_{i=1}^{N} \mu_i \alpha_i \left(\frac{1}{3} + \beta_i\right)$$

However, unlike the regular Ogden model, in the Ogden compressible foam model, the volumetric and deviatoric terms are *tightly coupled*. Hence, *this model is meant to model highly compressible rubber behavior*.

... Ogden Foam Model Guidelines

The Ogden compressible foam model behaves in a similar fashion to the regular Ogden model for incompressible rubber:

- **Larger negative values of α_i affect behavior in compression more drastically.**
- **Conversely, larger positive values of α_i affects tensile behavior (significant hardening)**

... Blatz-Ko Model

The *Blatz-Ko model* is specifically for compressible polyurethane foam rubber with the following form:

$$W = \frac{\mu}{2}\left(\frac{I_2}{I_3} + 2\sqrt{I_3} - 5\right)$$

where μ is the shear modulus.

- The bulk modulus κ is defined as $5\mu/3$.
  This implies $\nu = 0.25$.
- Note that I_2 and I_3 are regular (not deviatoric) second and third strain invariants.
- This model was proposed by Blatz and Ko for a 47% volume percent polyurethane foam-type rubber.

$$\kappa = \frac{2(1+\nu)}{3(1-2\nu)}\mu$$

$$\kappa = \frac{2(1+0.25)}{3(1-2\cdot 0.25)}\mu$$

$$\kappa = \frac{5}{3}\mu$$

... Blatz-Ko Model

The Blatz-Ko model can be thought of as a subset of Ogden compressible foam model, with N=1, $\mu_1=-\mu$, $\alpha_1=-2$, $\beta_1=0.5$.

$$W = \frac{\mu_1}{\alpha}\left(J^{\alpha/3}\left(\bar{\lambda}_1^{\alpha} + \bar{\lambda}_2^{\alpha} + \bar{\lambda}_3^{\alpha}\right) - 3\right) + \frac{\mu_1}{\alpha\beta}\left(J^{-\alpha\beta} - 1\right)$$

$$W = \frac{-\mu}{-2}\left(J^{-2/3}\left(\bar{\lambda}_1^{-2} + \bar{\lambda}_2^{-2} + \bar{\lambda}_3^{-2}\right) - 3\right) + \frac{-\mu}{-2\cdot 0.5}\left(J^{2\cdot 0.5} - 1\right)$$

$$W = \frac{\mu}{2}\left(\lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} - 3\right) + \frac{\mu}{2}\left(2J - 2\right)$$

$$W = \frac{\mu}{2}\left(\frac{I_2}{I_3} - 3\right) + \frac{\mu}{2}\left(2\sqrt{I_3} - 2\right)$$

$$W = \frac{\mu}{2}\left(\frac{I_2}{I_3} + 2\sqrt{I_3} - 5\right)$$

As will be shown later, the effective Poisson's ratio can also be determined from β, which again leads to the assumption of ν=0.25 for Blatz-Ko.

ANSYS ... Incompressibility Considerations

The *Ogden Compressible Foam* and *Blatz-Ko* models are for *compressible foam-type rubbers*. The deviatoric and volumetric terms of strain energy are tightly coupled.

For the *nearly incompressible* rubber models, the volumetric term is often presented as one of three forms, as shown on the right:

$$W_b^1 = \sum_{i=1}^{N} \frac{1}{d_i}(J-1)^{2i}$$

$$W_b^2 = \frac{1}{d}(J-1)^2$$

$$W_b^3 = \frac{1}{d}\left(\frac{J^2\text{-}1}{2} - \ln J\right)$$

- **Recall that the term J is ratio of current to original volume. Undeformed state is J=1.**
- **For cases of W_b^1, only d_1 is usually considered (= W_b^2).**
- **The selections of W_b and the bulk modulus value (κ=2/d) do not tend to affect results much unless the model is significantly stretched (leading to finite volume change) or highly confined.**

For the *fully incompressible* case with d=0, this volumetric term W_b is ignored (J=1, volume preserved).

... Incompressibility Considerations

Considerations for Incompressibility:

- **All rubber-like materials have some very small compressibility. However, assuming full incompressibility is usually a suitable approximation. The choice of treatment of material as nearly- or fully-incompressible is decided by the user and data available.**
  - **Without any data, κ is sometimes approximated anywhere from 500*μ (ν=0.499) to 2000*μ (ν=0.49975).**
- **For 18x lower-order elements, use B-Bar as first choice for nearly incompressible problems**
  - **If shear locking exists, switch to Enhanced Strain.**
  - **If volumetric locking occurs with very high ν, use Mixed u-P.**
- **If the material is fully incompressible, 18x elements with Mixed u-P *must* be used. Set d=0 and KEYOPT(6) > 0 for fully incompressible problems.**

... Poisson's Ratio

For nearly- or fully-incompressible materials, the material compressibility parameter *d* can be estimated as follows:

- **The initial bulk modulus can be estimated and written in terms of the initial shear modulus**
- **The material compressibility parameter is proportional to the inverse of the initial bulk modulus**
- **The initial bulk modulus is provided in the previous slides for each of the hyperelastic materials**
- **The material compressibility parameter can therefore be written in terms of the initial bulk modulus as shown on the right, assuming *nearly-* or *fully-incompressible* behavior.**

$$\mu = \frac{E}{2(1+\nu)}$$

$$\kappa = \frac{E}{3(1-2\nu)}$$

$$= \frac{2\mu(1+\nu)}{3(1-2\nu)}$$

$$\approx \frac{\mu}{(1-2\nu)} \quad \text{for } \nu \approx 0.5$$

$$d = \frac{2}{\kappa_o}$$

$$\approx \frac{2(1-2\nu)}{\mu_o}$$

ANSYS ... Poisson's Ratio

The material compressibility parameter *d* is not present in *compressible models* since the volumetric term is coupled:

- **For the Blatz-Ko compressible foam model, the Poisson's ratio is assumed to be ν=0.25**
- **For the compressible Ogden model, Poisson's ratio can be calculated as follows, assuming β_i is constant (β):**

$$\mu = \frac{E}{2(1+\nu)}$$

$$\kappa = \frac{E}{3(1-2\nu)}$$

$$\frac{\kappa}{\mu} = \frac{2(1+\nu)}{3(1-2\nu)}$$

$$\mu_o = \frac{\sum_{i=1}^{N} \mu_i \alpha_i}{2}$$

$$\kappa_o = \sum_{i=1}^{N} \mu_i \alpha_i \left(\frac{1}{3} + \beta_i\right)$$

$$\frac{\kappa_o}{\mu_o} = 2\left(\frac{1}{3} + \beta\right)$$

$$\frac{2(1+\nu)}{3(1-2\nu)} = 2\left(\frac{1}{3} + \beta\right)$$

$$(1+\nu) = (1-2\nu)(1+3\beta)$$

$$\nu + 2\nu + 6\beta\nu = 3\beta$$

$$\nu = \frac{\beta}{1+2\beta}$$

$$\beta = \frac{\nu}{1-2\nu}$$

References for Material Data Input

ANSYS Online Help References:

1. ANSYS Elements Reference, Section 4.7
2. ANSYS Structural Analysis Guide, Section 8.4.1.3
3. ANSYS, Inc. Theory Reference, Section 4.6