Basics of Antenna Arrays

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Ansys

Sources

The material presented herein is from the following sources:

"Engineering Electromagnetics," by Nathan Ida, 3rd ed. (2015)
"Antenna Theory," by Constantine A. Balanis, 4th ed. (2016)

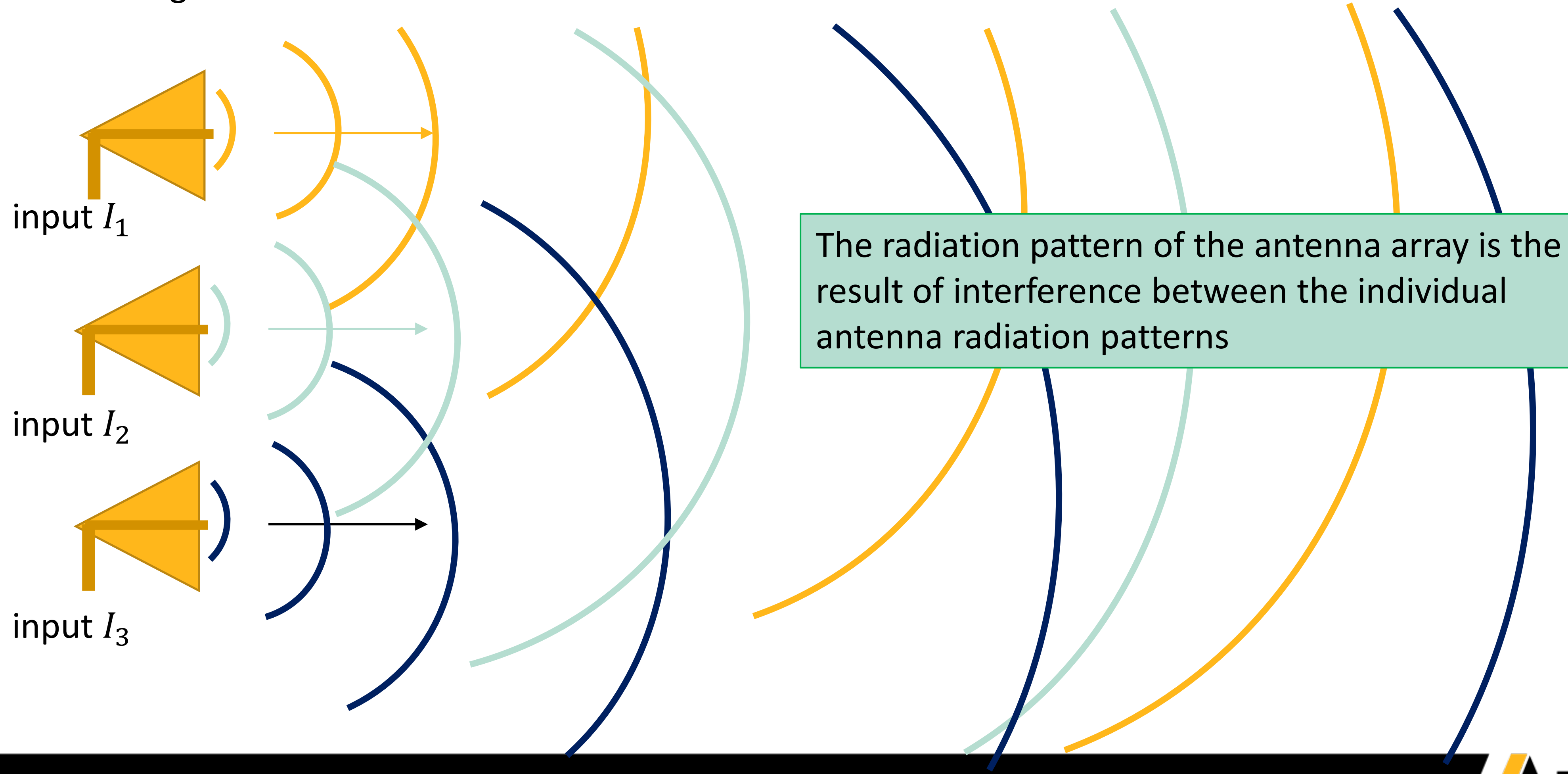
Agenda

When two or more radiating elements are operated in close proximity to one another, the interference of their radiated fields can become a major factor in determining their collective radiation pattern. Antenna engineers leverage this fact to their advantage by creating *antenna arrays* – collections of proximate antennas that are designed to operate cohesively to produce a desired pattern of radiation.

In this module we will focus on *uniform* arrays – arrays consisting of a collection of identical elements, all driven with the same signal amplitude, having equal spacing between neighboring elements and a progressively-stepped phase shift along the length of the array. Our focus will be primarily on linear arrays – where the elements are arranged along a single dimension – but we will also briefly look at planar arrays, which consist of a 2D grid of radiating elements.

What is an Antenna Array?

An antenna array is a collection of antennas which are operated in close proximity to one another, and designed to work in tandem. It leverages the interference pattern of the fields radiating from the various elements to achieve a desired radiation pattern. Antenna arrays are often used to enable tighter directivity than could be achieved with a single-element design.

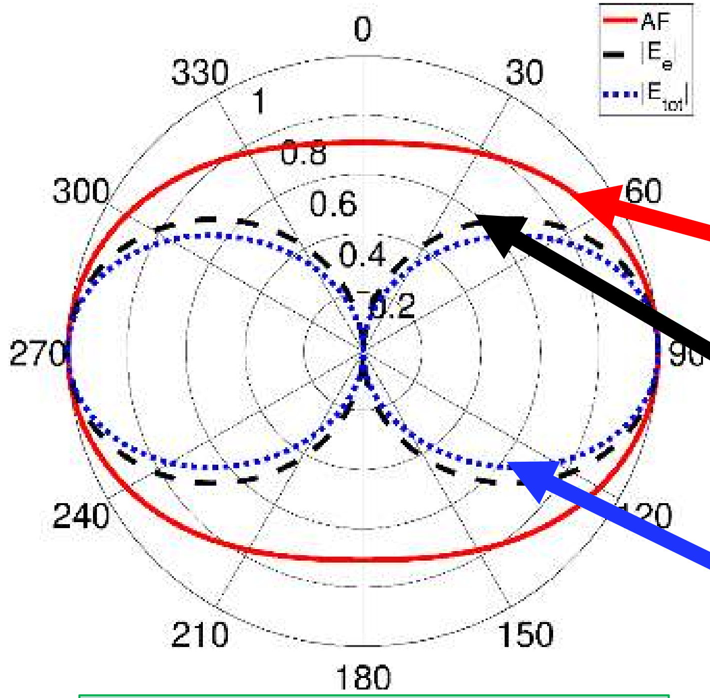


The radiation pattern of the antenna array is the result of interference between the individual antenna radiation patterns

The Array Factor

Antenna arrays consisting of a collection of **identical** elements are usually characterized in terms of their “Array Factor.” The Array Factor of any given array of identical antenna elements is a product of the number of elements, the relative input signals (magnitude and phase) of the various elements, and the geometric arrangement of the elements.

The total electric field, E_{tot}, radiated from an antenna array is equal to the product of the array factor, AF, and the electric radiation pattern of a single antenna element, located at the origin, E_e.



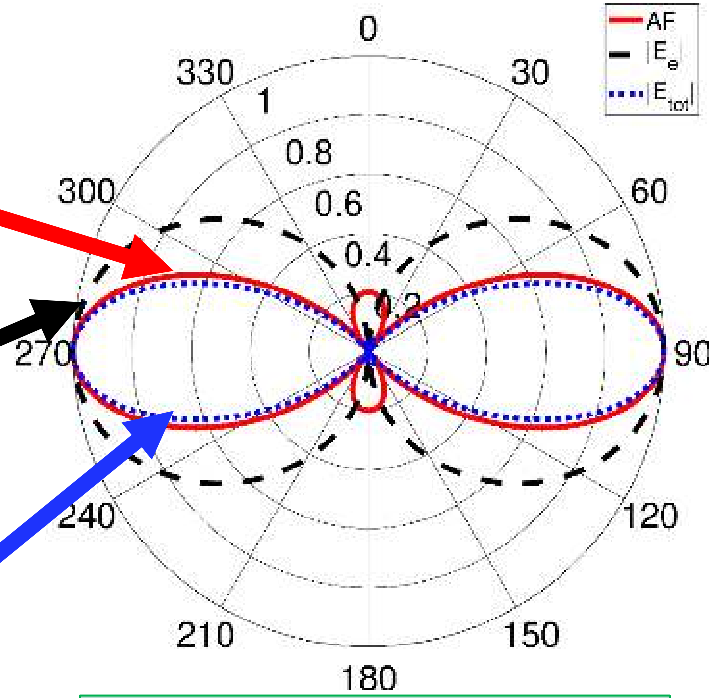
Example 1: N = 2

$$E_{tot} = E_e * AF$$

Array factor of an N element uniform linear array with 0.25λ spacing between elements and $\psi = 0$ (normalized)

Electric field pattern of a single dipole antenna (normalized)
Note: identical in both cases

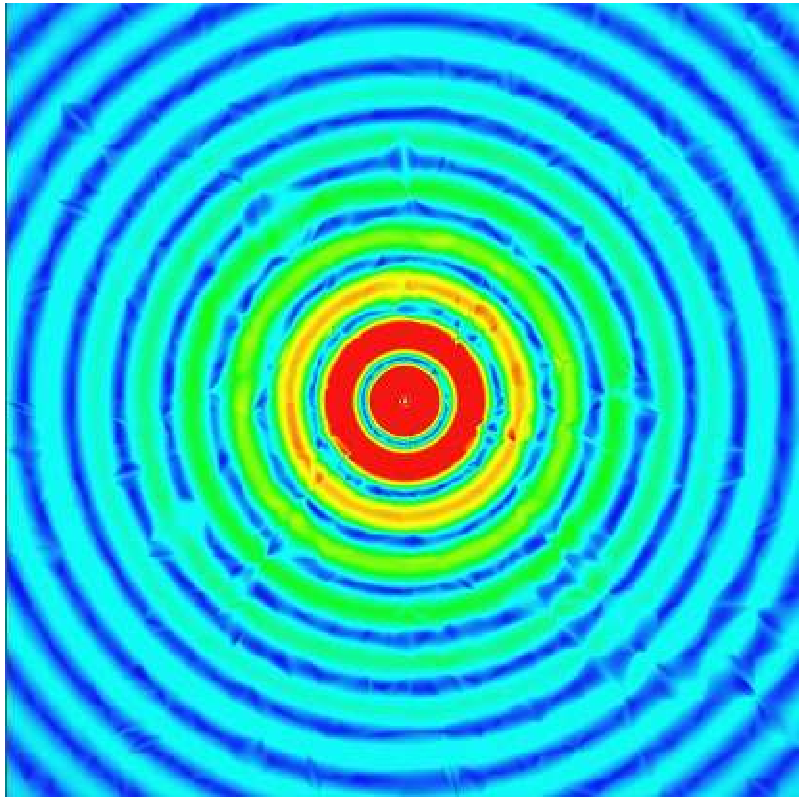
Electric field pattern of an array of N dipole antenna elements (normalized)



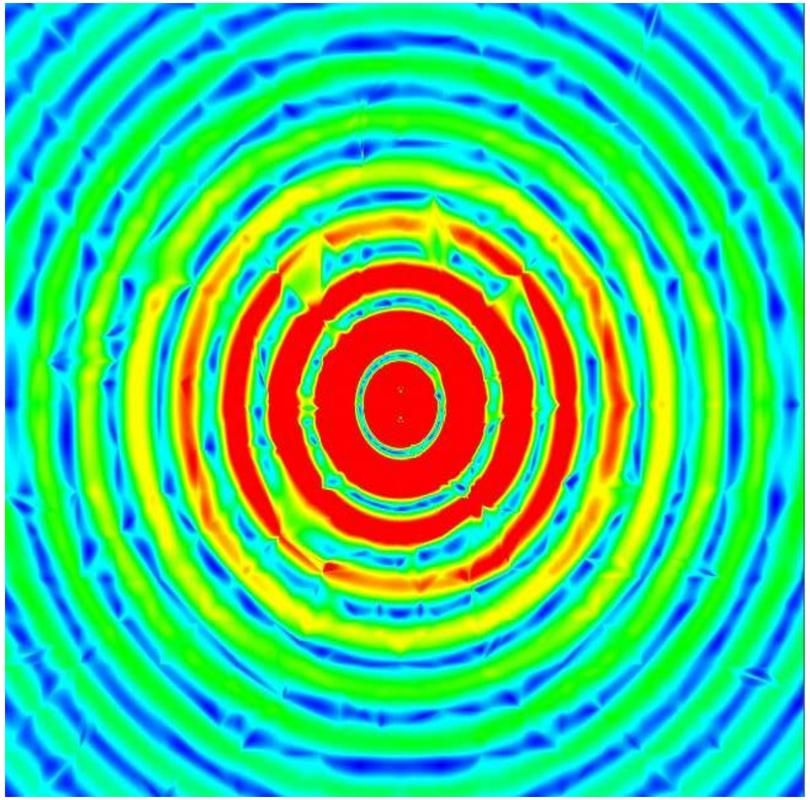
Example 2: N = 5

Interference Pattern Of Proximate Isotropic Radiators

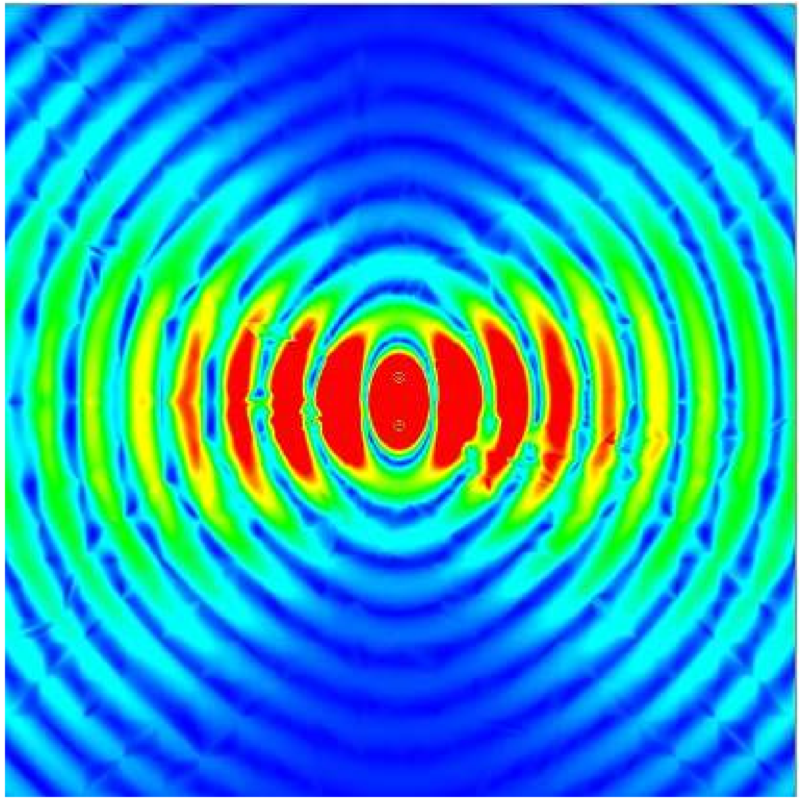
The array factor is determined by the interference pattern of its constituent antenna elements. To examine the array factor in isolation, let us consider isotropically radiating point sources (sources that radiate equally in all directions). In this case the total radiated electric field is exactly the same as the array factor ($|E_e| = 1$ in all directions).



A Single Isotropic Radiator



Array of Two Isotropic Radiators, vertical separation = $\lambda/4$



Array of Two Isotropic Radiators, vertical separation = $\lambda/2$

Uniform Antenna Arrays

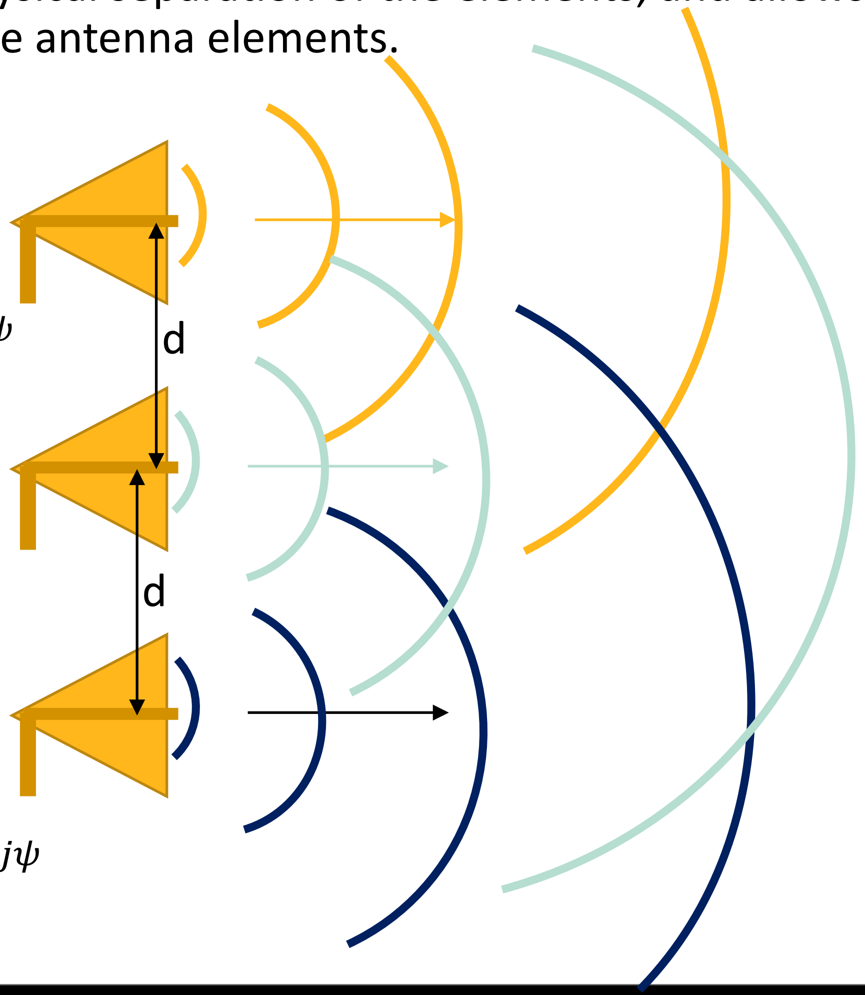
A uniform antenna array is an array of **identical** elements, all driven with the **same signal amplitude**, having **equal spacing** between neighboring elements and a **progressively-stepped phase shift** along the length of the array. This electronic phase shift augments the phase shift effected by the physical separation of the elements, and allows dynamic adjustment of the array factor without physical rearrangement of the antenna elements.

In a uniform array, each antenna is driven with the same *magnitude* signal, but with a progressively-stepped *phase shift*.

input $I_1 = I_\circ(\omega t)e^{j\psi}$

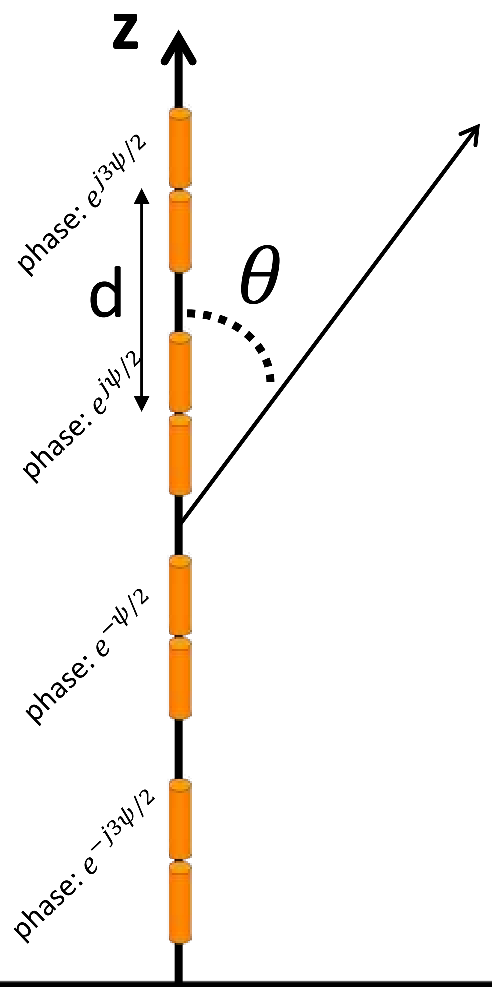
input $I_2 = I_\circ(\omega t)$

input $I_3 = I_\circ(\omega t)e^{-j\psi}$



Array Factor of Uniform Antenna Arrays

The array factor of an N-element uniform antenna array, arranged in a line along the z-axis, centered at the origin, and viewed from the far field, is given by:



$$AF = \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\psi)}$$

where k is the wavenumber, equal to $2\pi/\lambda$, *d* is the separation between elements, and ψ is the phase difference between neighboring elements.

This may also be written:

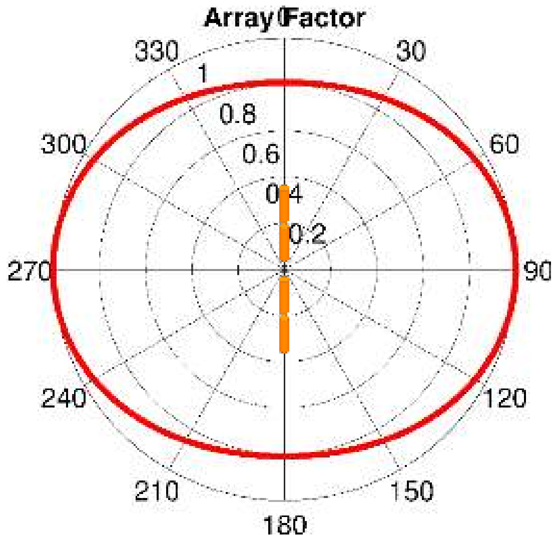
$$AF = \left[\frac{\sin\left(\frac{N(kd\cos\theta+\phi)}{2}\right)}{\sin\left(\frac{kd\cos\theta+\phi}{2}\right)}\right]$$

or, in the special case where $N = 2$,

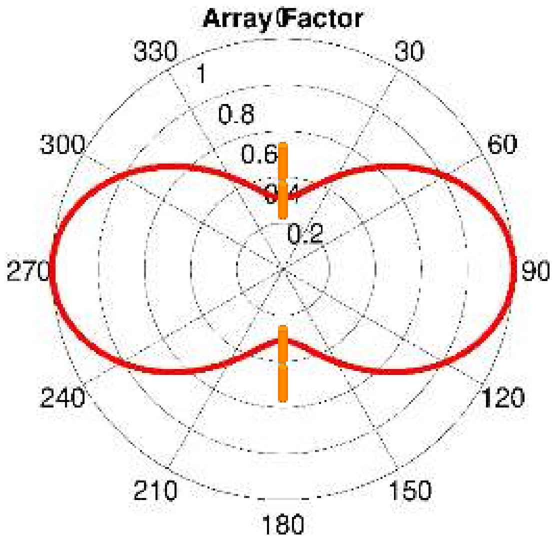
$$AF_2 = 2\cos\left[\frac{1}{2}(\mathrm{kd}\cos\theta + \phi)\right]$$

Array Factor of Uniform Antenna Arrays

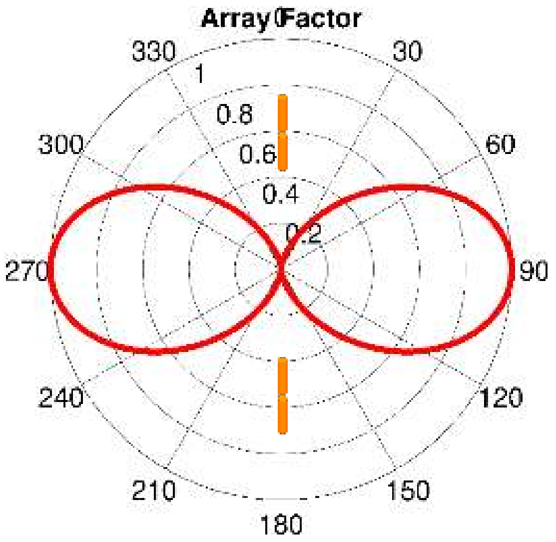
The spatial separation between elements in an antenna array increases the *apparent electrical size* of the effective radiator. This increase in electrical size enables a compression of the radiation pattern in the dimension of the array. For a fixed number of elements, increasing the separation d between elements increases the observed compression of the primary lobes of the radiation pattern.



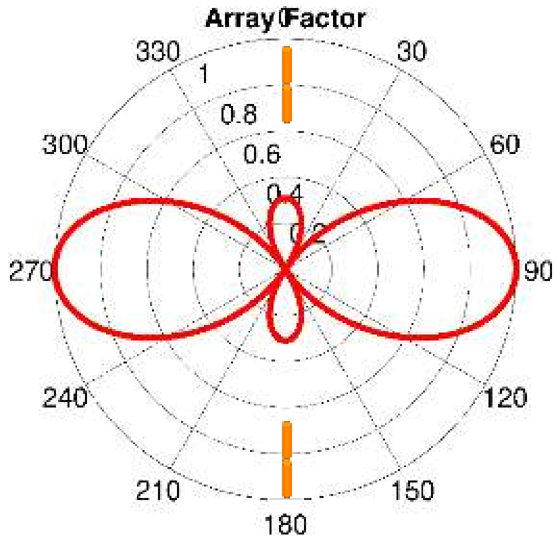
AF: N=2, d = 0.1λ, $\psi = 0°$



AF: N=2, d = 0.4λ, $\psi = 0°$



AF: N=2, d = 0.5λ, $\psi = 0°$



AF: N=2, d = 0.6λ, $\psi = 0°$

As shown above, a two-element array has progressively more compressed radiation as the two elements move further apart. However, as d increases beyond 0.5λ, a secondary set of lobes appears, pointing in the dimension of the array.

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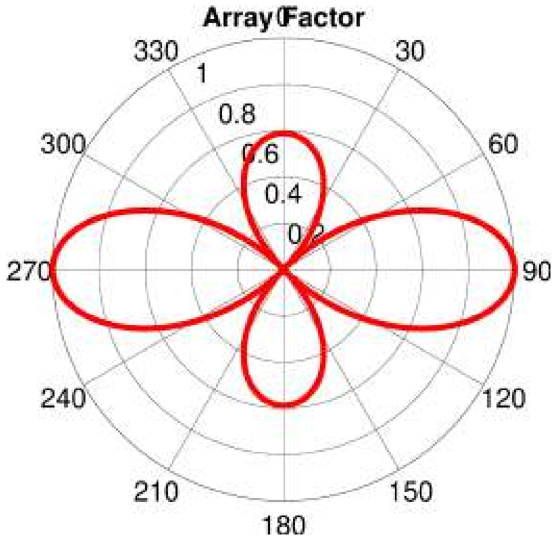
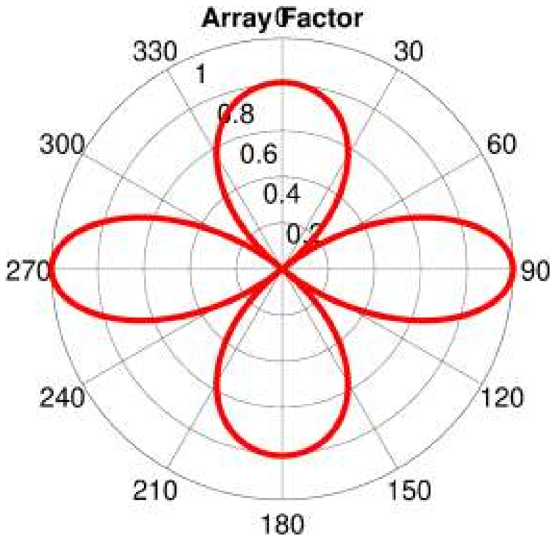
Array Factor of Uniform Antenna Arrays

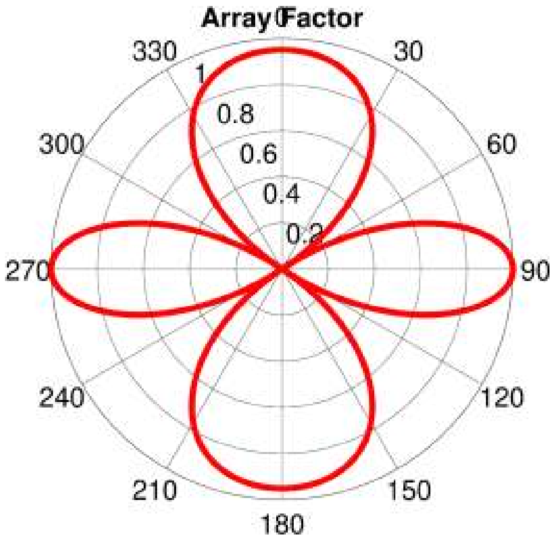
As shown below, the secondary lobes of a two-element array increase in strength, and the primary lobes narrow, as d increases from 0.5λ and approaches λ.



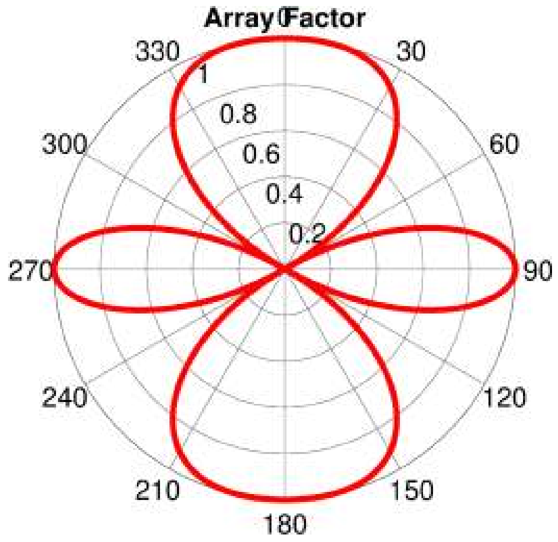
AF: N=2, d = 0.7λ, $\psi = 0°$



AF: N=2, d = 0.8λ, $\psi = 0°$



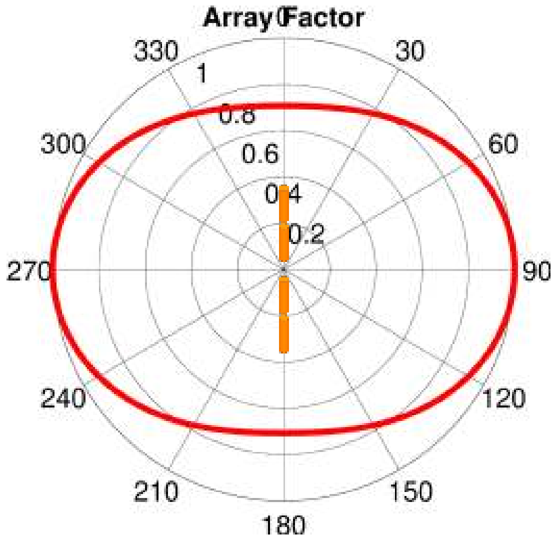
AF: N=2, d = 0.5λ, $\psi = 0°$



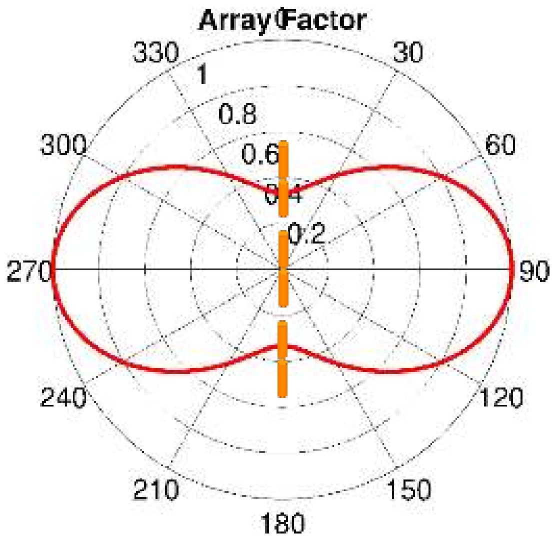
AF: N=2, d = λ, $\psi = 0°$

Array Factor of Uniform Antenna Arrays

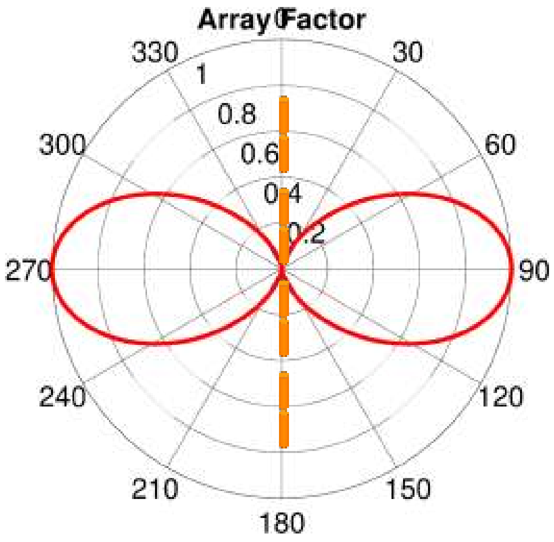
For an array with a fixed spatial separation between elements, increasing the number of elements similarly results in compression of the radiation pattern. The array factor patterns shown below have a fixed element spacing of $d = 0.25\lambda$, but a progressively increasing number of elements N.



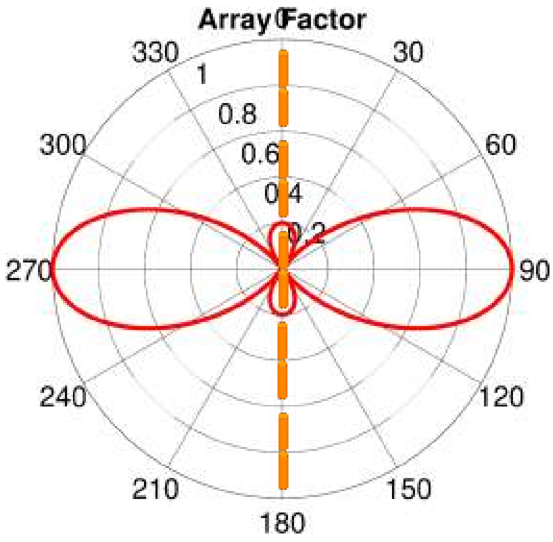
AF: N=2, d = 0.25λ, $\psi = 0°$



AF: N=3, d = 0.25λ, $\psi = 0°$



AF: N=4, d = 0.25λ, $\psi = 0°$

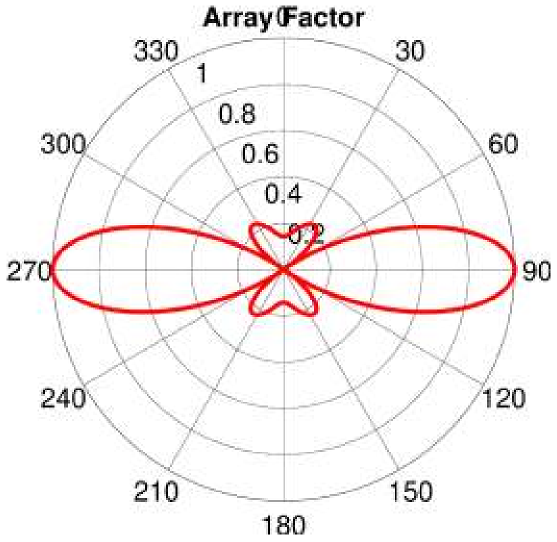


AF: N=5, d = 0.25λ, $\psi = 0°$

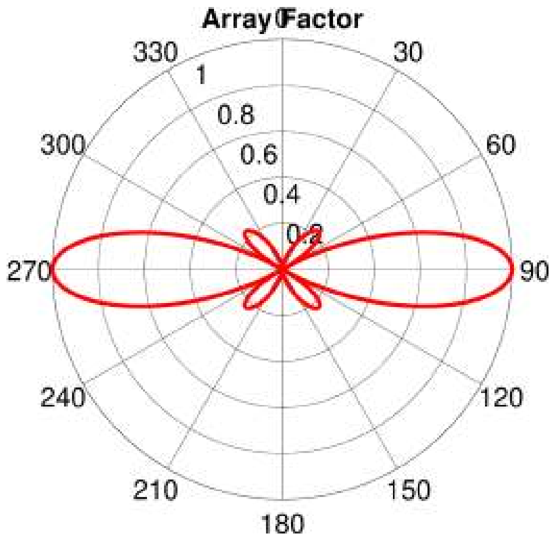
As shown above, an array with $d = 0.25\lambda$ has progressively more compressed radiation as the number of elements increases.

Array Factor of Uniform Antenna Arrays

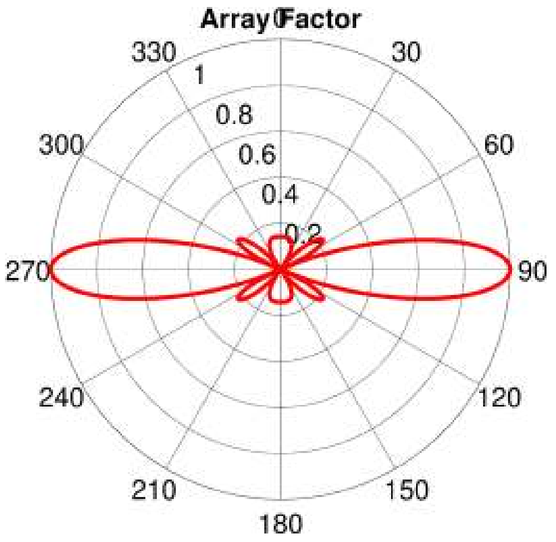
For an array with a fixed spatial separation of $d = 0.25\lambda$ between elements, increasing the number of elements increasingly narrows the focus of the radiation, while also introducing small side lobes.



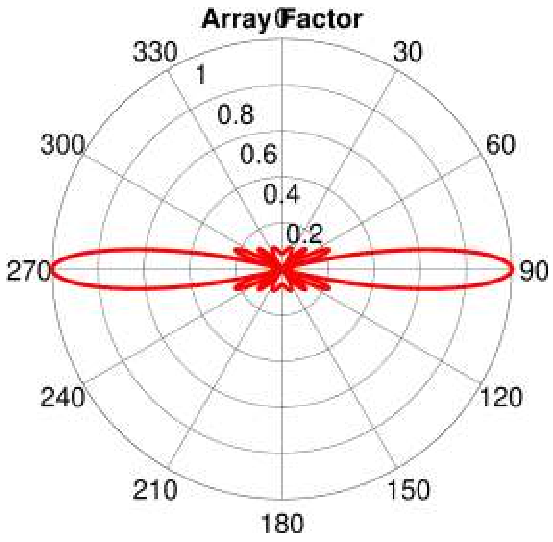
AF: N=7, d = 0.25λ, $\psi = 0°$



AF: N=8, d = 0.25λ, $\psi = 0°$



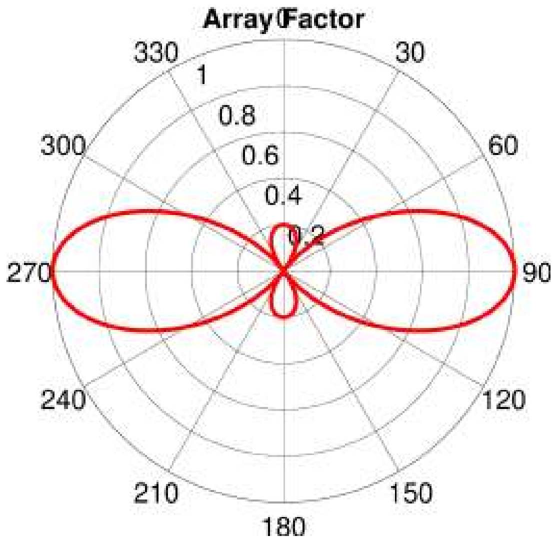
AF: N=10, d = 0.25λ, $\psi = 0°$



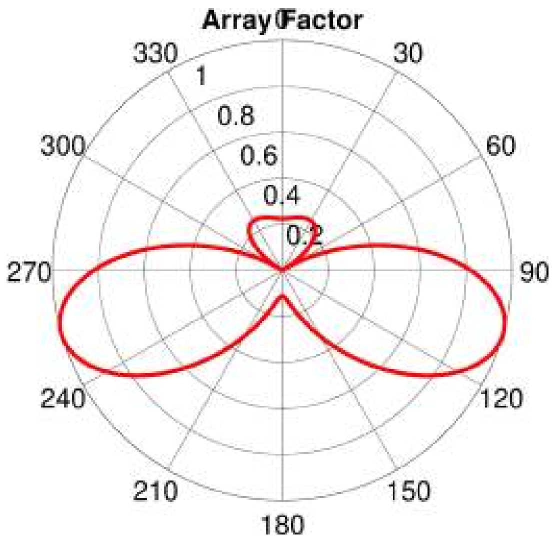
AF: N=15, d = 0.25λ, $\psi = 0°$

Phased Arrays

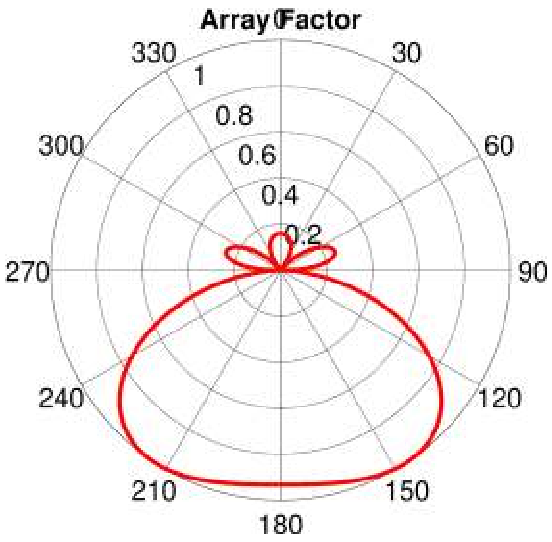
Another valuable control variable at our disposal is the linearly-stepped phase difference ψ between neighboring elements in the array. An array that utilizes such a variation in the phase of the driven signal is called a "phased array", and can be used to dynamically steer the beam without physically rearranging the elements.



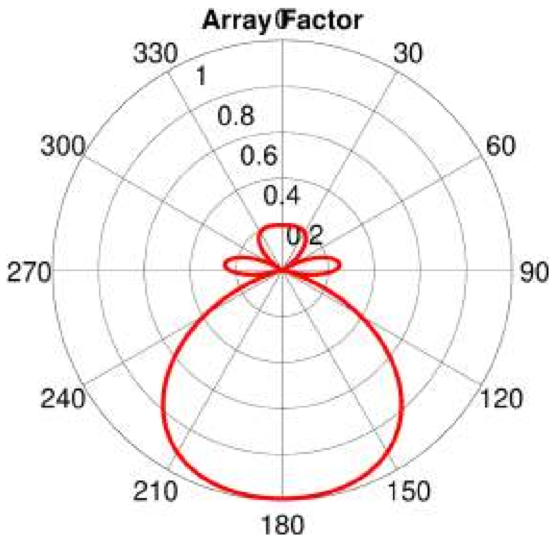
AF: N=5, d = 0.25λ, $\psi = 0°$



AF: N=5, d = 0.25λ, $\psi = 25°$



AF: N=5, d = 0.25λ, $\psi = 75°$

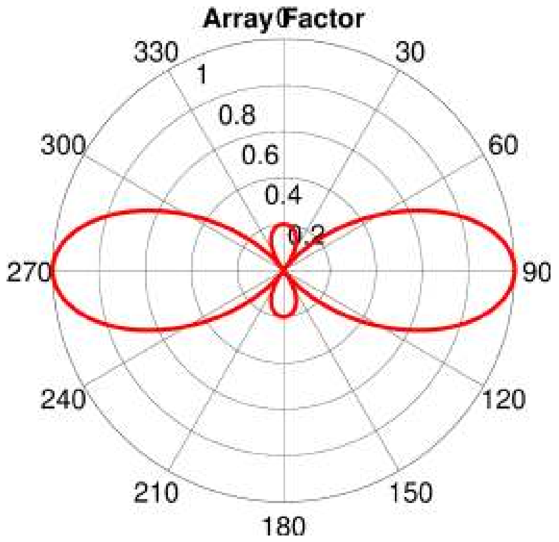


AF: N=5, d = 0.25λ, $\psi = 95°$

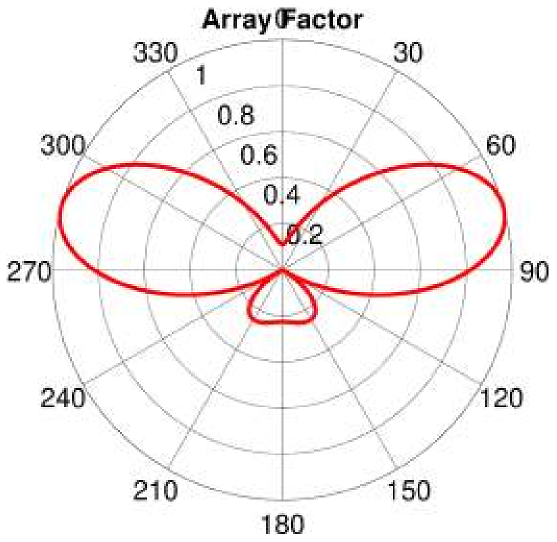
As shown above, varying values of ψ may be used to steer the beam radiated by a linear array of isotropic radiators off broadside.

Phased Arrays

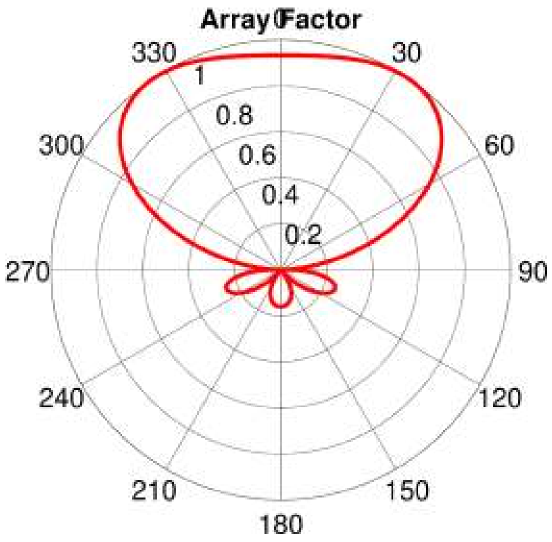
This slide shows the same progression as the previous, but for negative values of ψ.



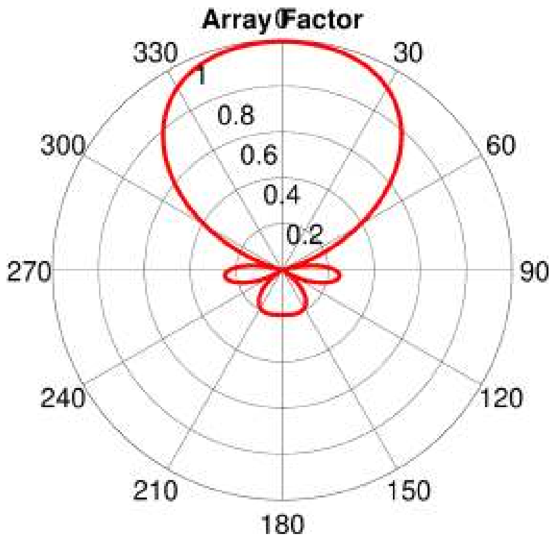
AF: N=5, d = 0.25λ, $\psi = 0°$



AF: N=5, d = 0.25λ, $\psi = -25°$



AF: N=5, d = 0.25λ, $\psi = -75°$

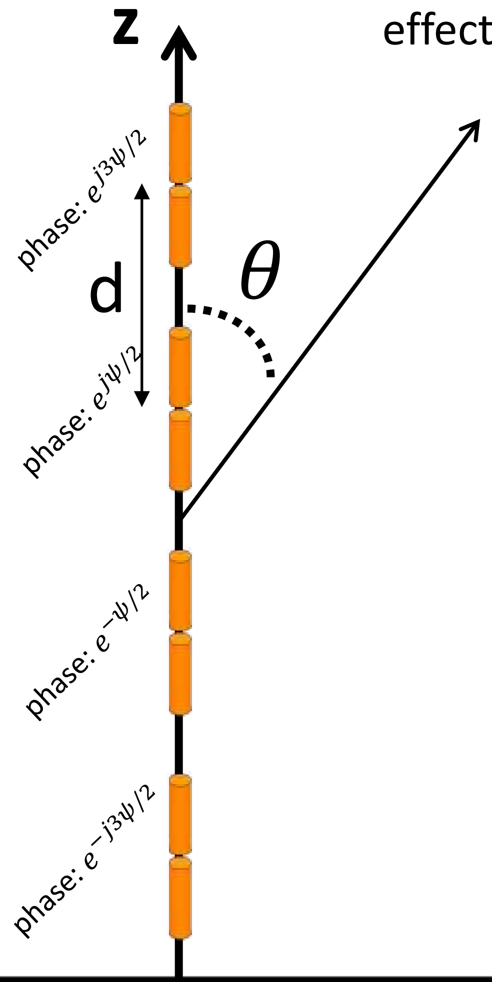


AF: N=5, d = 0.25λ, $\psi = -95°$

As shown above, varying values of ψ may be used to steer the beam radiated by a linear array of isotropic radiators off broadside.

Normalized Array Factor of Uniform Antenna Arrays

For an array where each element is driven with the same current magnitude, the strength of the radiation will naturally increase as the number of radiating elements increases. However, this does not necessarily reflect a system improvement, since the higher power output is proportional to a higher power input. In order to avoid conflating this effect with an actual gain improvement, the array factor is often reported as a *normalized* value.



$$AF_n = \frac{1}{N}\sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\psi)}$$

which may also be written:

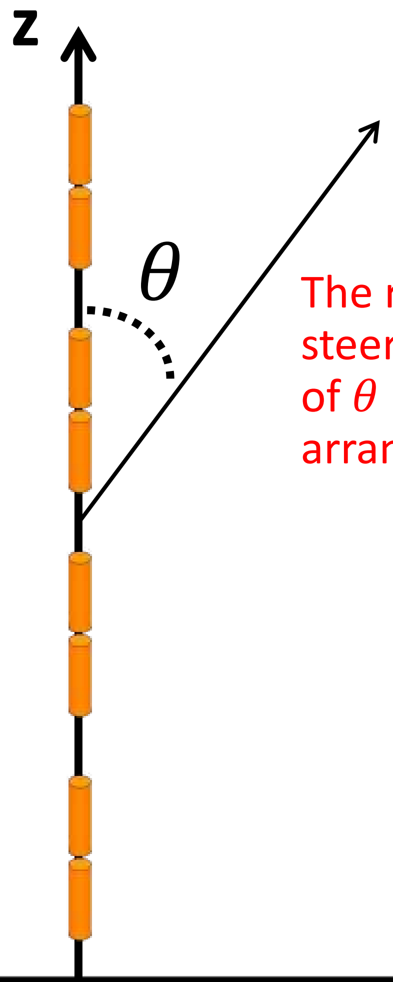
$$AF_n = \frac{1}{N}\left[\frac{\sin\left(\frac{N(kd\cos\theta+\phi)}{2}\right)}{\sin\left(\frac{kd\cos\theta+\phi}{2}\right)}\right]$$

or, in the special case where $N = 2$,

$$AF_{2n} = \cos\left[\frac{1}{2}(\mathrm{kd}\cos\theta+\phi)\right]$$

1D vs. 2D Arrays

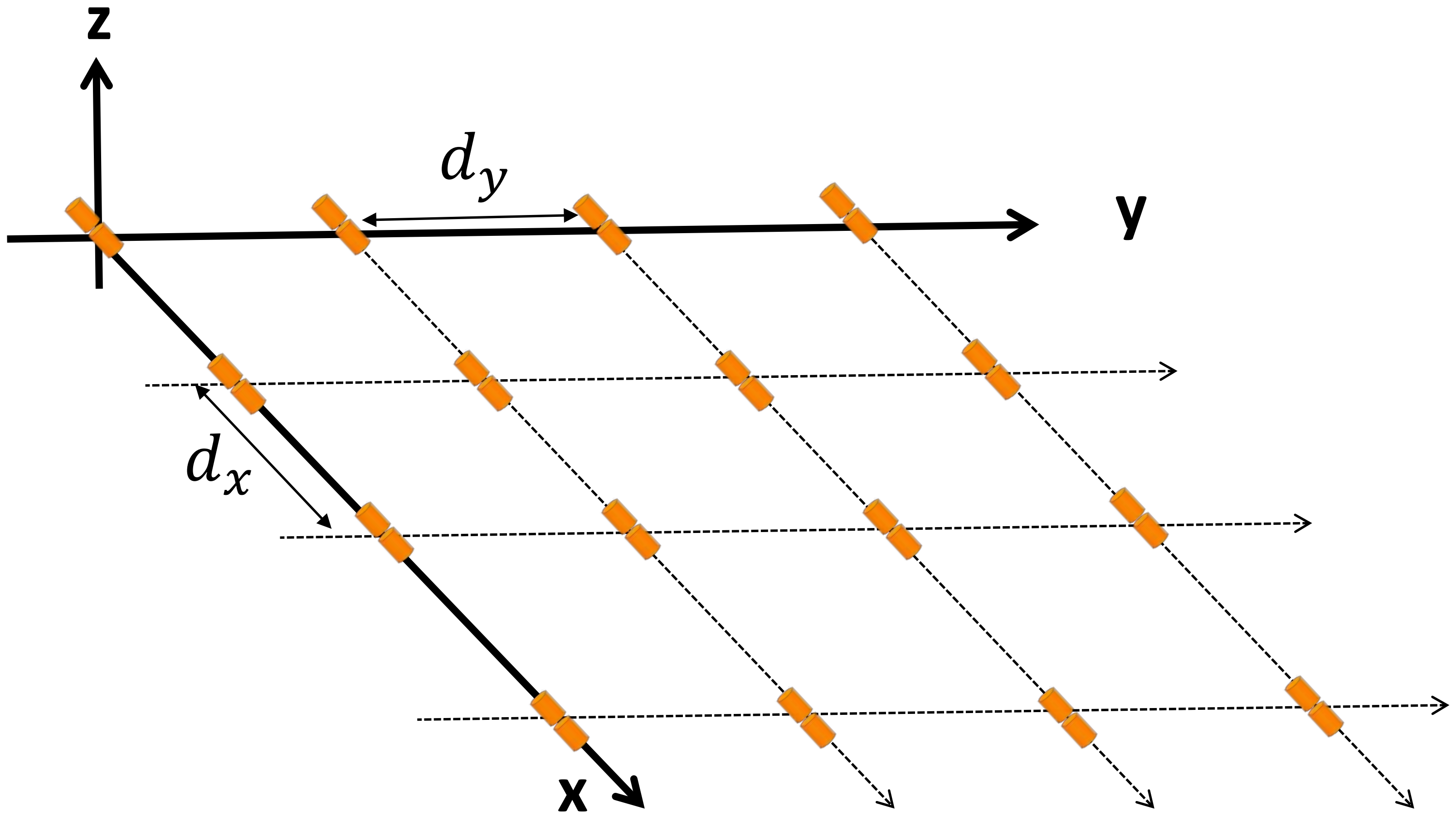
As we've seen, a 1D antenna array enables control of the radiation in the direction of the array. For instance, an array of isotropic radiators arranged along the z-axis may be used to steer the beam in the θ direction, angled from the z-axis.



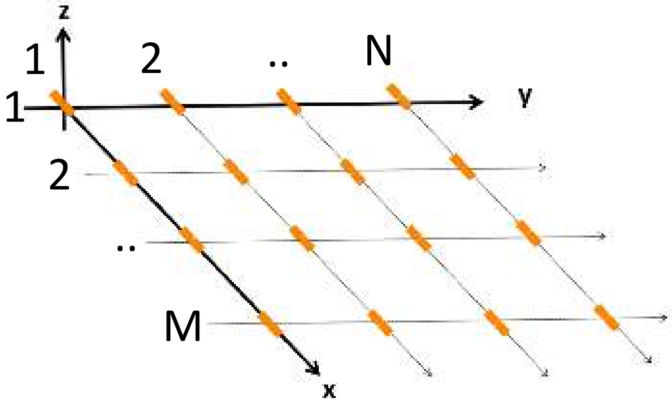
The radiated beam may be steered to any desired angle of θ using a linear array arranged along z.

1D vs. 2D Arrays

A *planar* array, in which elements are arranged along *two* dimensions, may be used to enable steering around a second axis. Here, the x-directed spacing between elements is d_x, and each column of elements arranged in the x-direction has a relative phase difference between neighbors of ψ_x. Similarly, the y-directed spacing between elements is d_y, and each row of elements arranged in the y-direction has a relative phase difference between neighbors of ψ_y.



1D vs. 2D Arrays



For the case of a uniform planar array with [M × N] elements distributed in the x-y plane, as shown, the normalized array factor becomes:

$$AF_n = \frac{1}{MN}\sum_{m=1}^{M} e^{j(m-1)(kd_x \sin\theta\cos\phi+\psi_x)} \sum_{n=1}^{N} e^{j(n-1)(kd_y \sin\theta\sin\phi+\psi_y)}$$

which may also be written:

$$AF_n = \left[\frac{1}{M}\frac{\sin\left(\frac{M(kd_x \sin\theta\cos\phi+\psi_x)}{2}\right)}{\sin\left(\frac{kd_x \sin\theta\cos\phi+\psi_x}{2}\right)}\right]\left[\frac{1}{N}\frac{\sin\left(\frac{N(kd_y \sin\theta\sin\phi+\psi_y)}{2}\right)}{\sin\left(\frac{kd_y \sin\theta\sin\phi+\psi_y}{2}\right)}\right]$$

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