How to Use the Incompressible ideal gas law for Natural Convection

Problem/Description:

In the analysis of natural convection of a gas, the incompressible ideal gas law is used because the Boussinesq model is improper to use for a large temperature change. This solution outlines how the incompressible ideal gas law operates and an example.

Solution:

When analyzing natural convection, the ideal gas models (incompressible and compressible) can be used for a gas. The others (e.g. Boussinesq, piecewise linear etc.) are used for liquids or gases. The ideal gas model (compressible) is applied to high-speed flow such as shock waves (natural convection is neglected) and the numerical solution is unstable. The formula is:

$$\rho = \frac{P_{op} + P}{\frac{R}{M_w} T}$$

where
R is the universal gas constant
M_w is the molecular weight of the gas
P_{op} is the reference pressure
P is the local gauge pressure

The incompressible ideal gas law applies to slow flows such as natural convection and the solution is stable. The formula is given by:

$$\rho = \frac{P_{op}}{\frac{R}{M_w} T}$$

where the variables are the same as that of the compressible ideal gas law.

For gaseous natural convection, the incompressible ideal gas law is recommended, but setting the reference pressure is important to correctly calculate the density. In relation to the Boussinesq model, the ideal gas model can be used for any temperature range and in conjunction with chemical transport and chemical reaction.

The volume force (driving force of the flow) is given by

$$(\rho - \rho_0) g$$

where ρ_0 is the reference density. To ensure numerical prediction accuracy, the reference density must be set. Generally, it is recommended that the user specify the density corresponding to the temperature of the system. If not, the density is automatically specified as the average density calculated for each iteration (numerically unstable).

To use the incompressible ideal gas law, it should be set for density in the Materials panel (Figure 1). The gravitational acceleration and reference density must also be set in the Operating conditions panel (Figure 2).

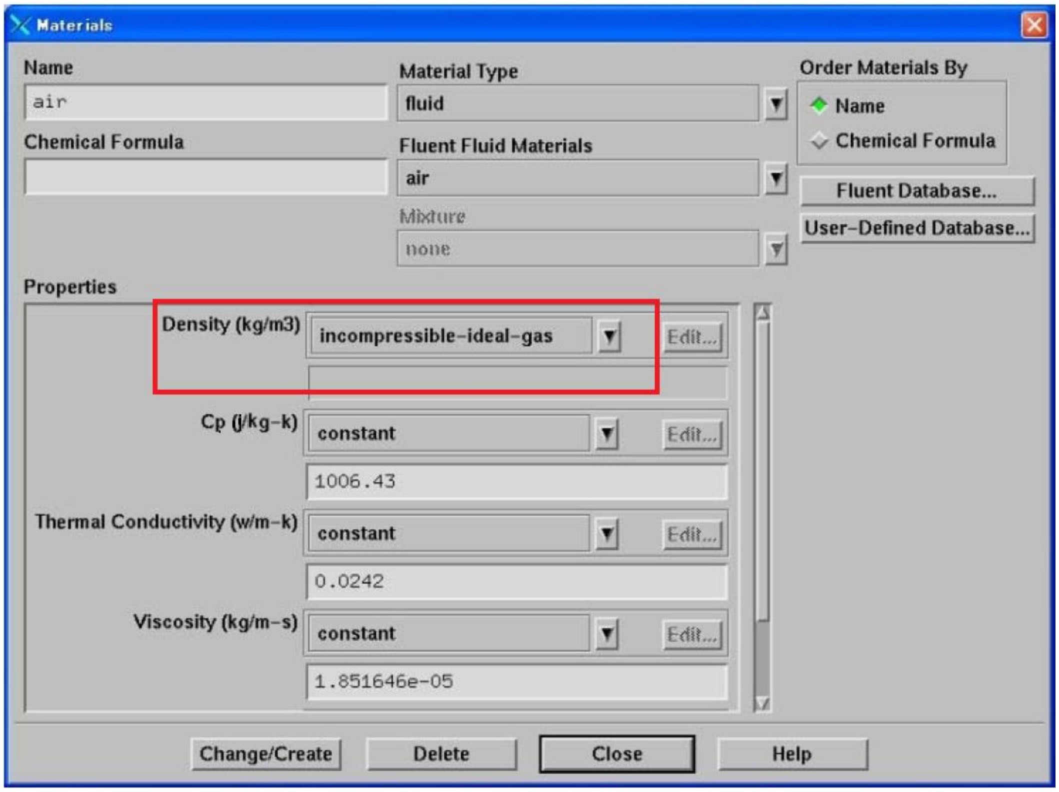

Figure 1: Materials panel with incompressible ideal gas selected for density

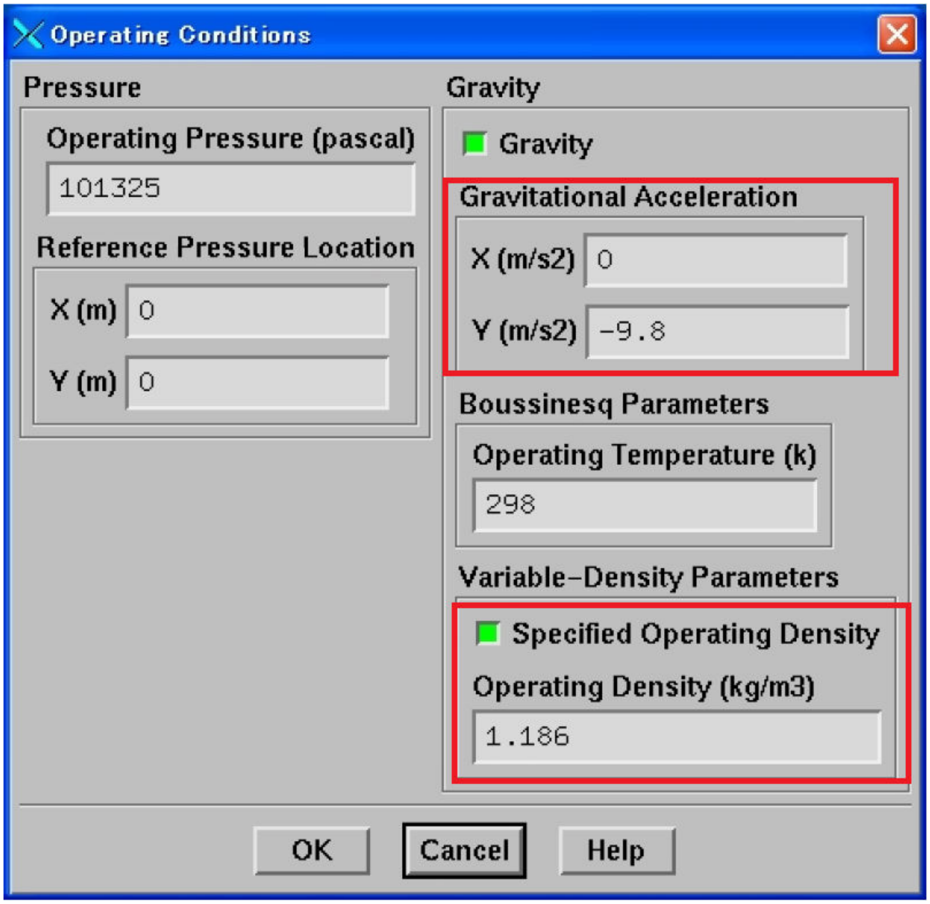

Figure 2: Operating conditions panel for incompressible ideal gas model

Consider the 2D, steady calculation of natural convection of air using the incompressible ideal gas model with reference pressure 101325 [Pa] and density 1.186 [kg/m^3]. The temperature and velocity distributions are shown in Figures 3 and 4 respectively.

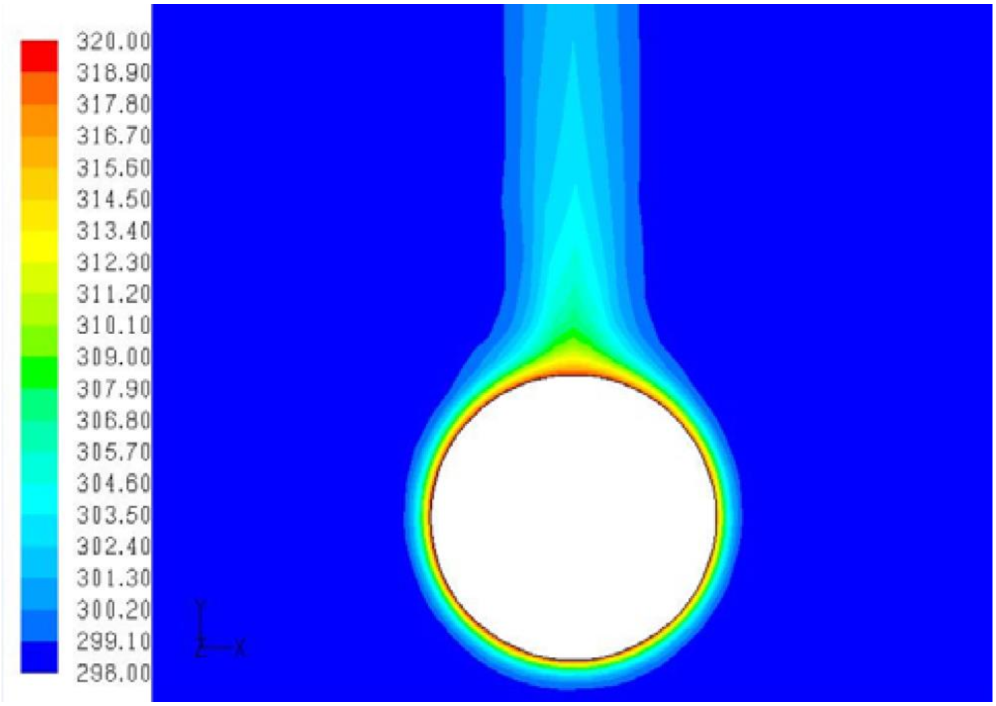

Figure 3: Temperature distribution using incompressible ideal gas law

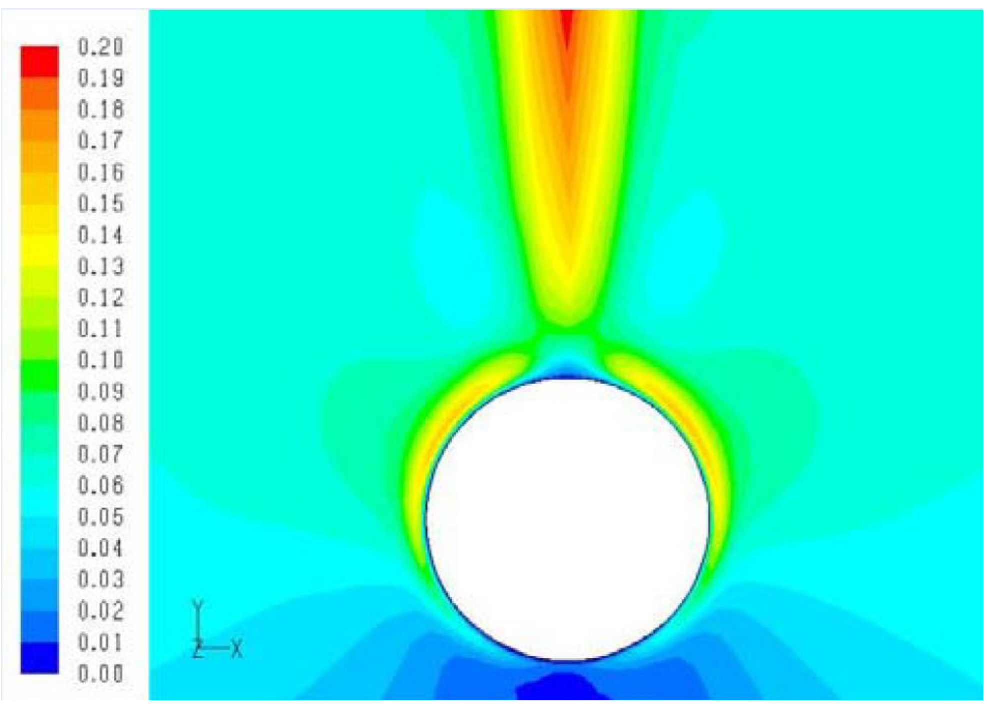

Figure 4: Velocity distribution using incompressible ideal gas law

The residual charts when the reference density is not specified is shown in Figure 5 and Figure 6 is when density is specified. This shows that specifying the reference density has a significant impact on convergence.

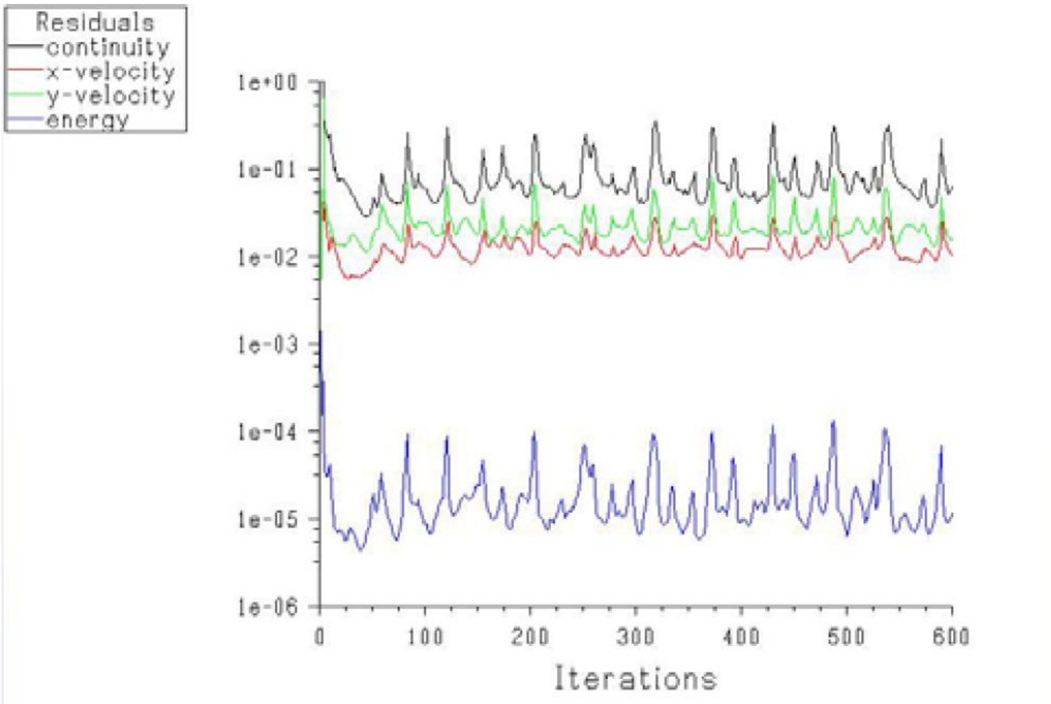

Figure 5: Residuals when density is not specified

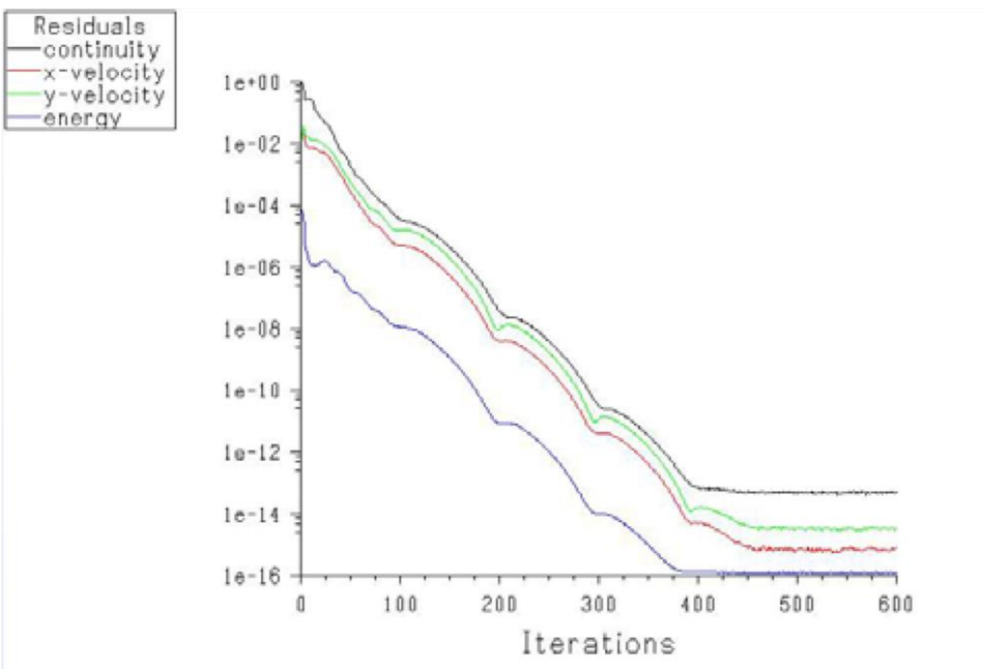

Figure 6: Residuals when density is specified