Basics of S-parameters

What are S-parameters?

Developed by Kathryn L. Smith, PhD

Ansys

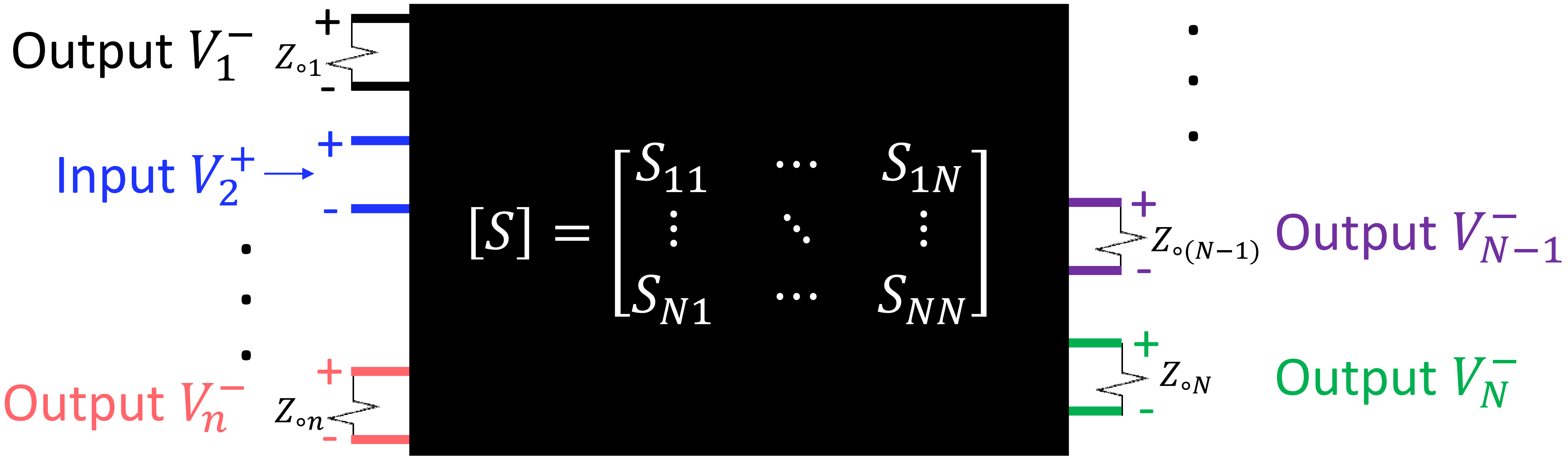
Sources

The material presented herein is from the following sources:

"Engineering Electromagnetics," by Nathan Ida, 3rd ed. (2015)
"Antenna Theory," by Constantine A. Balanis, 4th ed. (2016)
HFSS 2019 Help Files
"Elements of Electromagnetics," by Matthew N.O Sadiku, 5th ed. (2010)
"Microwave Engineering," by David Pozar, 4th ed. (2012)

What are S-Parameters?

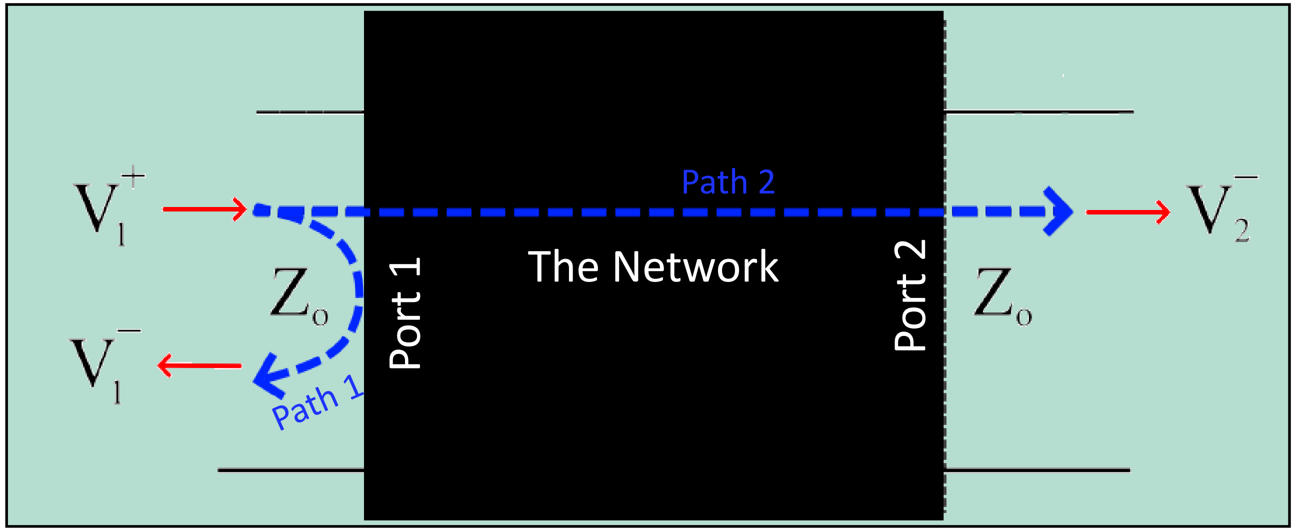
[S]-parameters, or “scattering parameters,” characterize a network in terms of its **reflected and transmitted voltages** resulting from known **input voltages**.



What are S-Parameters?

For a two-port network with a known voltage wave input at port 1, there are two possible paths the signal may take.

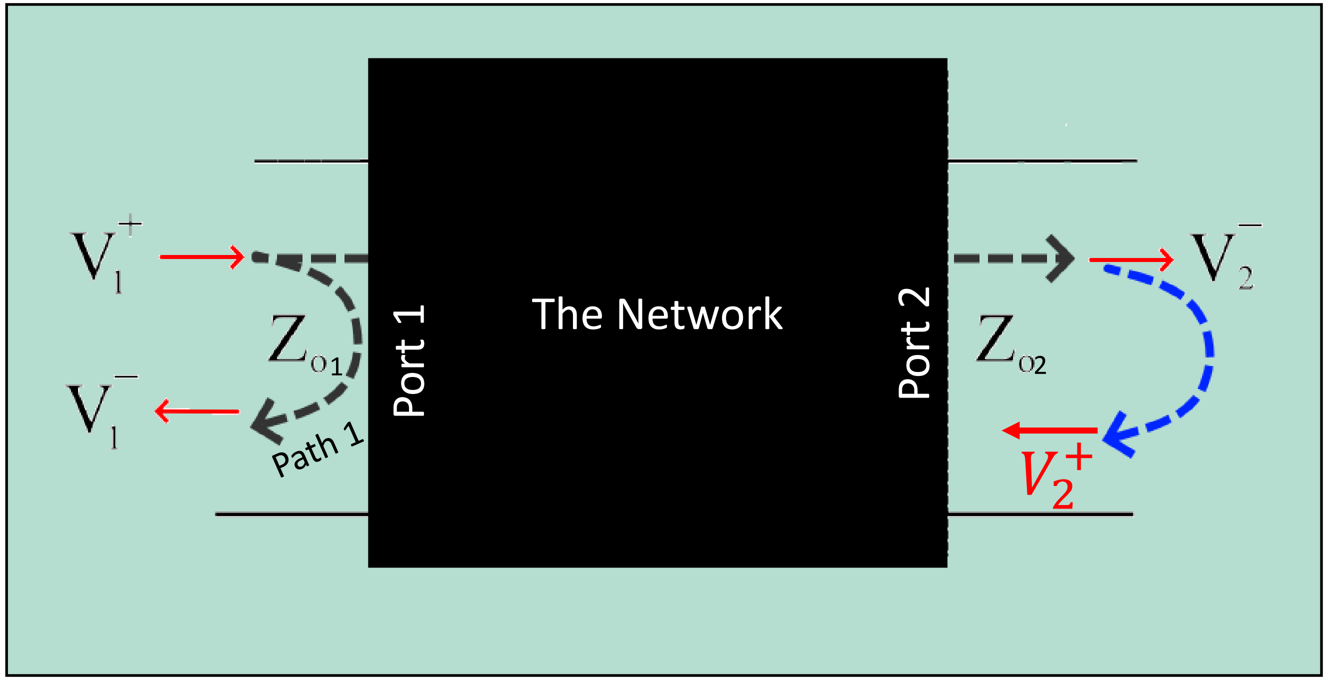
1) It may reflect at the input to the system. This happens when there is a mismatch between the characteristic impedance of the input port (Z_{o1}) and the input impedance to the network.
2) It may transmit through the network and exit at port 2.



Each of these corresponds to a single element of the S-matrix. The elements are denoted using two subscripts, the first of which indicates the port at which the signal **exited** the system, and the second of which indicates the port at which the signal **entered** the system. Thus, the signal that enters at port 1 and immediately exits at port 1 (path 1 above) corresponds to S_{11}, while the signal that enters at port 1 and exits at port 2 (path 2 above) corresponds to S_{21}.

What are S-Parameters?

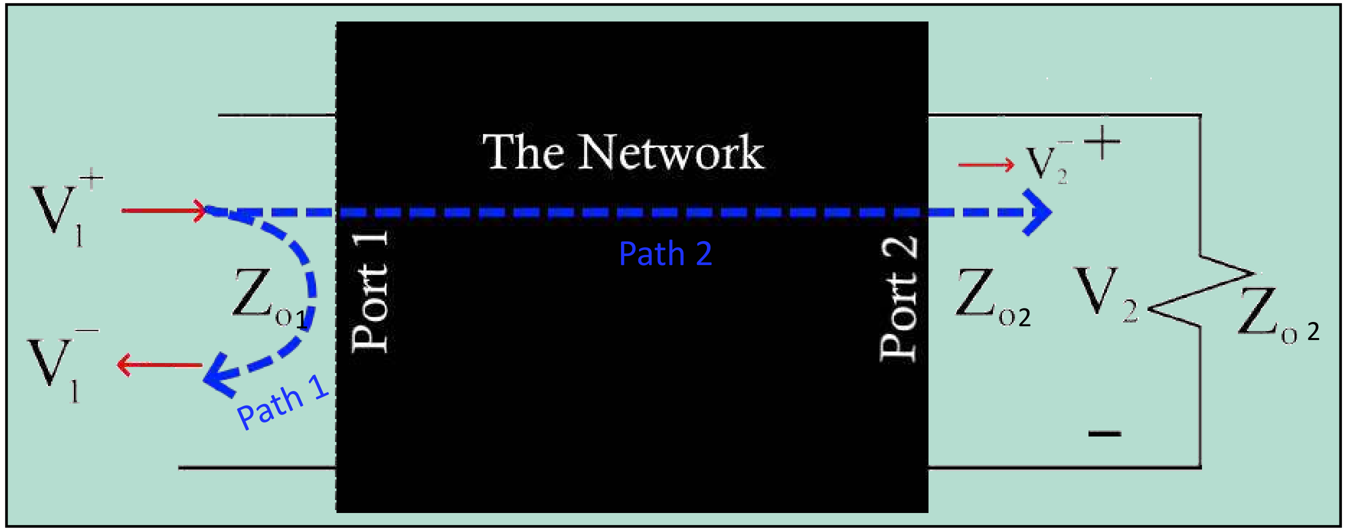
Note that it is possible for the signal exiting at port 2 to reflect off something external to the network and **re-enter** the system, as a second input (V_2^+). This will also experience its own reflections and transmissions.



Since a voltage measurement only tells you the total voltage present at the node, and cannot determine where the signal came from, this external reflection would make it impossible to accurately measure the effect of the input at port 1. Therefore, **in order to measure S-parameters, all ports other than the input port must be match-terminated, or loaded with their own characteristic impedance.** This prevents external reflections, and allows accurate measurement of the voltages exiting the system.

What are S-Parameters?

With the second port match-terminated, and a known input V_1^+, all other voltages in this diagram may be determined through measurement.



The total voltage measured at port 1 is the sum of the input signal V_1^+ and the reflected signal V_1^-. So,

$$V_1 = V_1^+ + V_1^- \quad \text{but} \quad V_1^- = \Gamma_{in1} V_1^+ \quad \text{so} \quad V_1 = V_1^+ (1 + \Gamma_{in1})$$

where V_1 is the total measured (or applied) port voltage at port 1 and Γ_{in1} is the reflection coefficient looking into port 1.

Since port 2 is loaded with its own characteristic impedance, nothing will reflect back into the network as V_2^+, and the entire port voltage observed at port 2 (V_2) is due to the portion of V_1^+ that was transmitted. So,

$$V_2 = V_2^+$$

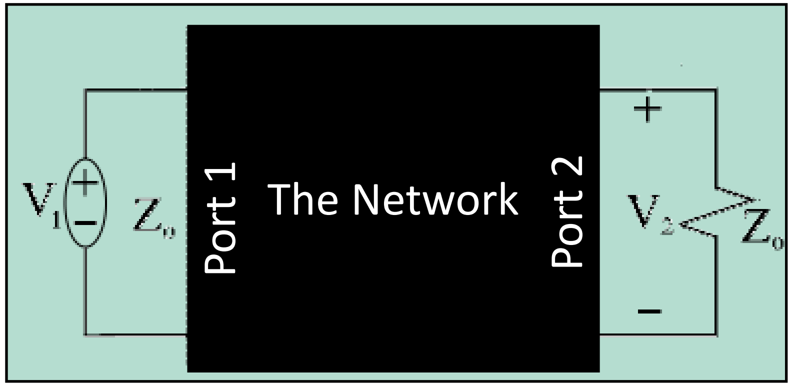
This method can be repeated with input at port 2 to obtain the other two elements of the S-matrix.

What are S-Parameters?

$$S_{ij} = \frac{\sqrt{Z_{\circ j}}}{\sqrt{Z_{\circ i}}}\frac{V_i^-}{V_j^+}\Bigg|_{V_k^+=0,k\neq j}$$

For our example two-port network, the two required measurements are as follows:

Measurement 1:

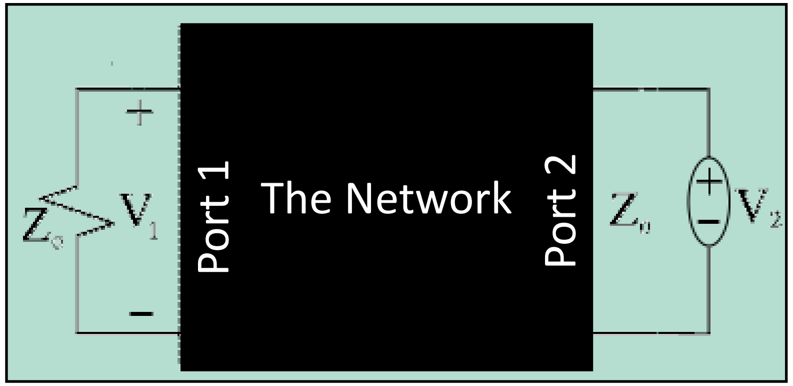


This measurement consists of a test voltage input at port 1 with port 2 short-circuited. Output currents are measured at port 1 and port 2. This will give us two parameters of the network:

$$S_{11} = \frac{V_1^-}{V_1^+} \quad \text{and} \quad S_{21} = \frac{\sqrt{Z_{\circ 1}}}{\sqrt{Z_{\circ 2}}}\frac{V_2^-}{V_1^+}$$

where $V_2 = V_2^-$ and $V_1 = V_1^+(1+\Gamma_{in1})$, where Γ_{in1} is the reflection coefficient looking into the network from port 1.

Measurement 2



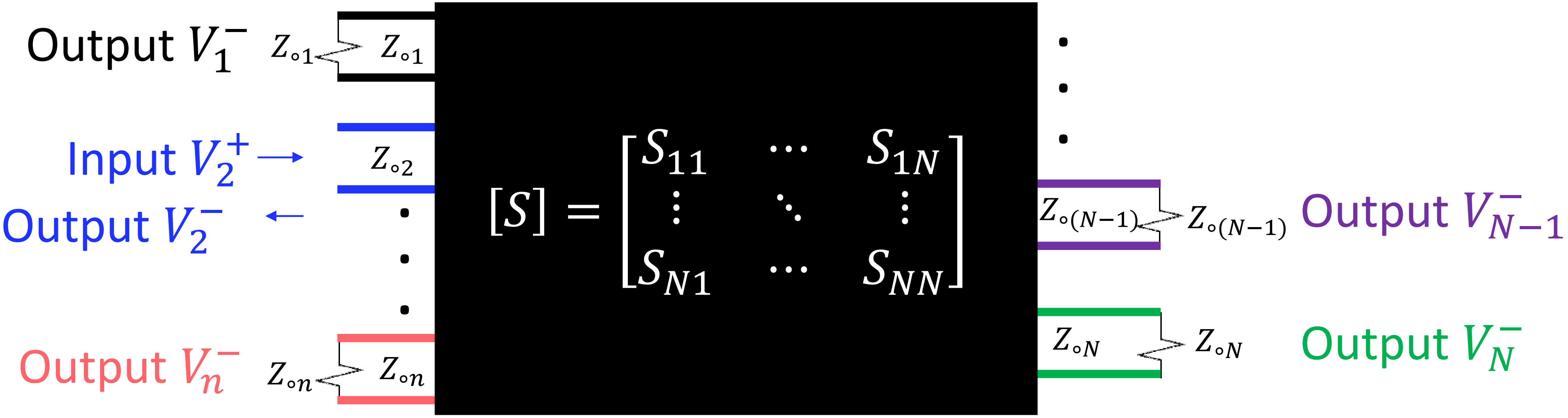
This measurement consists of a test voltage input at port 2 with port 1 short-circuited. Output currents are measured at port 1 and port 2. This will give us the other two network parameters:

$$S_{12} = \frac{\sqrt{Z_{\circ 2}}}{\sqrt{Z_{\circ 1}}}\frac{V_1^-}{V_2^+} \quad \text{and} \quad S_{22} = \frac{V_2^-}{V_2^+}$$

where $V_1 = V_1^-$ and $V_2 = V_2^+(1+\Gamma_{in2})$, where Γ_{in2} is the reflection coefficient looking into the network from port 2.

What are S-Parameters?

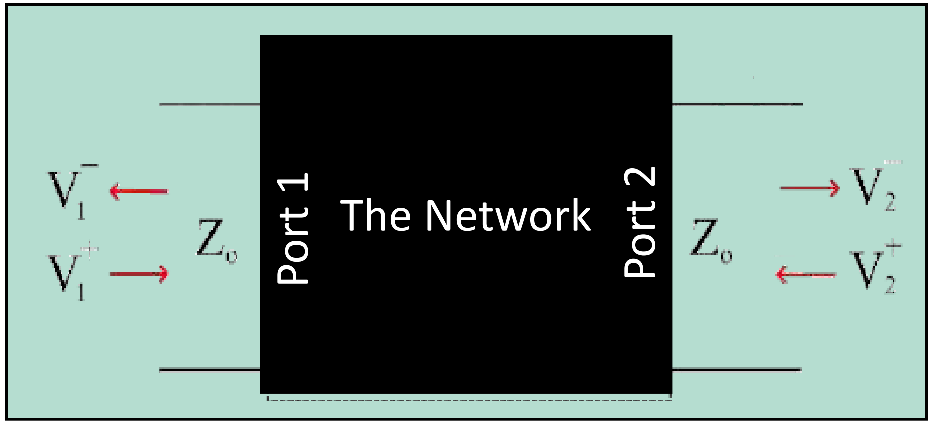
This method may be extended for an N-port network. Characterization of an N-port network requires N measurements, each involving input at a single port, and match-termination of all other ports.



What are S-Parameters?

For an N-port network, each element of the [S]-matrix may be found using the following formula:

$$S_{ij} = \frac{\sqrt{Z_{\circ j}}}{\sqrt{Z_{\circ i}}} \frac{V_i^-}{V_j^+}\bigg|_{V_k^+=0, k\neq j}$$



An example 2-port network

Note that the voltages with the '+' superscript are defined **entering** the system, and the voltages with the '-' superscript are defined **exiting** the system.

Note that, if port i is the voltage-input port and port j is a voltage-output port, the criteria $V_k^+ = 0, k \neq j$ is met by match-terminating all non-driven ports.

Ansys