Constraint Equations

Lesson 2
Constraint Equations

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Kinematic Constraint notation

$$\mathbf{\Phi}^{\mathrm{K}}(\mathbf{q})$$

Driving Constraint or motion in Ansys motion

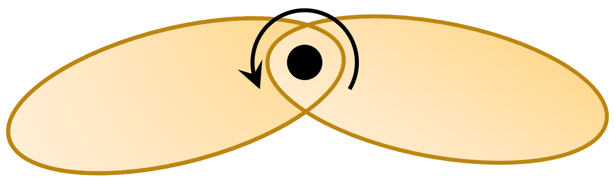
$$\mathbf{\Phi}^{\mathrm{D}}(\mathbf{q}, t) = x - \sin(t) = 0$$

Combined Constraint equations

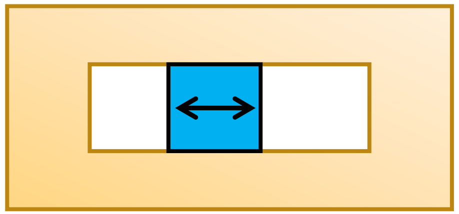
$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}^{\mathrm{K}}(\mathbf{q}) \\ \mathbf{\Phi}^{\mathrm{D}}(\mathbf{q}, t) \end{bmatrix} = \mathbf{0}$$

Joint Library

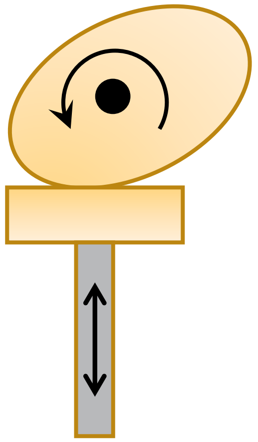
1. Revolute joint



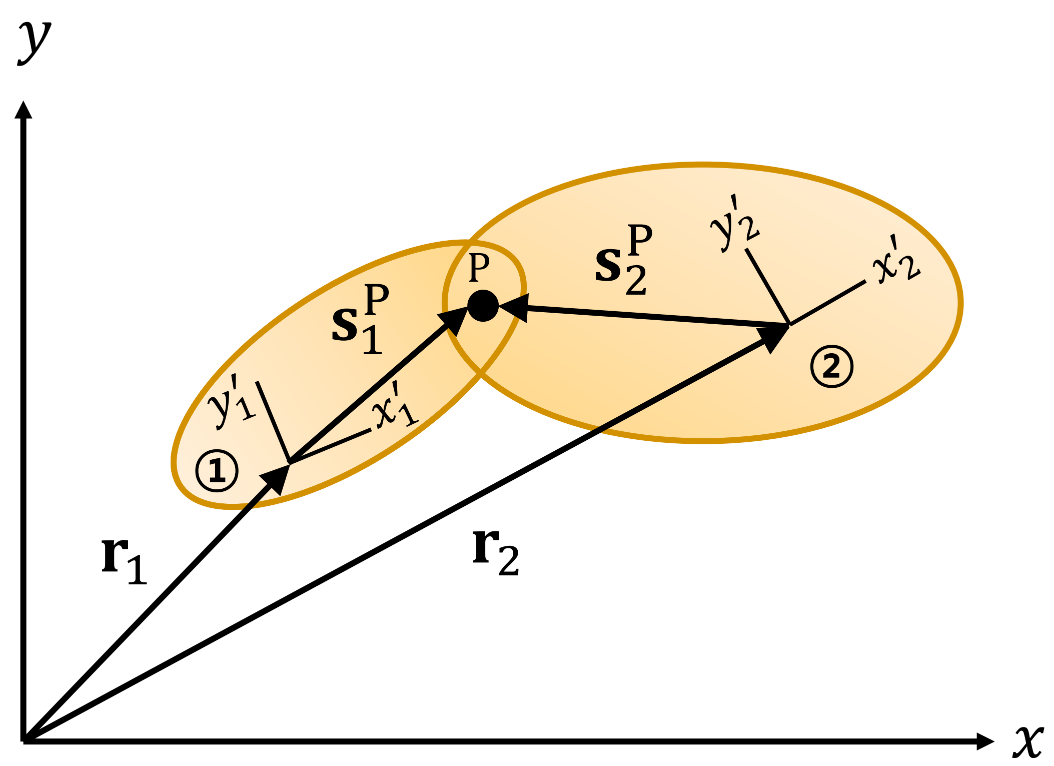
2. Translate joint



3. Cam



Revolute Joint: Position Level Constraint



$$\mathbf{\Phi}^{r(1,2)} = \mathbf{r}_1 + \mathbf{s}_1^{\mathrm{P}} - \mathbf{r}_2 - \mathbf{s}_2^{\mathrm{P}}$$

$$= \mathbf{r}_1 + \mathbf{A}_1\mathbf{s}'^{\mathrm{P}}_1 - \mathbf{r}_2 - \mathbf{A}_2\mathbf{s}'^{\mathrm{P}}_2 = \mathbf{0}$$

Revolute Joint: Velocity Level Constraint

$$\mathbf{\Phi} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \mathbf{A}(\theta_1)\mathbf{s}_1' - \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} - \mathbf{A}(\theta_2)\mathbf{s}_2' = \mathbf{0}$$

Differentiating the constraint

$$\dot{\mathbf{\Phi}} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} + \mathbf{B}(\theta_1)\mathbf{s}_1'\dot{\theta}_1 - \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} - \mathbf{B}(\theta_2)\mathbf{s}_2'\dot{\theta} = 0$$

$$= \begin{bmatrix} 1 & 0 & -\sin\theta_1\xi_1 - \cos\theta_1\eta_1 & -1 & 0 & \sin\theta_2\xi_2 + \cos\theta_2\eta_2 \\ 0 & 1 & \cos\theta_1\xi_1 - \sin\theta_1\eta_1 & 0 & -1 & -\cos\theta_2\xi_2 + \sin\theta_2\eta_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{\theta}_2 \end{bmatrix} = \mathbf{0}$$

Jacobian Matrix → **J**

$$\frac{\partial \mathbf{A}}{\partial \theta} = \mathbf{B} = \begin{bmatrix} -\sin\theta & -\cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$$

Revolute Joint: Acceleration Level Constraint

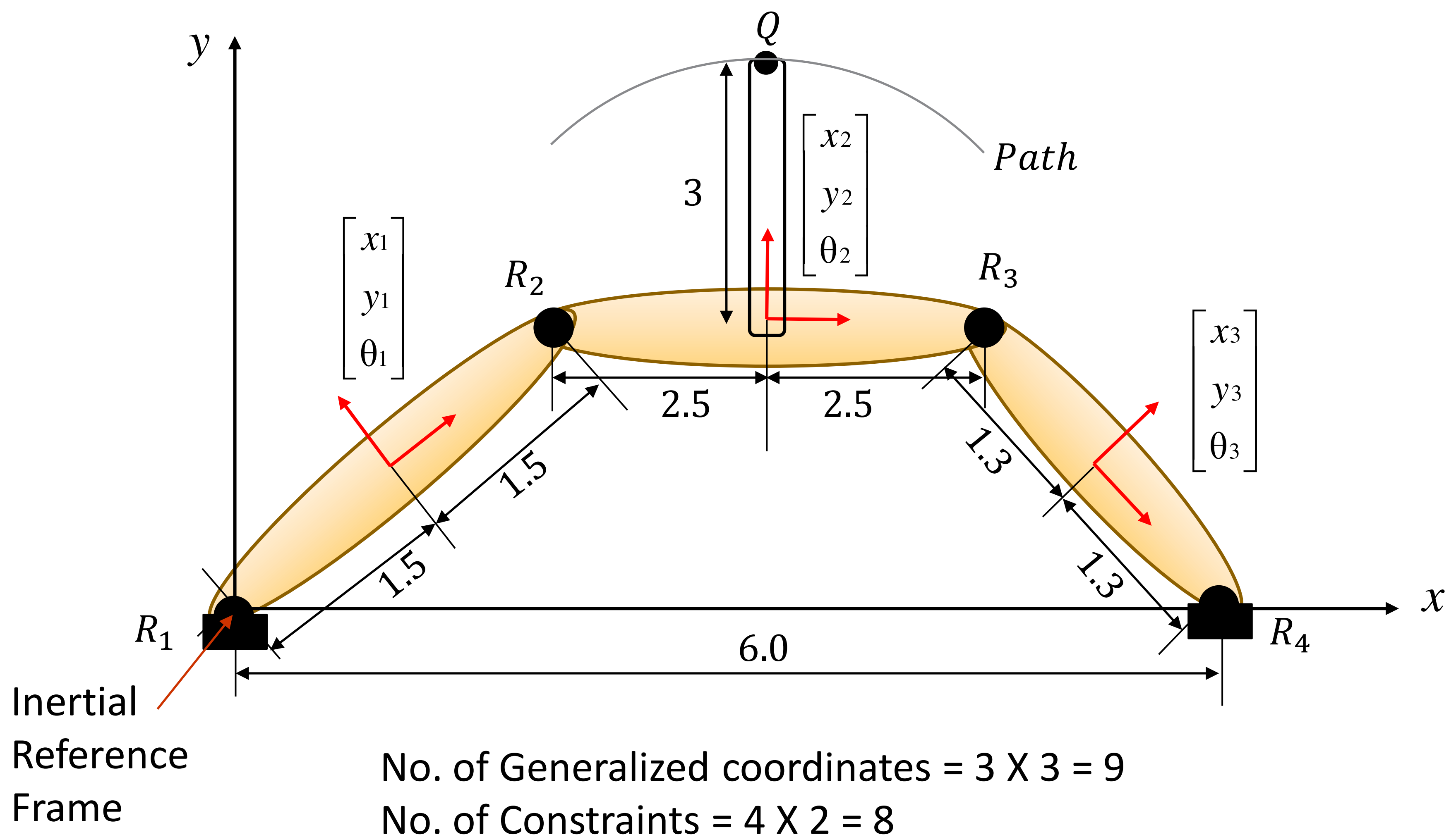
Taking the second time derivative yields

$$\ddot{\boldsymbol{\Phi}} = \mathbf{J}\ddot{\mathbf{q}} + \begin{bmatrix} 0 & -\mathbf{A}(\theta_1)\mathbf{s}_1'\dot{\theta}_1 & 0 & \mathbf{A}(\theta_2)\mathbf{s}_2'\dot{\theta}_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\theta}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{\theta}_2 \end{bmatrix} = \mathbf{0}$$

$\boldsymbol{\gamma}$ is defined as

$$\mathbf{J}\ddot{\mathbf{q}} = \mathbf{A}(\theta_1)\mathrm{s}_1'\dot{\theta}_1^{\,2} - \mathbf{A}(\theta_2)\mathrm{s}_2'\dot{\theta}_2^{\,2} \equiv \boldsymbol{\gamma}$$

A Four Bar Mechanism



No. of Generalized coordinates = 3 X 3 = 9

No. of Constraints = 4 X 2 = 8

D. O. F = 9 - 8 = 1

Constraints for the Four Bar Mechanism

Constraints

$$R_1: \mathbf{\Phi}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \mathbf{A}(\theta_1)\begin{bmatrix} -1.5 \\ 0 \end{bmatrix} = \mathbf{0}$$

$$R_2: \mathbf{\Phi}_2 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \mathbf{A}(\theta_1)\begin{bmatrix} 1.5 \\ 0 \end{bmatrix} - \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} - \mathbf{A}(\theta_2)\begin{bmatrix} -2.5 \\ 0 \end{bmatrix} = \mathbf{0}$$

$$R_3: \mathbf{\Phi}_3 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \mathbf{A}(\theta_2)\begin{bmatrix} 2.5 \\ 0 \end{bmatrix} - \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} - \mathbf{A}(\theta_3)\begin{bmatrix} -1.3 \\ 0 \end{bmatrix} = \mathbf{0}$$

$$R_4: \mathbf{\Phi}_4 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} + \mathbf{A}(\theta_3)\begin{bmatrix} 1.3 \\ 0 \end{bmatrix} - \begin{bmatrix} 6.0 \\ 0 \end{bmatrix} = \mathbf{0}$$

※ Driving Constraint (Motion)

$$\mathbf{\Phi}^D = \theta_1 - (t + \frac{\pi}{3}) = 0 \quad \text{, t time} \longrightarrow \quad \text{Total 9 constraints}$$

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