Frequency Characteristics of The Half-Wave Dipole

Input Impedance

Developed by Kathryn L. Smith, PhD

Ansys

Sources

The material presented herein is from the following source:

"Antenna Theory," by Constantine A. Balanis, 3rd ed. (2005)

Input Impedance Over Frequency

Formally, the input impedance of a sinusoidally-driven dipole antenna is given by $Z_{in} = R_{in} + jX_{in}$, where the input resistance R_{in} may be calculated from the radiation resistance R_r and the electrical length kl of the antenna using the equation:

$$R_{in} = \frac{R_r}{\left(\sin\left({}^{kl}/_{2}\right)\right)^2}$$

where $k = 2\pi/\lambda$ is the wavenumber and l is the length of the dipole in meters. The radiation resistance may be calculated from:

$$R_r = \frac{\eta}{2\pi}\left\{C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)\left[S_i(2kl) - 2S_i(kl)\right] + \frac{1}{2}\cos(kl)\left[C + \ln\left({}^{kl}/_{2}\right) + C_i(2kl) - 2C_i(kl)\right]\right\}$$

where $C = 0.5772$ is Euler's constant, η is the intrinsic impedance of the medium into which the antenna is radiating (for free space, 377Ω), and $S_i(x)$ and $C_i(x)$ are the sine and cosine integrals, respectively. These are defined by:

$$S_i(x) = \int_0^x \frac{\sin y}{y}\,dy\,, \qquad C_i(x) = \int_\infty^x \frac{\cos y}{y}\,dy$$

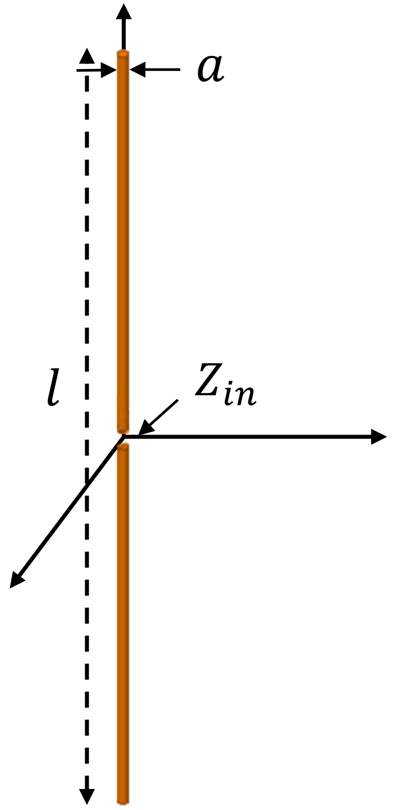
Input Impedance Over Frequency

The imaginary part of the input impedance, X_{in}, may also be calculated, as:

$$X_{in} = \frac{\eta}{4\pi}\left\{2S_i(kl) + \cos(kl)\left[2S_i(kl) - S_i(2kl)\right] - \sin(kl)\left[2C_i(kl) - C_i(2kl) - C_i\left(\frac{2ka^2}{l}\right)\right]\right\}$$

where a is the radius of the dipole, in meters.

Note that all of these equations depend on electrical length, kl, rather than directly on the physical length of the antenna, l. This means that the input impedance of the antenna depends on the relationship between the dipole length and the driving frequency, rather than on either of those factors taken individually.

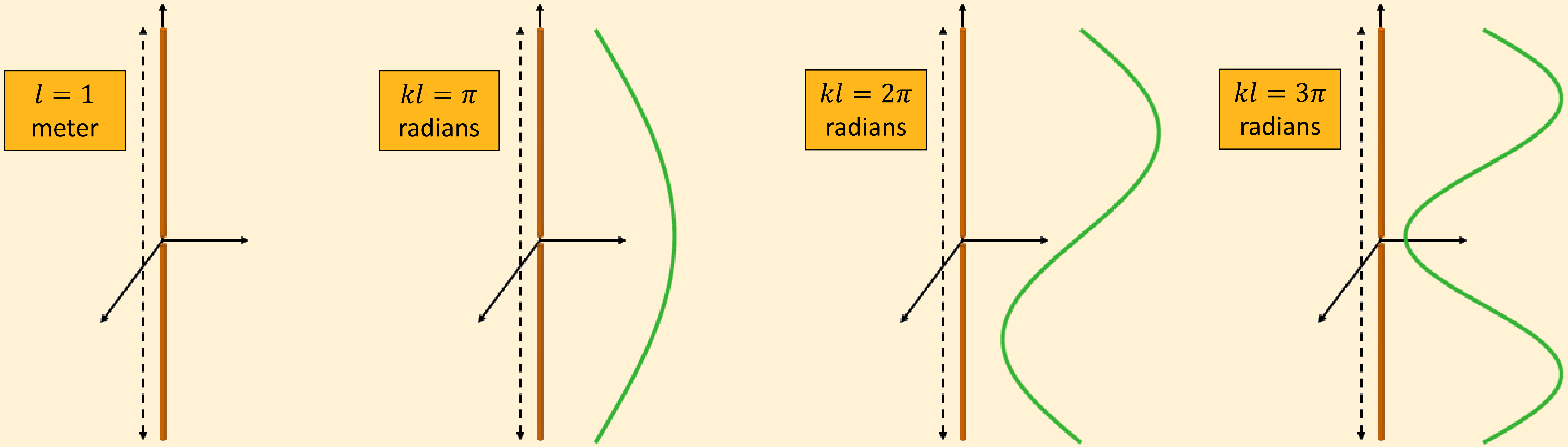


Input Impedance Over Frequency

Electrical Length is the length of the antenna, expressed in wavelengths or radians of the driven signal.

Example: This dipole antenna has a physical length of 1 meter. This *does not* depend on the driving frequency. However, its electrical length is calculated as kl, or $\frac{2\pi l}{\lambda}$, which is the length of the antenna in radians, and which *does* depend on the driving frequency.

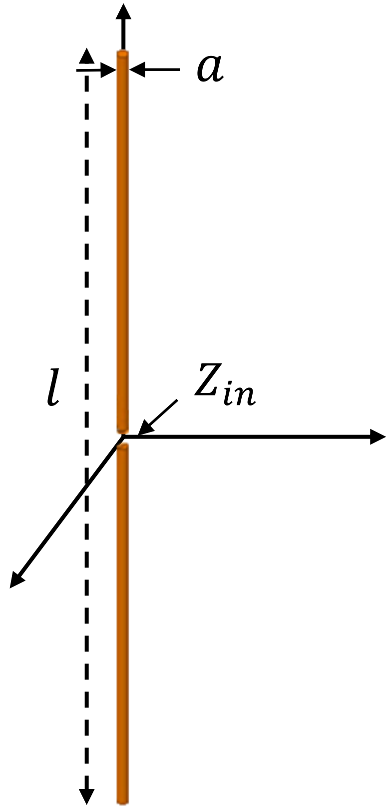
Physical Length	Electrical Length at $f = 150MHz$	Electrical Length at $f = 300MHz$	Electrical Length at $f = 450MHz$



Input Impedance Over Frequency

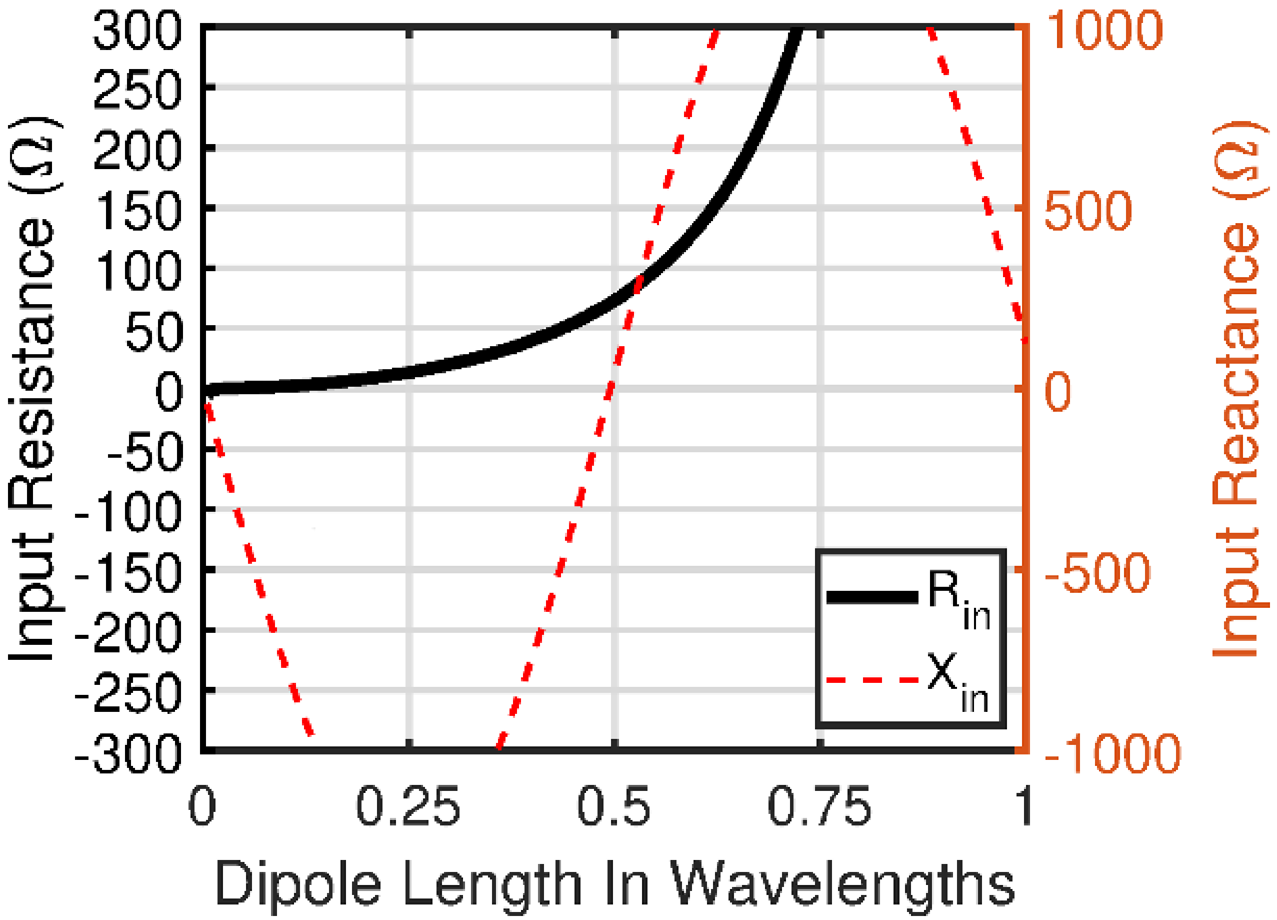
You can easily write a program in your calculator, or in Matlab, which can calculate input impedance for a dipole of arbitrary electrical length and radius. This would be a good thing to do, to help you develop your understanding of the operation of linear antennas.

For the purposes of this class, we will simply highlight a few important features.



Input Impedance Over Frequency

The graph below shows the input impedance R_{in} and the input reactance X_{in} for a thin ($a \ll \lambda$) dipole, vs the length of that dipole measured in wavelengths.



Note that the x-axis corresponds to increasing frequency, from left to right.

When the antenna is a half-wave dipole (at 0.5λ on the x-axis), it has an input impedance of

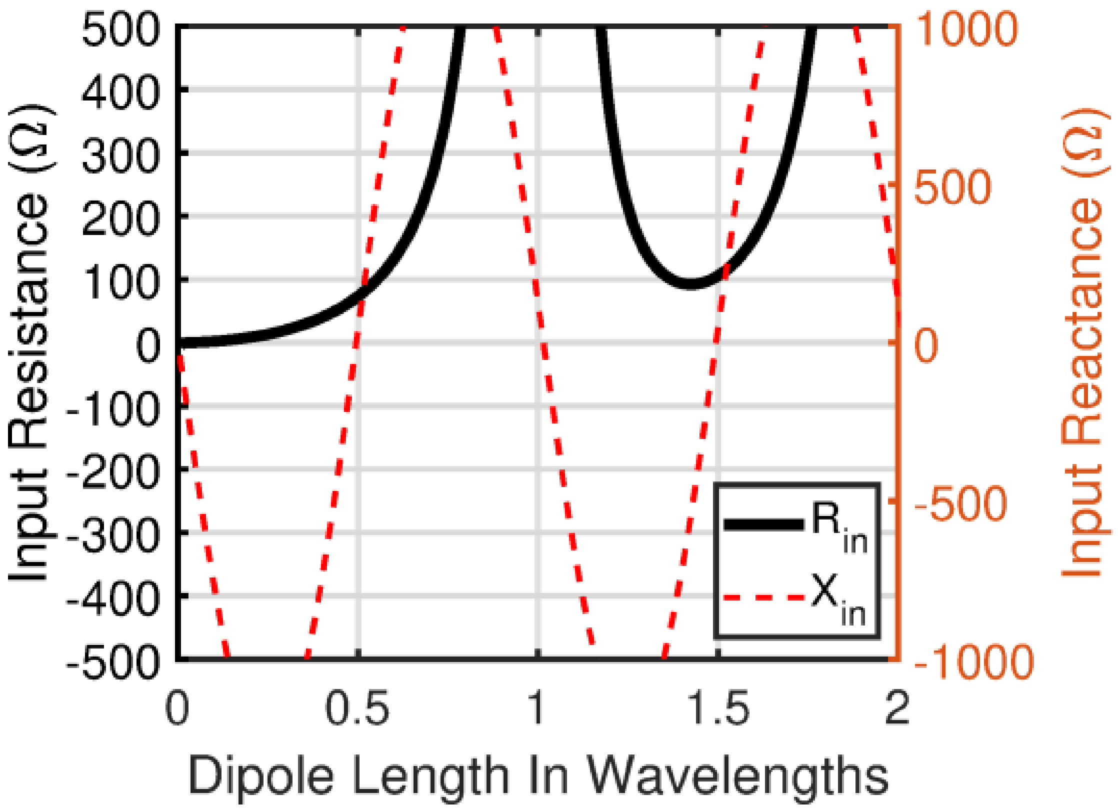
$$Z_{in,0.5\lambda} \approx 73 + j42.5\Omega$$

while its actual resonance $(X_{in} = 0)$ occurs where the antenna is approximately 0.495λ long.

The antenna is *capacitive* at frequencies below this first resonance, and *inductive* at frequencies slightly above.

Input Impedance Over Frequency

This graph is the same as the one on the previous slide, but shows a wider range of frequencies.



The antenna has a second resonance (its first *harmonic*) at the frequency where its length is approximately equal to λ (at 1λ on the x-axis). However, at this frequency the input resistance is very high.

The third resonance (second harmonic) occurs at $l \approx 1.5\lambda$, and here the input impedance is approximately 104Ω.

The antenna is *capacitive* at frequencies between the first and second resonance, *inductive* at frequencies between the second and third resonance, *capacitive* at frequencies between the third and fourth resonance, and so on.

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