

Basics of Antenna Arrays

Manipulating the Array Factor

Developed by Kathryn L. Smith, PhD





Sources

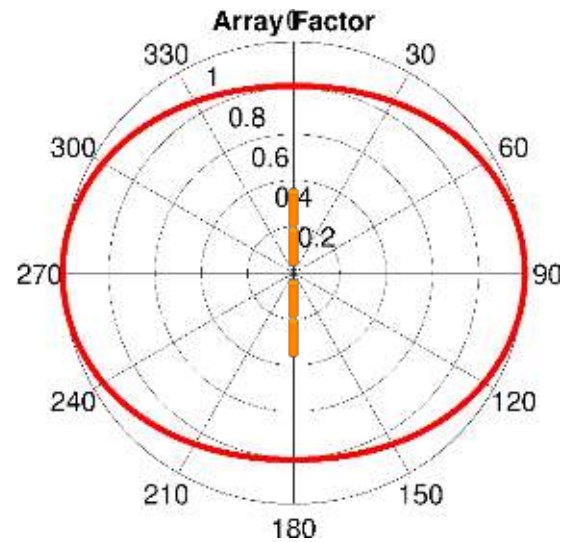
The material presented herein is from the following sources:

“Engineering Electromagnetics,” by Nathan Ida, 3rd ed. (2015)

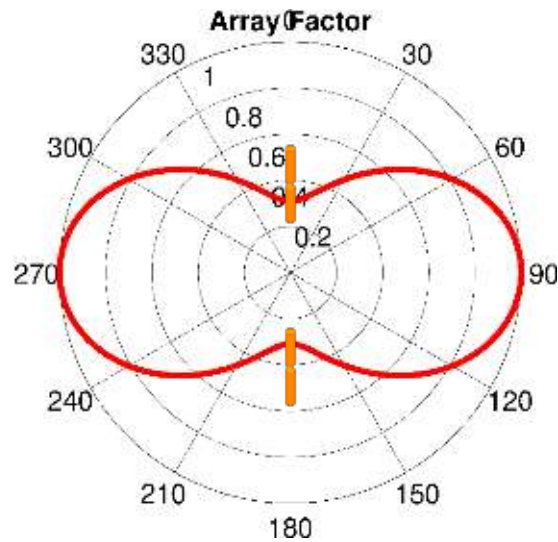
“Antenna Theory,” by Constantine A. Balanis, 4th ed. (2016)

Array Factor of Uniform Antenna Arrays

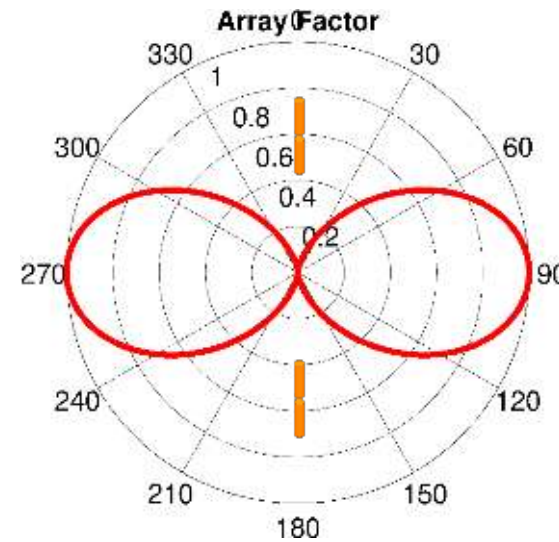
The spatial separation between elements in an antenna array increases the *apparent electrical size* of the effective radiator. This increase in electrical size enables a compression of the radiation pattern in the dimension of the array. For a fixed number of elements, increasing the separation d between elements increases the observed compression of the primary lobes of the radiation pattern.



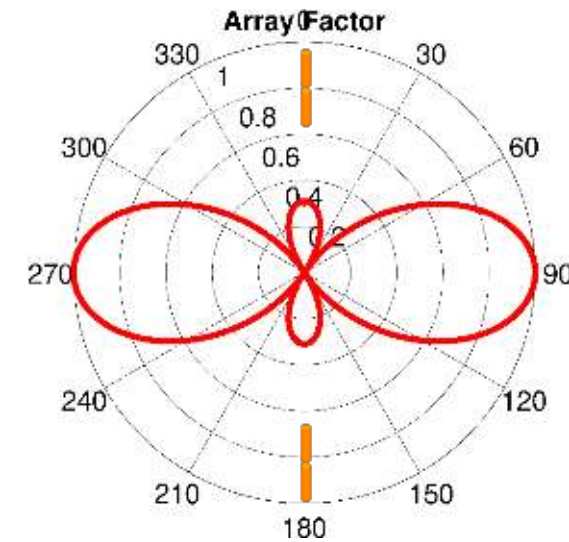
AF: $N=2$, $d = 0.1\lambda$,
 $\psi = 0^\circ$



AF: $N=2$, $d = 0.4\lambda$,
 $\psi = 0^\circ$



AF: $N=2$, $d = 0.5\lambda$,
 $\psi = 0^\circ$

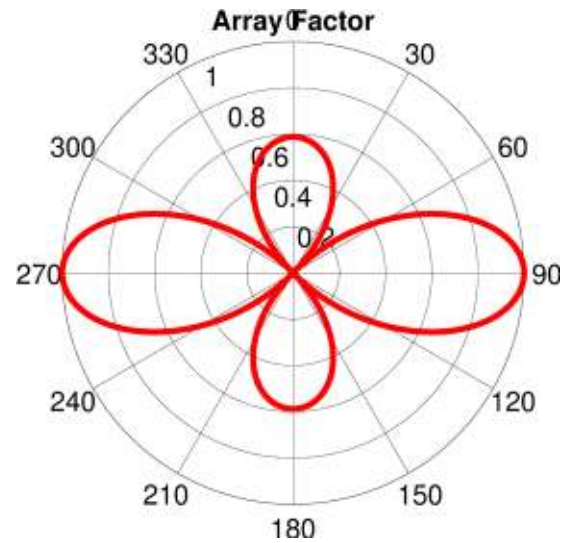


AF: $N=2$, $d = 0.6\lambda$,
 $\psi = 0^\circ$

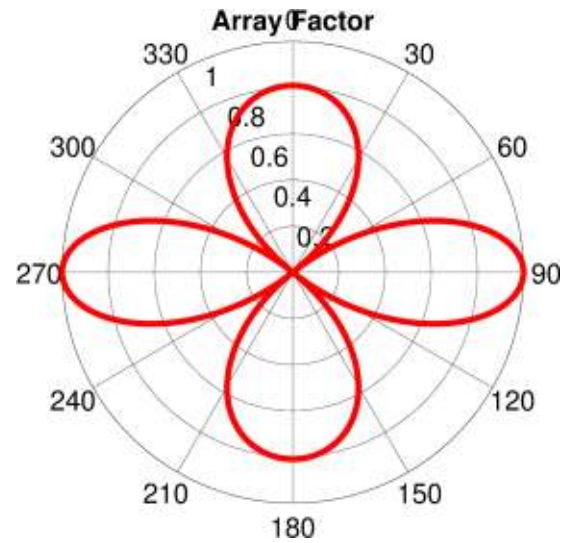
As shown above, a two-element array has progressively more compressed radiation as the two elements move further apart. However, as d increases beyond 0.5λ , a secondary set of lobes appears, pointing in the dimension of the array.

Array Factor of Uniform Antenna Arrays

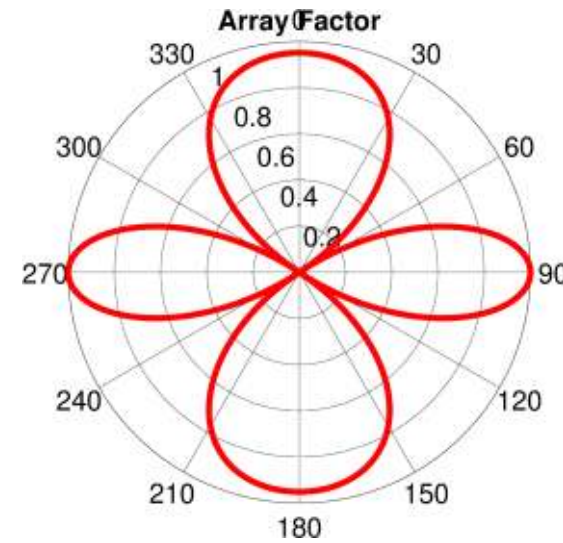
As shown below, the secondary lobes of a two-element array increase in strength, and the primary lobes narrow, as d increases from 0.5λ and approaches λ .



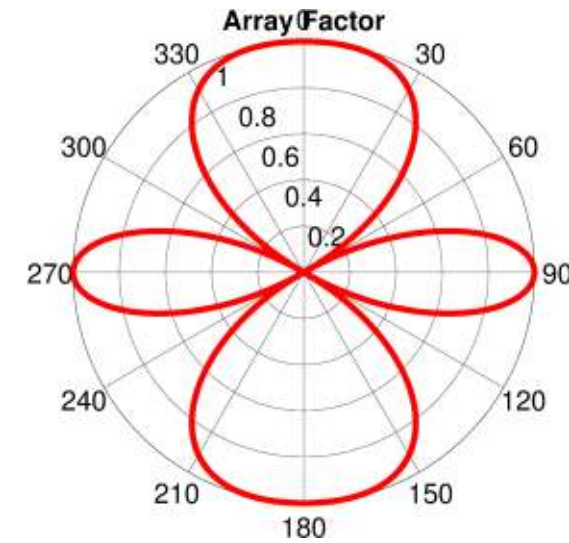
AF: $N=2$, $d = 0.7\lambda$,
 $\psi = 0^\circ$



AF: $N=2$, $d = 0.8\lambda$,
 $\psi = 0^\circ$



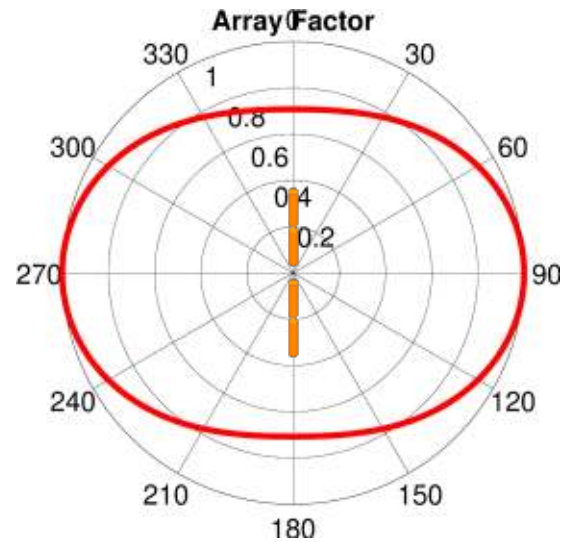
AF: $N=2$, $d = 0.5\lambda$,
 $\psi = 0^\circ$



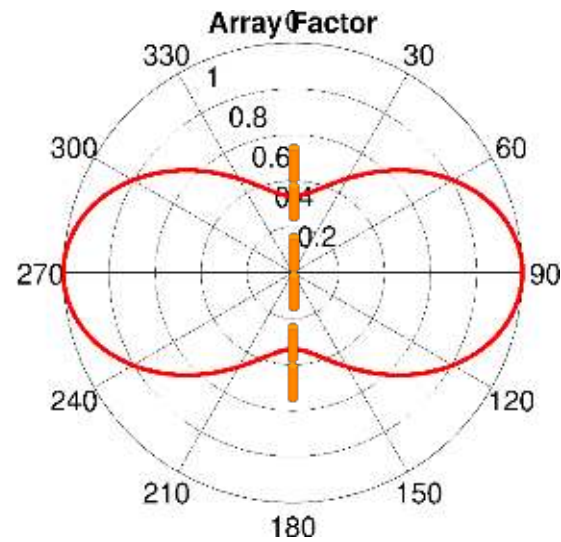
AF: $N=2$, $d = \lambda$,
 $\psi = 0^\circ$

Array Factor of Uniform Antenna Arrays

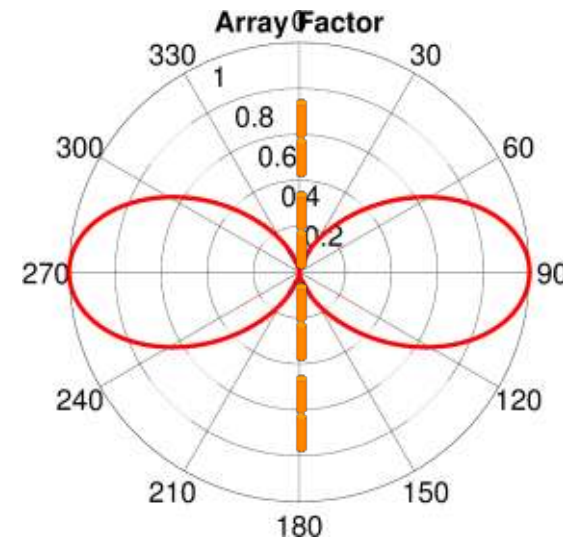
For an array with a fixed spatial separation between elements, increasing the number of elements similarly results in compression of the radiation pattern. The array factor patterns shown below have a fixed element spacing of $d = 0.25\lambda$, but a progressively increasing number of elements N .



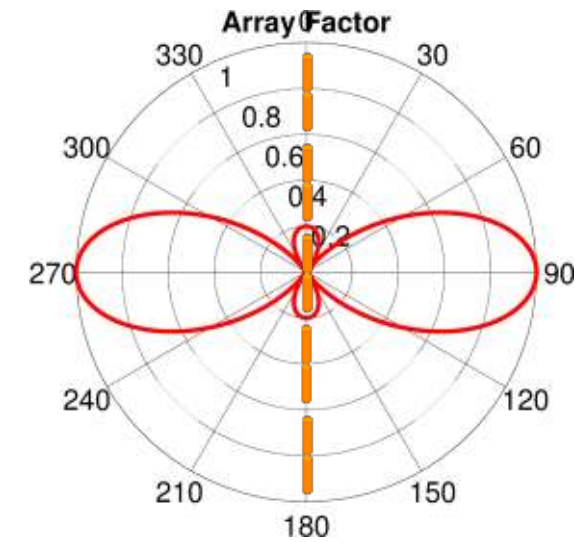
AF: $N=2$, $d = 0.25\lambda$,
 $\psi = 0^\circ$



AF: $N=3$, $d = 0.25\lambda$,
 $\psi = 0^\circ$



AF: $N=4$, $d = 0.25\lambda$,
 $\psi = 0^\circ$

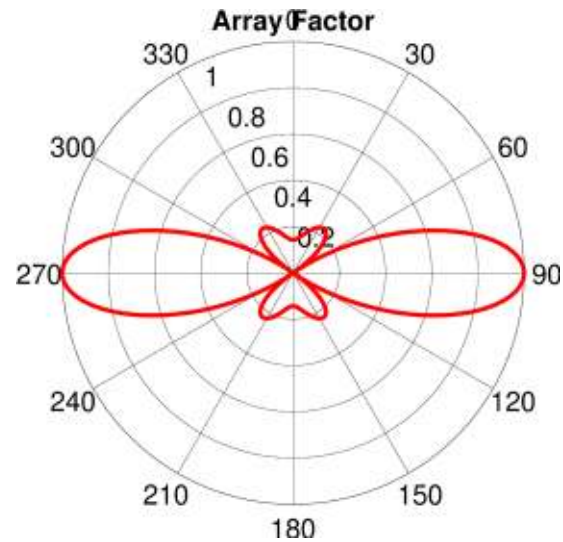


AF: $N=5$, $d = 0.25\lambda$,
 $\psi = 0^\circ$

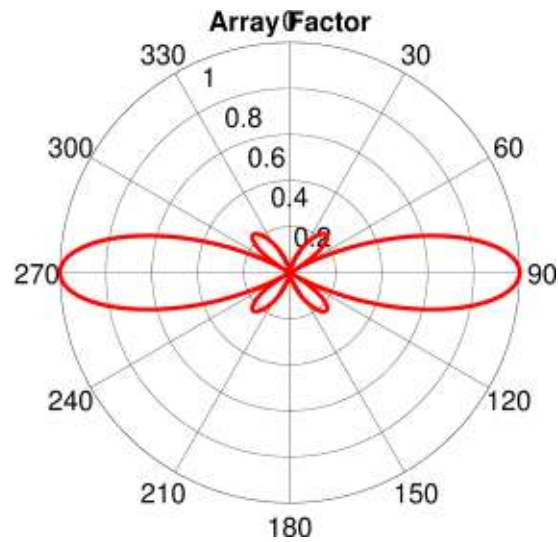
As shown above, an array with $d = 0.25\lambda$ has progressively more compressed radiation as the number of elements increases.

Array Factor of Uniform Antenna Arrays

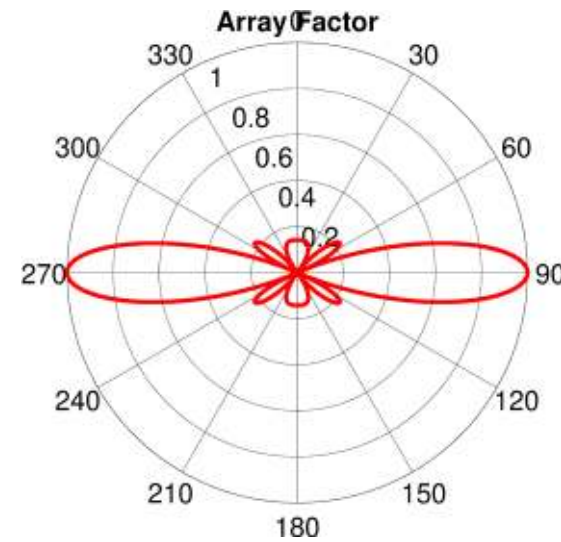
For an array with a fixed spatial separation of $d = 0.25\lambda$ between elements, increasing the number of elements increasingly narrows the focus of the radiation, while also introducing small side lobes.



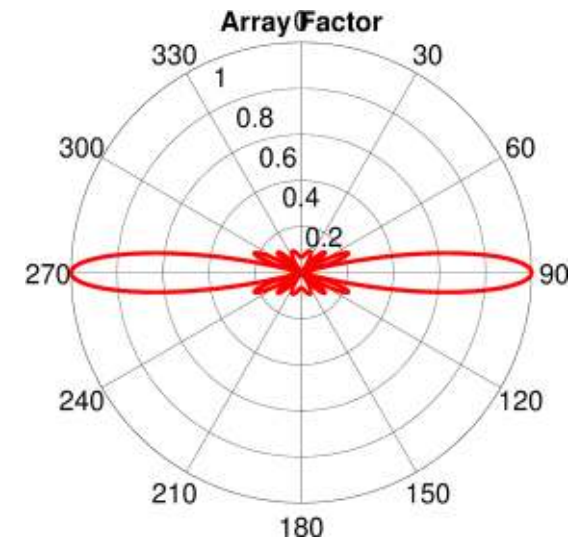
AF: $N=7$, $d = 0.25\lambda$,
 $\psi = 0^\circ$



AF: $N=8$, $d = 0.25\lambda$,
 $\psi = 0^\circ$



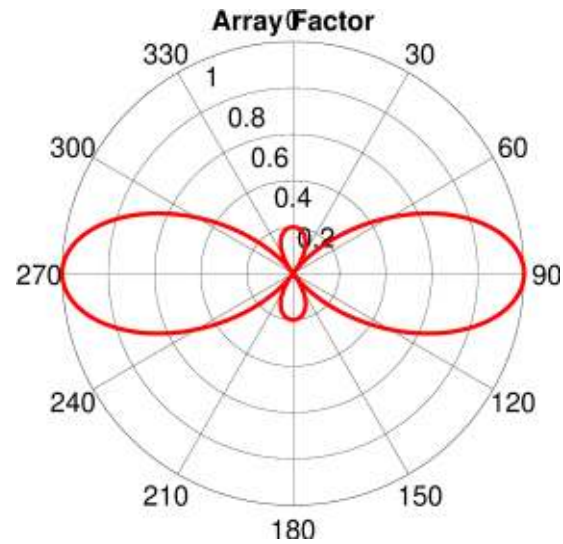
AF: $N=10$, $d = 0.25\lambda$,
 $\psi = 0^\circ$



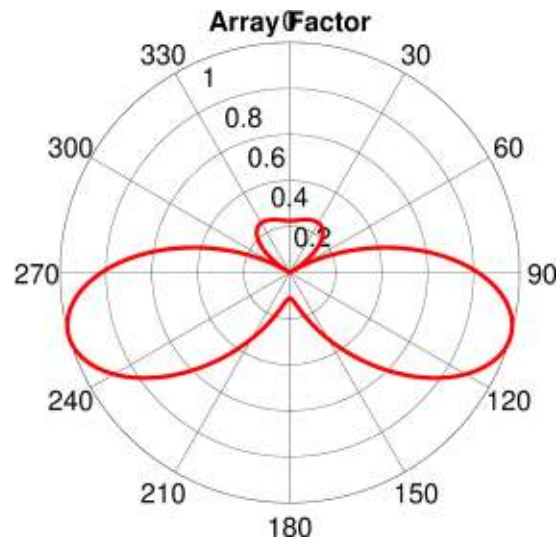
AF: $N=15$, $d = 0.25\lambda$,
 $\psi = 0^\circ$

Phased Arrays

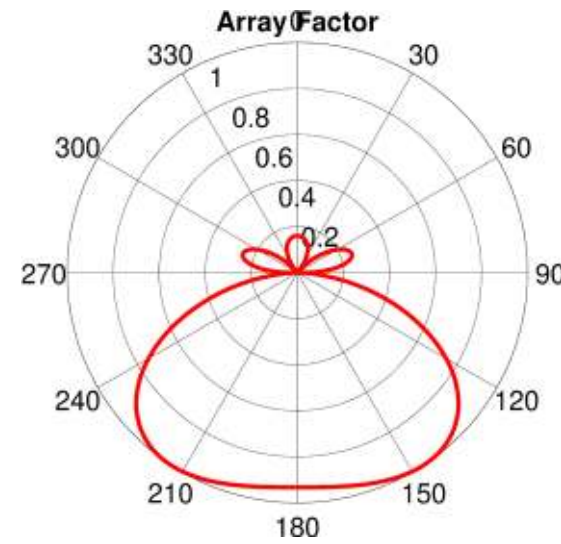
Another valuable control variable at our disposal is the linearly-stepped phase difference ψ between neighboring elements in the array. An array that utilizes such a variation in the phase of the driven signal is called a “phased array”, and can be used to dynamically steer the beam without physically rearranging the elements.



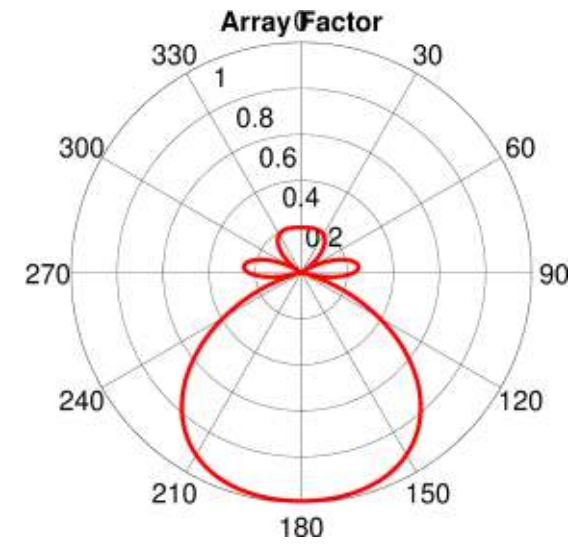
AF: $N=5$, $d = 0.25\lambda$,
 $\psi = 0^\circ$



AF: $N=5$, $d = 0.25\lambda$,
 $\psi = 25^\circ$



AF: $N=5$, $d = 0.25\lambda$,
 $\psi = 75^\circ$

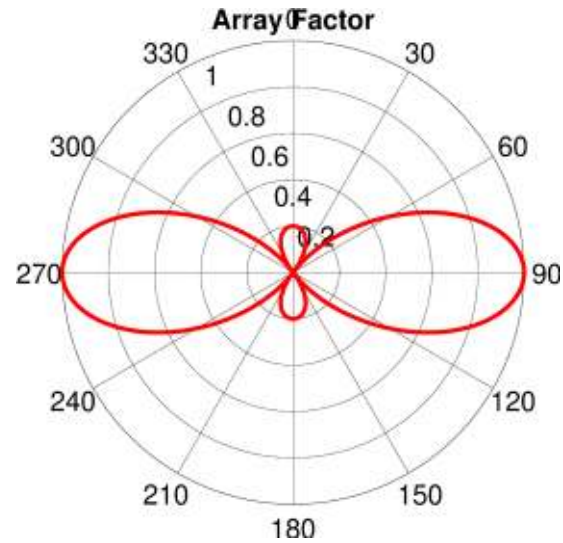


AF: $N=5$, $d = 0.25\lambda$,
 $\psi = 95^\circ$

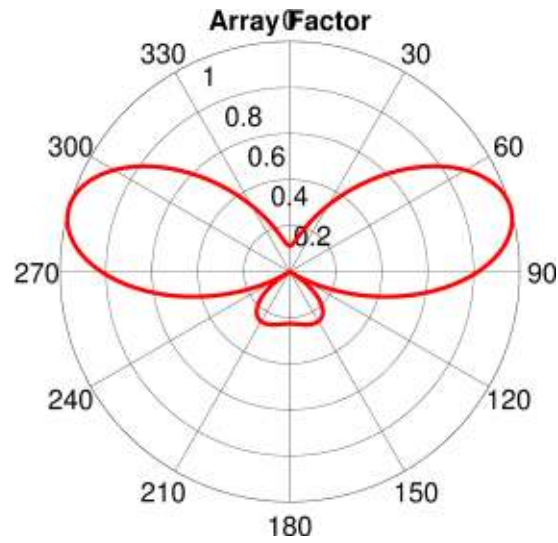
As shown above, varying values of ψ may be used to steer the beam radiated by a linear array of isotropic radiators off broadside.

Phased Arrays

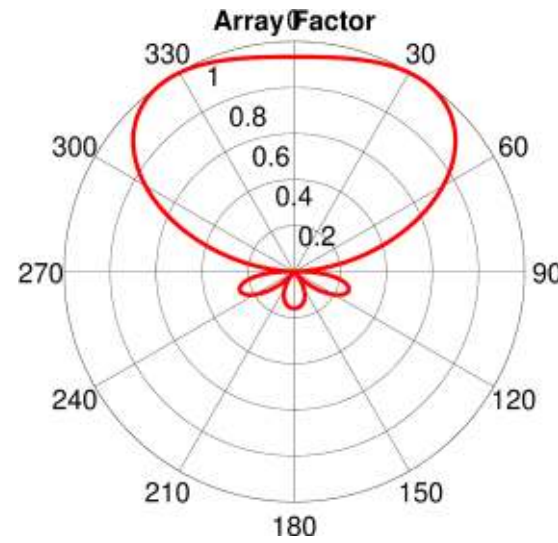
This slide shows the same progression as the previous, but for negative values of ψ .



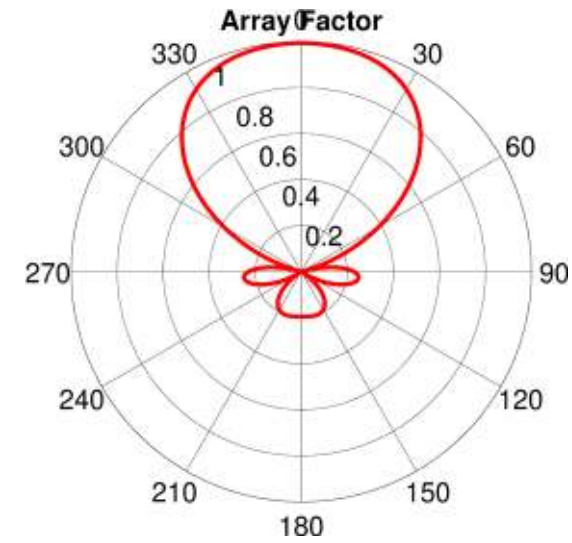
AF: $N=5$, $d = 0.25\lambda$,
 $\psi = 0^\circ$



AF: $N=5$, $d = 0.25\lambda$,
 $\psi = -25^\circ$



AF: $N=5$, $d = 0.25\lambda$,
 $\psi = -75^\circ$

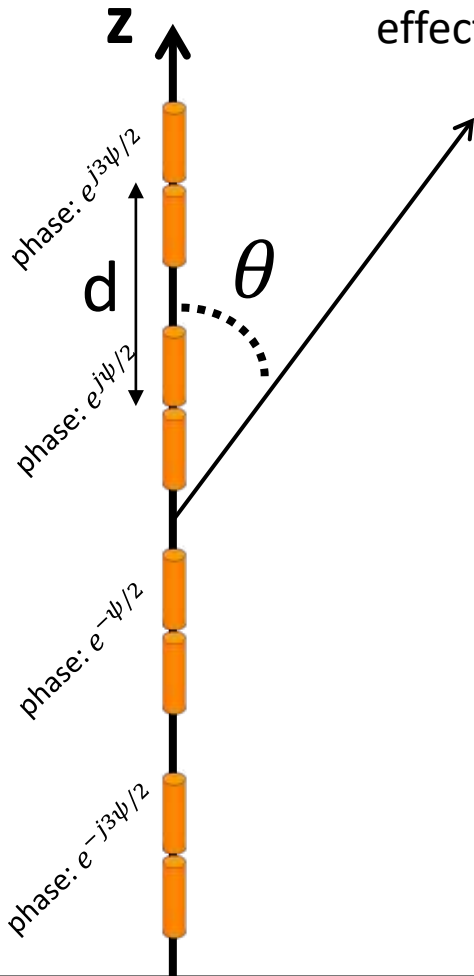


AF: $N=5$, $d = 0.25\lambda$,
 $\psi = -95^\circ$

As shown above, varying values of ψ may be used to steer the beam radiated by a linear array of isotropic radiators off broadside.

Normalized Array Factor of Uniform Antenna Arrays

For an array where each element is driven with the same current magnitude, the strength of the radiation will naturally increase as the number of radiating elements increases. However, this does not necessarily reflect a system improvement, since the higher power output is proportional to a higher power input. In order to avoid conflating this effect with an actual gain improvement, the array factor is often reported as a *normalized* value.



$$AF_n = \frac{1}{N} \sum_{n=1}^N e^{j(n-1)(kd \cos \theta + \psi)}$$

which may also be written:

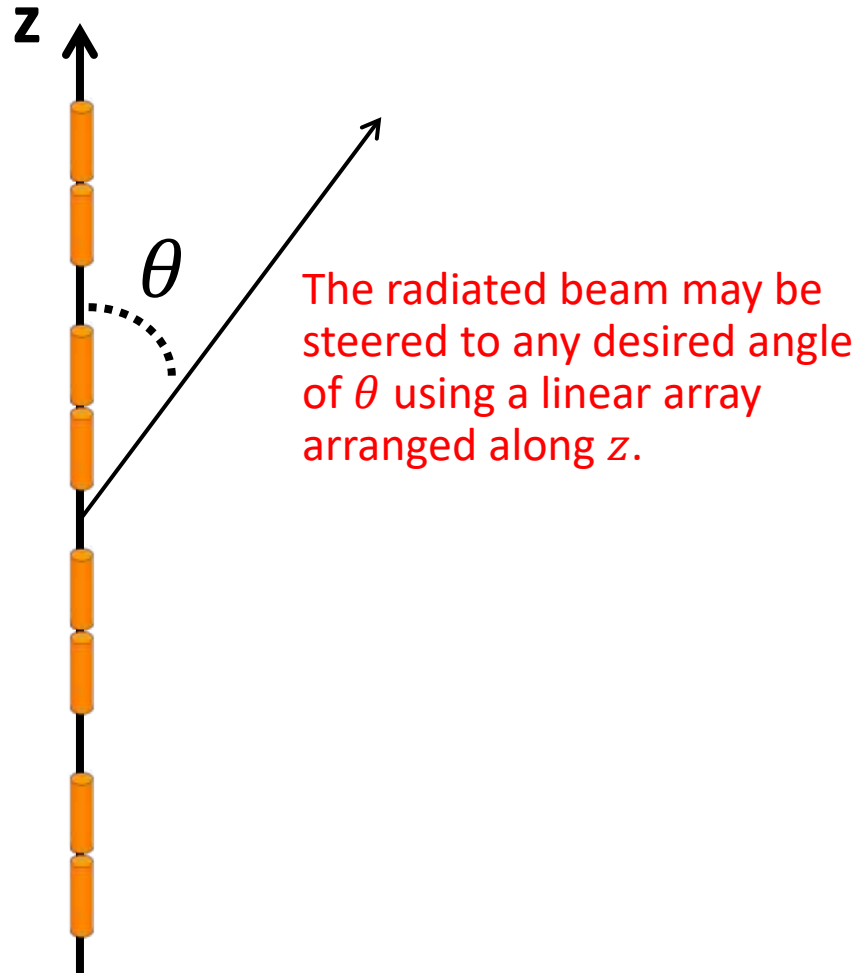
$$AF_n = \frac{1}{N} \left[\frac{\sin \left(\frac{N(kd \cos \theta + \phi)}{2} \right)}{\sin \left(\frac{kd \cos \theta + \phi}{2} \right)} \right]$$

or, in the special case where $N = 2$,

$$AF_{2n} = \cos \left[\frac{1}{2} (kd \cos \theta + \phi) \right]$$

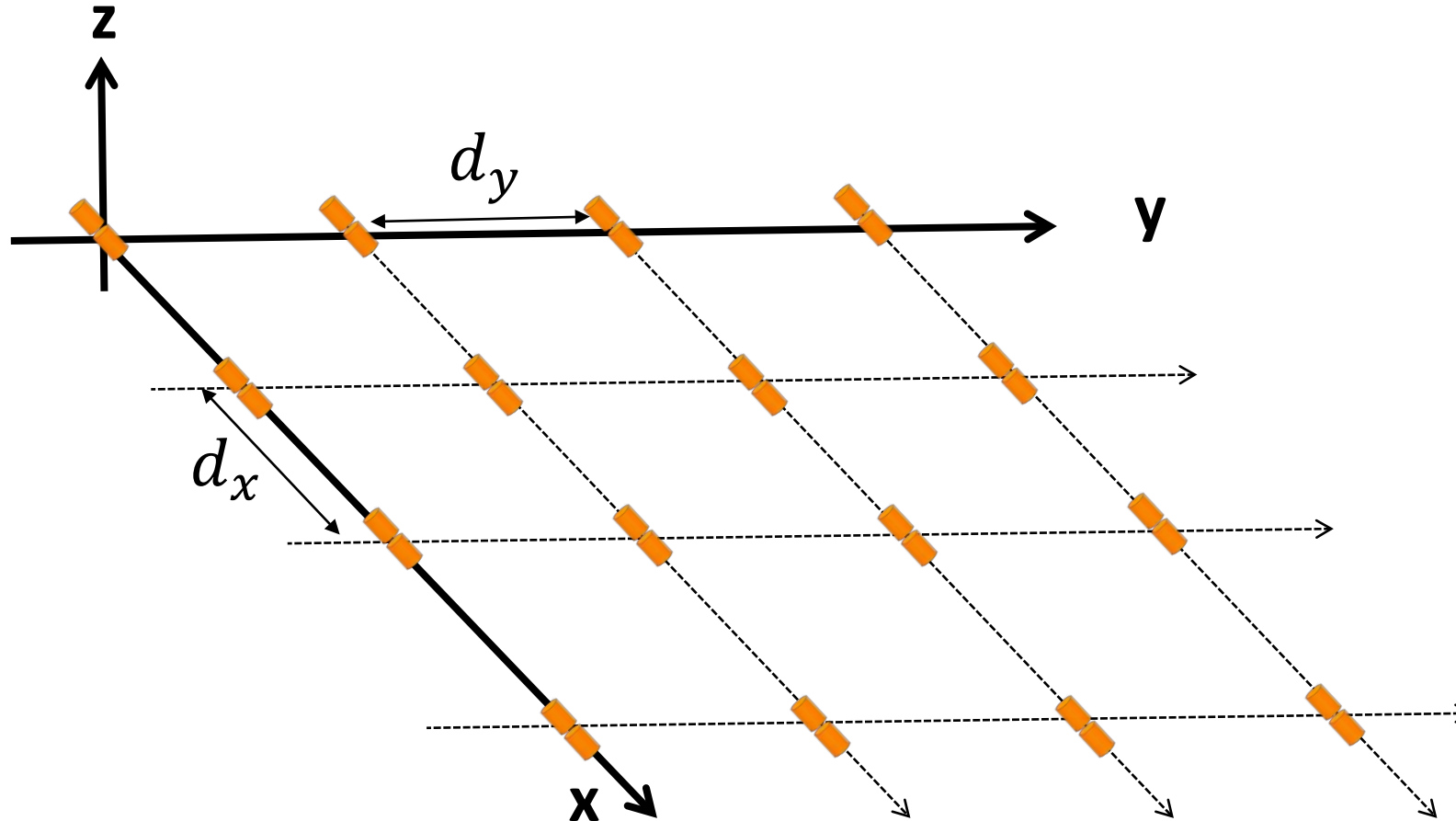
1D vs. 2D Arrays

As we've seen, a 1D antenna array enables control of the radiation in the direction of the array. For instance, an array of isotropic radiators arranged along the z-axis may be used to steer the beam in the θ direction, angled from the z-axis.

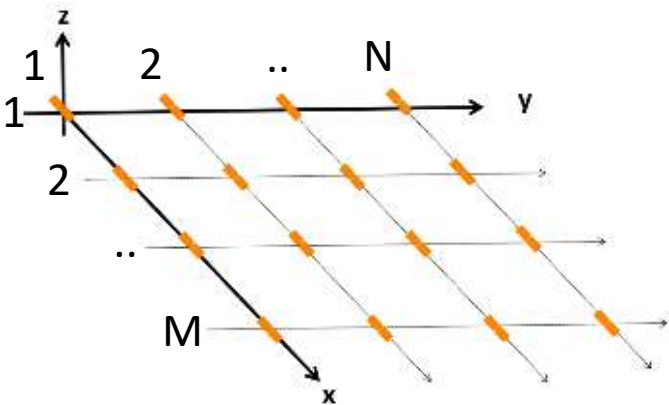


1D vs. 2D Arrays

A *planar* array, in which elements are arranged along *two* dimensions, may be used to enable steering around a second axis. Here, the x-directed spacing between elements is d_x , and each column of elements arranged in the x-direction has a relative phase difference between neighbors of ψ_x . Similarly, the y-directed spacing between elements is d_y , and each row of elements arranged in the y-direction has a relative phase difference between neighbors of ψ_y .



1D vs. 2D Arrays



For the case of a uniform planar array with $[M \times N]$ elements distributed in the x-y plane, as shown, the normalized array factor becomes:

$$AF_n = \frac{1}{MN} \sum_{m=1}^M e^{j(m-1)(kd_x \sin \theta \cos \phi + \psi_x)} \sum_{n=1}^N e^{j(n-1)(kd_y \sin \theta \sin \phi + \psi_y)}$$

which may also be written:

$$AF_n = \left[\frac{1}{M} \frac{\sin\left(\frac{M(kd_x \sin \theta \cos \phi + \psi_x)}{2}\right)}{\sin\left(\frac{kd_x \sin \theta \cos \phi + \psi_x}{2}\right)} \right] \left[\frac{1}{N} \frac{\sin\left(\frac{N(kd_y \sin \theta \sin \phi + \psi_y)}{2}\right)}{\sin\left(\frac{kd_y \sin \theta \sin \phi + \psi_y}{2}\right)} \right]$$

 **Ansys**

