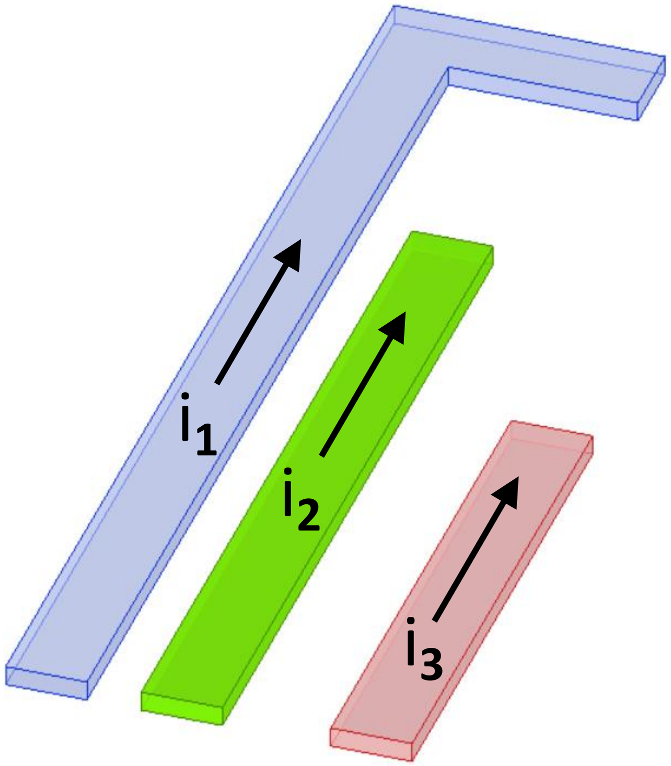
Module 5: Q3D Inductance Matrix Reduction

Release 2020 R1

Three Conductors and the Inductance Matrix



$$\varphi = \text{flux} = Li$$

$$\varphi_1 = L_{11}i_1 + L_{12}i_2 + L_{13}i_3$$

$$\varphi_2 = L_{12}i_1 + L_{22}i_2 + L_{23}i_3$$

$$\varphi_3 = L_{13}i_1 + L_{23}i_2 + L_{33}i_3$$

From these equations we can form the inductance matrix for the three-conductor system..

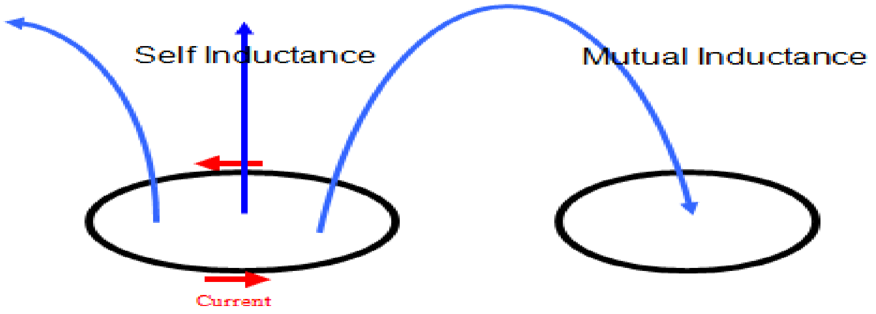
$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix}$$

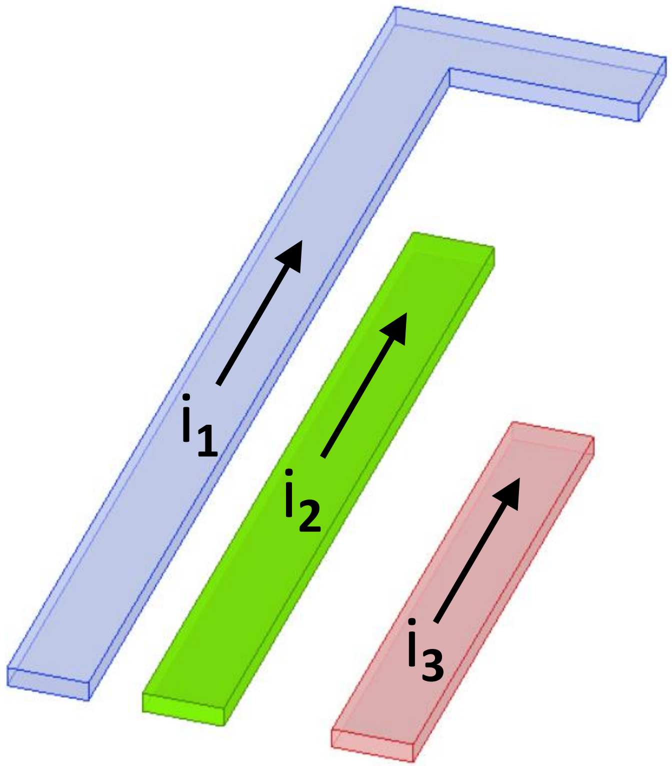
For technical background and reference, please see:
Circuit Matrix Reduction Operations by J. Eric Bracken
… and …
Matrix Reduction Operations In Q3D by Greg Pitner

Self Inductance and Mutual Inductance

The flux φ of any one conductor includes the mutual inductance from current in nearby conductors.

Q3D simulates self and mutual inductances.





$$\varphi_1 = L_{11}i_1 + L_{12}i_2 + L_{13}i_3$$

$$\varphi_2 = L_{12}i_1 + L_{22}i_2 + L_{23}i_3$$

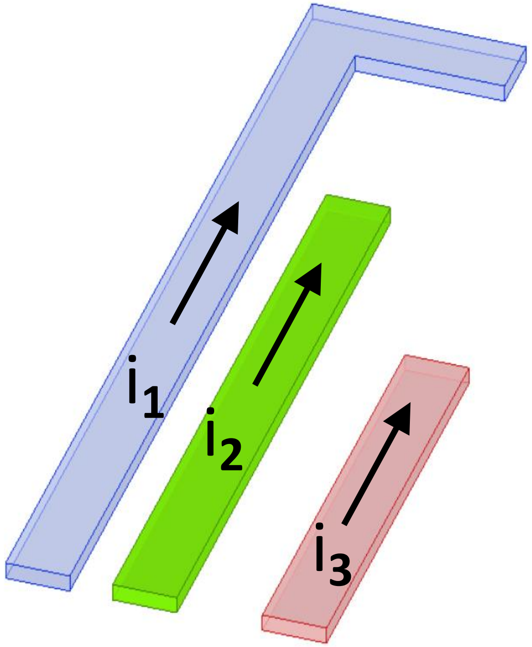
$$\varphi_3 = L_{13}i_1 + L_{23}i_2 + L_{33}i_3$$

$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix}$$

Self-inductance terms in dark red

Mutual inductance terms in green

The Inductance Matrix for the ACRL Q3D Simulation



Resistance Units: ohm Matrix

Inductance Units: nH Original

Self Terms

EMCDbar5 AC RL @ 0Hz

View | Format | Passivity | Export

	Bar1:Source1	Bar2:Source2	Bar3:Source3
Freq: 0Hz			
Bar1:Source1	33.05285	11.92634	4.82218
Bar2:Source2	11.92634	20.09579	5.65317
Bar3:Source3	4.82218	5.65317	11.82154

The **Bar3** mutual inductances are the smallest because **Bar3** is both the shortest conductor and the most distant from the others.

The diagonal terms of the inductance matrix are the self-inductances. In this case they are larger than the mutual inductances.

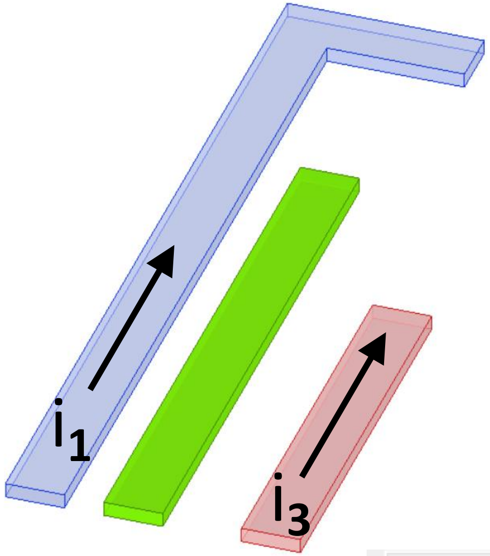
$$L = \begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix}$$

$$\varphi_1 = L_{11}i_1 + L_{12}i_2 + L_{13}i_3$$

$$\varphi_2 = L_{12}i_1 + L_{22}i_2 + L_{23}i_3$$

$$\varphi_3 = L_{13}i_1 + L_{23}i_2 + L_{33}i_3$$

Inductance Matrix - Floating a Conductor - ***Float Net…***



Floating an isolated conductor forces any current in the conductor to zero. Removing a current will remove a row and a column from the inductance matrix. This example floats the middle conductor 2. We are left with three equations and two unknowns.

$$\varphi_1 = L_{11}i_1 + L_{12}i_2 + L_{13}i_3$$

$$\varphi_2 = L_{12}i_1 + L_{22}i_2 + L_{23}i_3$$

$$\varphi_3 = L_{13}i_1 + L_{23}i_2 + L_{33}i_3$$

$$L = \begin{bmatrix} L_{11} & L_{13} \\ L_{13} & L_{33} \end{bmatrix}$$

Below is the 2 x 2 matrix from the ***Float Net Matrix Reduction*** operation.

Resistance Units: ohm Matrix

Inductance (checked) Units: nH Original

Self Terms

View | Format | Passivity | Export

EMCDbar5 AC RL

	Bar1:Source1	Bar2:Source2	Bar3:Source3
Freq: 100MHz			
Bar1:Source1	30.64371	11.98994	4.88161
Bar2:Source2	11.98994	18.15782	5.56616
Bar3:Source3	4.88161	5.56616	10.80609

Resistance Units: ohm Matrix

Inductance (checked) Units: nH FloatNetMatrix1

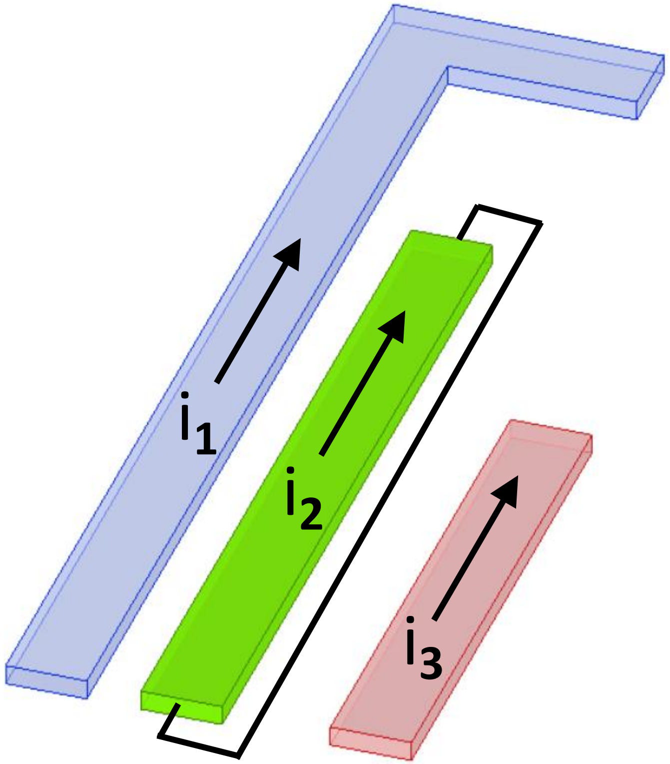
Self Terms

View | Format | Passivity | Export

EMCDbar5 AC RL

	Bar1:Source1	Bar3:Source3
Freq: 100MHz		
Bar1:Source1	30.64371	4.88161
Bar3:Source3	4.88161	10.80609

Inductance *Matrix Reduction* Grounding a Conductor - *Ground Net…*

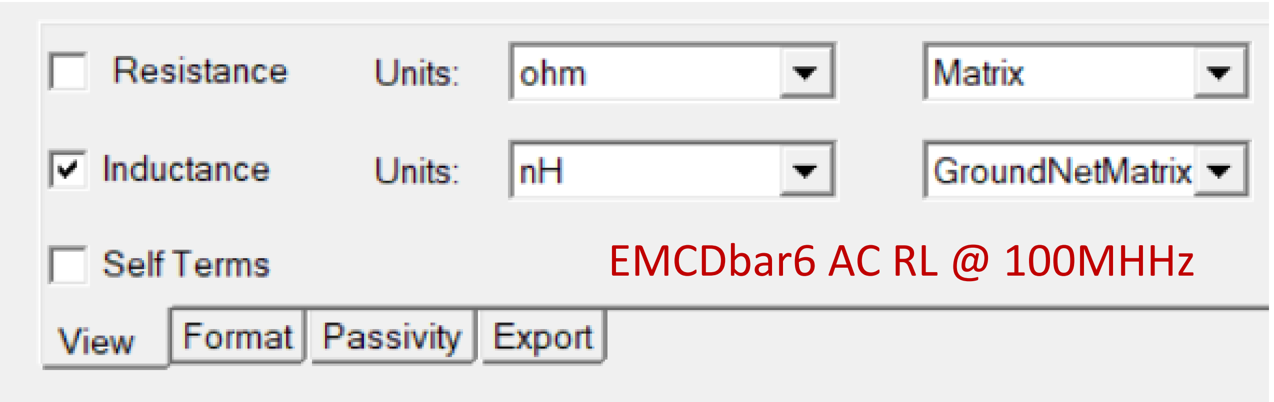


The effect on the inductance matrix, of grounding a conductor, is to zero the voltage across the conductor; current can still flow.

Ground Net… connects the sources and sinks of a net together; it grounds both ends of a net.

$$L=\begin{bmatrix} L_{11}-\frac{L_{12}^2}{L_{22}} & L_{13}-\frac{L_{12}L_{23}}{L_{22}} \\ L_{13}-\frac{L_{12}L_{23}}{L_{22}} & L_{33}-\frac{L_{23}^2}{L_{22}} \end{bmatrix}$$

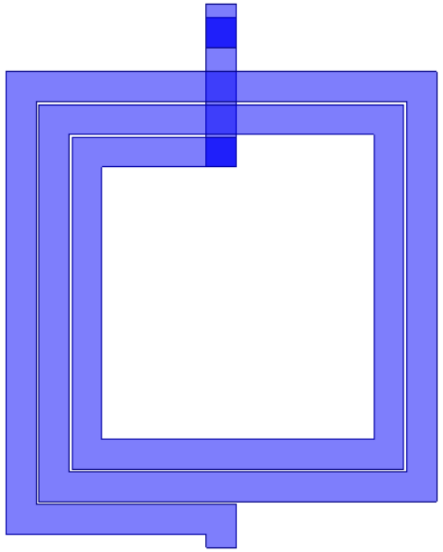
Notice how the four entries in the ***GroundNetMatrix*** are smaller than in the ***Original*** matrix. This matches the formula which subtracts a quantity from each term.



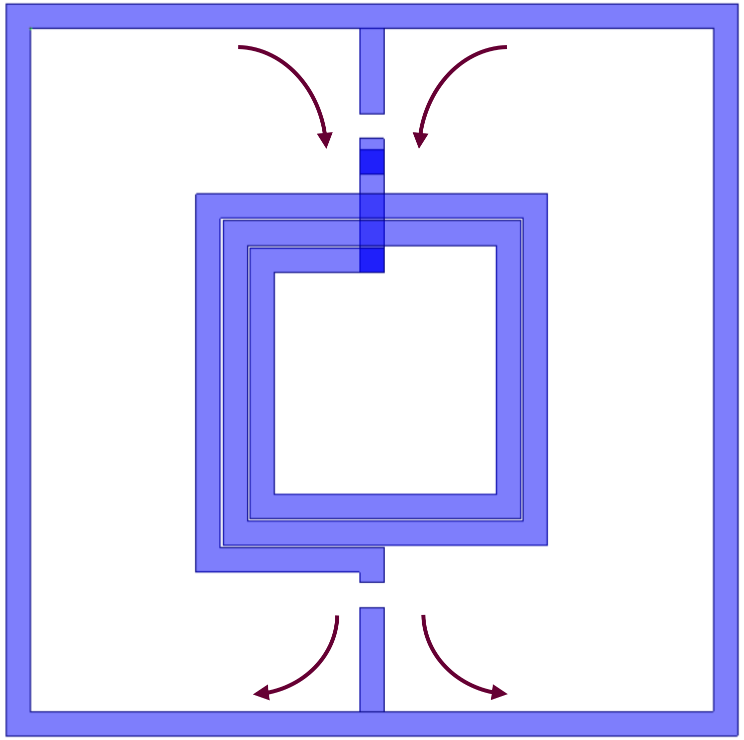
	Bar1:Source1	Bar3:Source3
Freq: 100MHz		
Bar1:Source1	22.72656	1.20618
Bar3:Source3	1.20618	9.09982

Partial Inductance and Loop Inductance

Inductance is defined, (and can be simulated), in terms of current loops. Q3D can calculate the partial inductance of an isolated section of current-carrying conductor. Many engineering challenges require consideration of the entire current and inductance loop. Q3D ***Matrix Reduction*** offers multiple ways to calculate current and inductance in complete or nearly complete loops.

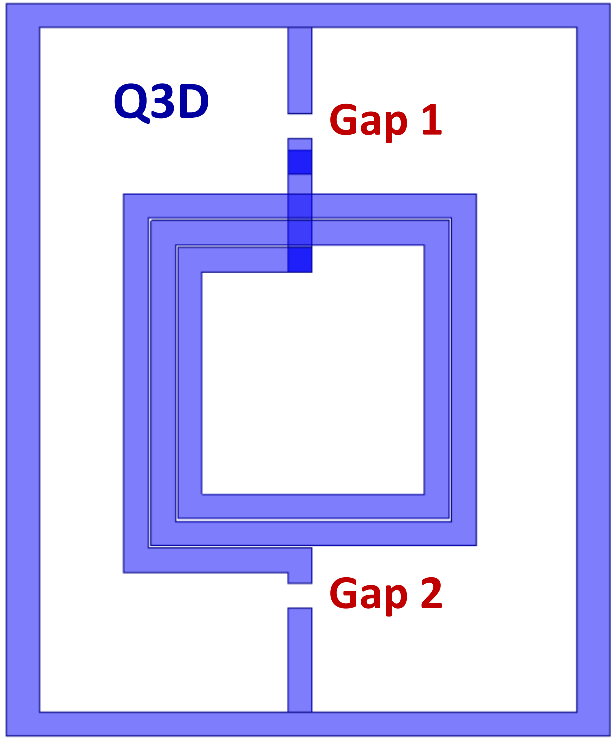


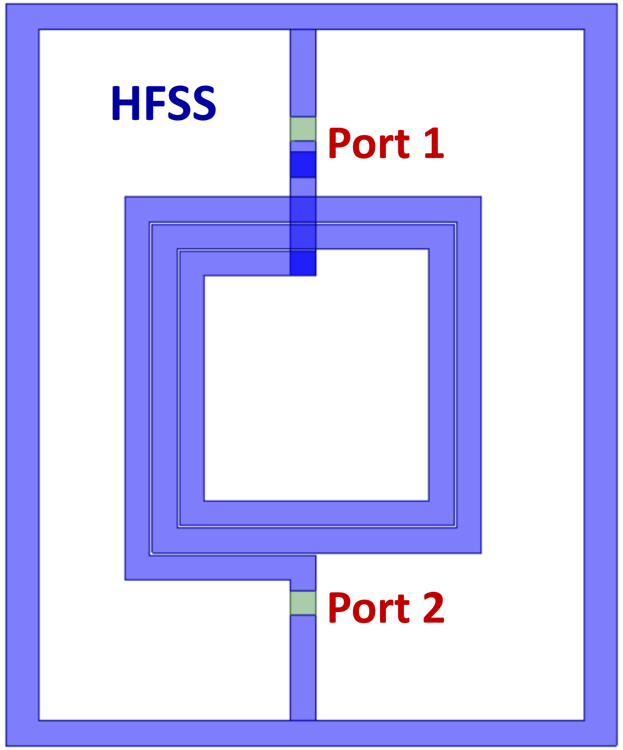
This isolated inductor has no current return path. Partial inductance does not include any distance or length that would be needed to complete a current loop.



Driven and measured by ports in the gaps, the inductance of these complete loops will take into account all of the space inside of the outer conductor loops.

Q3D Partial Inductance and HFSS Loop Inductance - Spiral



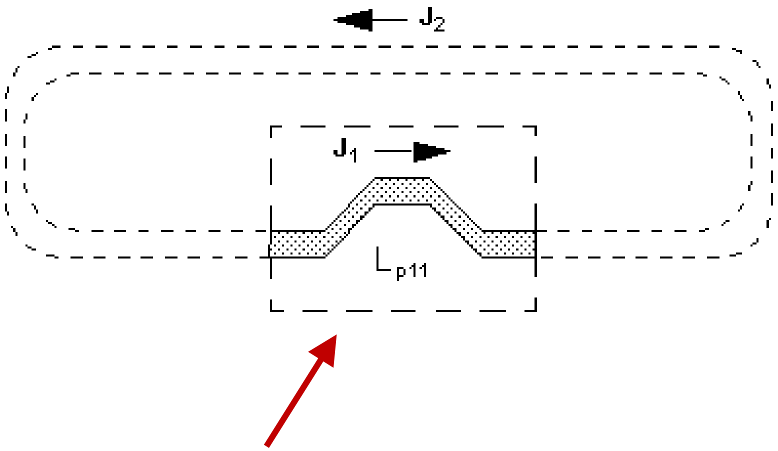


The Q3D setup of this spiral inductor places sources and sinks at the gaps. Q3D can simulate both nets, the spiral and the ring, separately. Q3D uses ***Matrix Reduction Join in Series…*** at one gap to calculate the complete loop inductance looking from the other gap.

In HFSS (both fully arbitrary 3D - FA3D - and 3D Layout) ports span the gap between the spiral geometry section and the ring, connecting the inductor geometry into complete loops.

Q3D Calculates Partial Inductance

- Inductance can be defined as partial inductance or loop inductance.
  - Loop inductance includes the entire area in the current loop.
  - Partial inductance includes only a small section of a conductor.
  - Loop inductance is what gets measured in a lab. (This is because real current always flows in a loop. You can't have an isolated section of current.)
  - Partial inductance represents the component of inductance that results only from the part of the current loop that is explicitly being modeled.
  - In multiconductor systems, both partial and loop inductance can have mutual inductance terms.
- Q3D Extractor calculates partial inductance
- Using ***Matrix Reduction***, to join sections of conductors together, Q3D offers several ways to calculate loop inductances. One of these ways is ***Join in Series…*** and another is ***Return Path.***



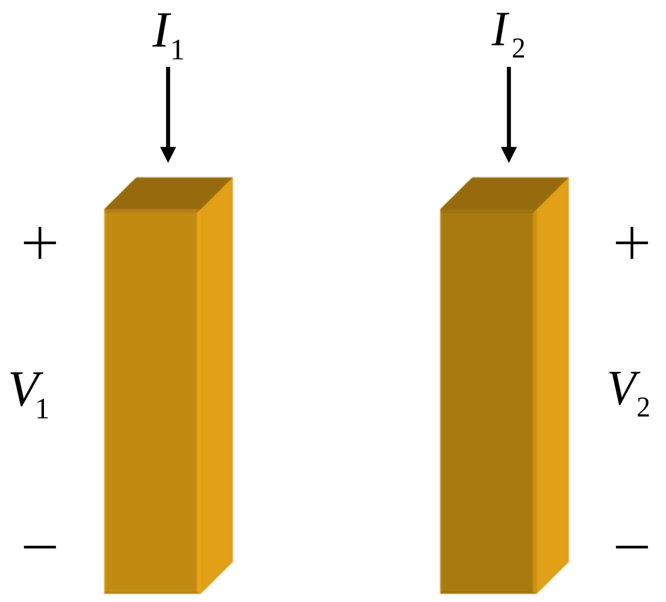
L_{p11} is the partial inductance of the short section of conductor of interest.

The dashed lines represent an idealized current loop.

Partial Inductance and Resistance Matrix

- Q3D computes "partial" RLC matrices.
- These matrices relate currents to voltage drops across <u>open</u> conductor segments

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{12} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + j\omega \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



These shapes represent two conductors close enough together to interact. The inductance matrix includes their self and mutual inductance terms.

In order to best model real structures and correlate with measurement, Q3D ***Matrix Reduction*** offers several ways to calculate the inductance of multiple conductors that can form complete current loops. ***Join in Series…*** is one way and ***Return Path…*** is another.

Ansys

Creating a Current Loop to Calculate Loop Inductance

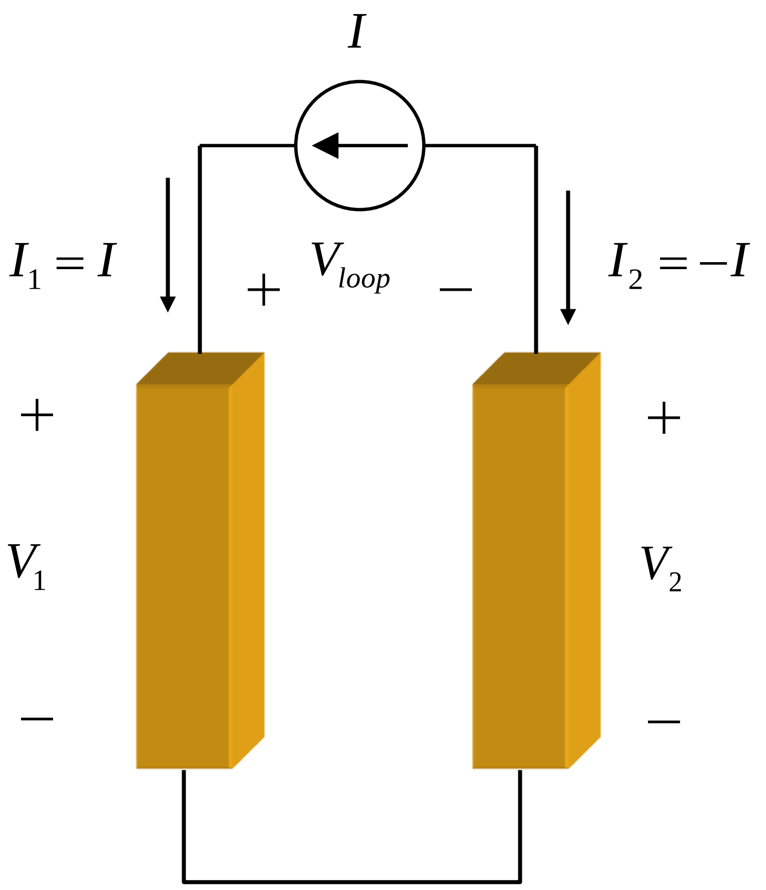
- In order to calculate a loop inductance, connect these two conductors in a current loop with a current source.
- Apply KVL and KCL:

$$
\begin{aligned}
V_{loop} &= V_1 - V_2 \\
&= (R_{11}I_1 + R_{12}I_2) + j\omega(L_{11}I_1 + L_{12}I_2) \\
&\quad - (R_{12}I_1 + R_{22}I_2) - j\omega(L_{12}I_1 + L_{22}I_2) \\
&= (R_{11} - 2R_{12} + R_{22})I + j\omega(L_{11} - 2L_{12} + L_{22})I
\end{aligned}
$$

Loop resistance **Loop inductance**

- The total loop inductance is the sum of the partial self and mutual inductances for each section of the loop.

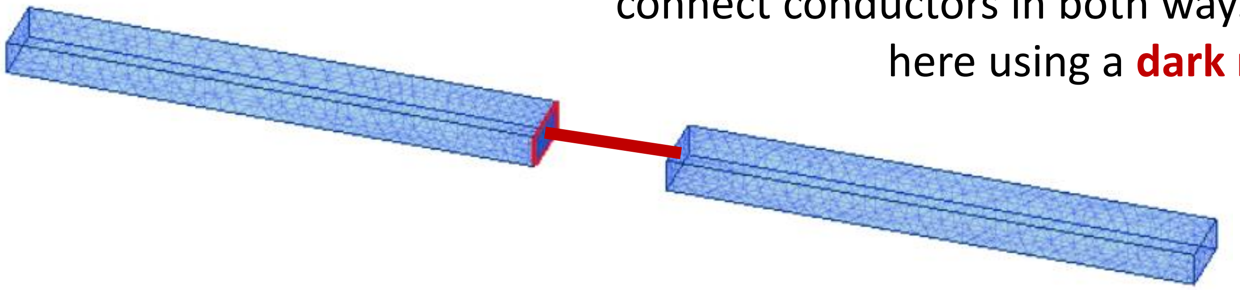
$L_{loop} = L_{11} - 2L_{12} + L_{22}$

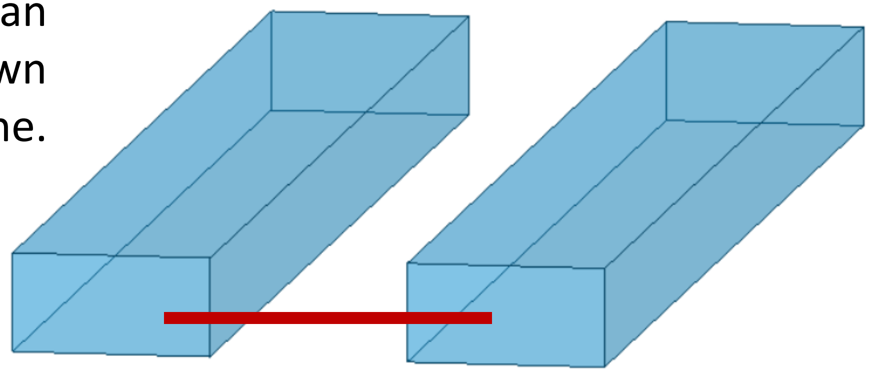


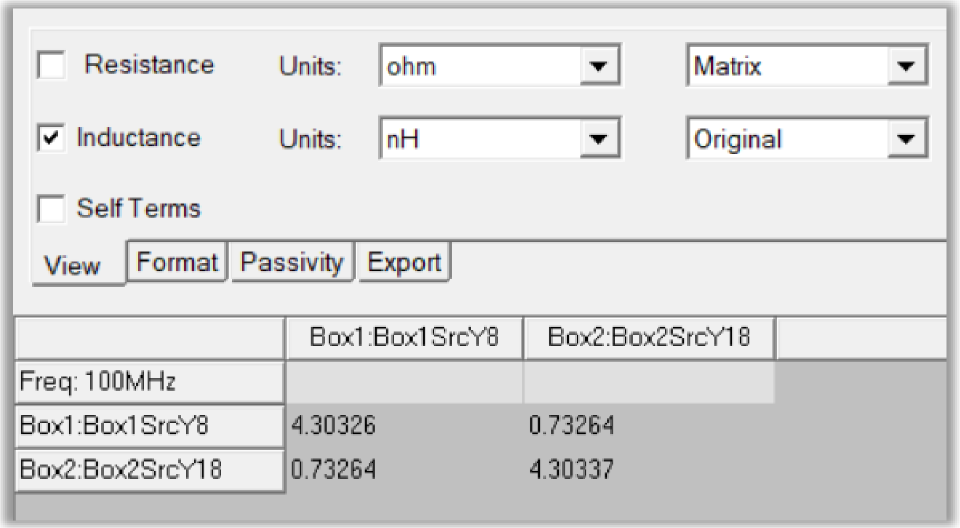
Ansys

Matrix Reduction Join in Series

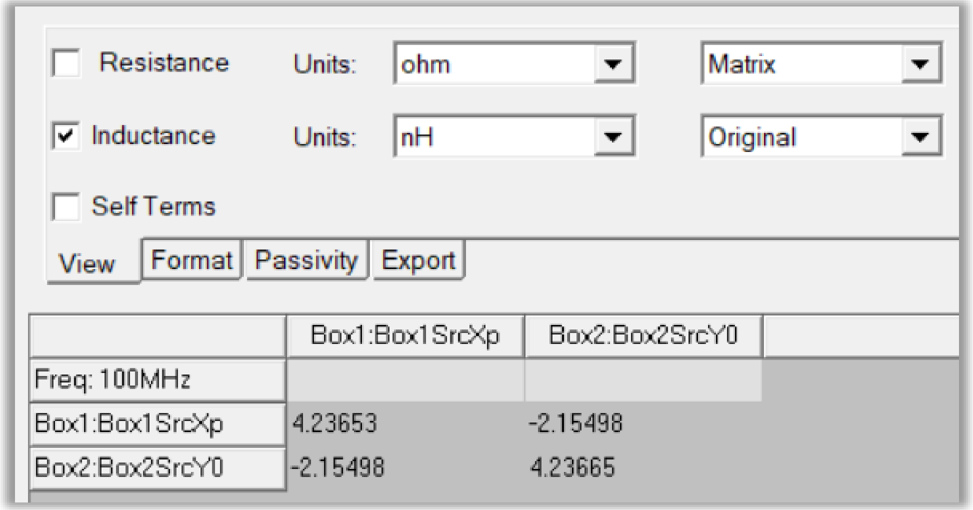
The ***Join in Series Matrix Reduction*** can connect conductors in both ways shown here using a **dark red** line.







	Box1:Box1SrcY8	Box2:Box2SrcY18
Freq: 100MHz		
Box1:Box1SrcY8	4.30326	0.73264
Box2:Box2SrcY18	0.73264	4.30337



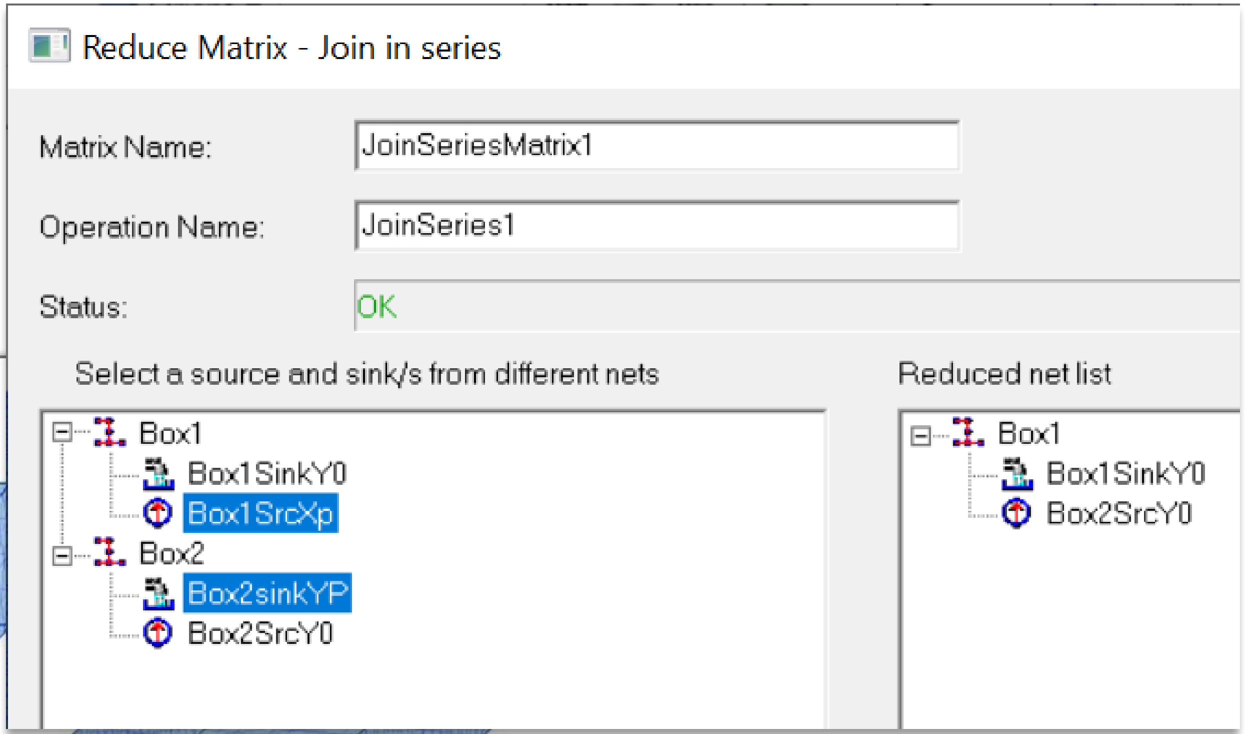
	Box1:Box1SrcXp	Box2:Box2SrcY0
Freq: 100MHz		
Box1:Box1SrcXp	4.23653	-2.15498
Box2:Box2SrcY0	-2.15498	4.23665

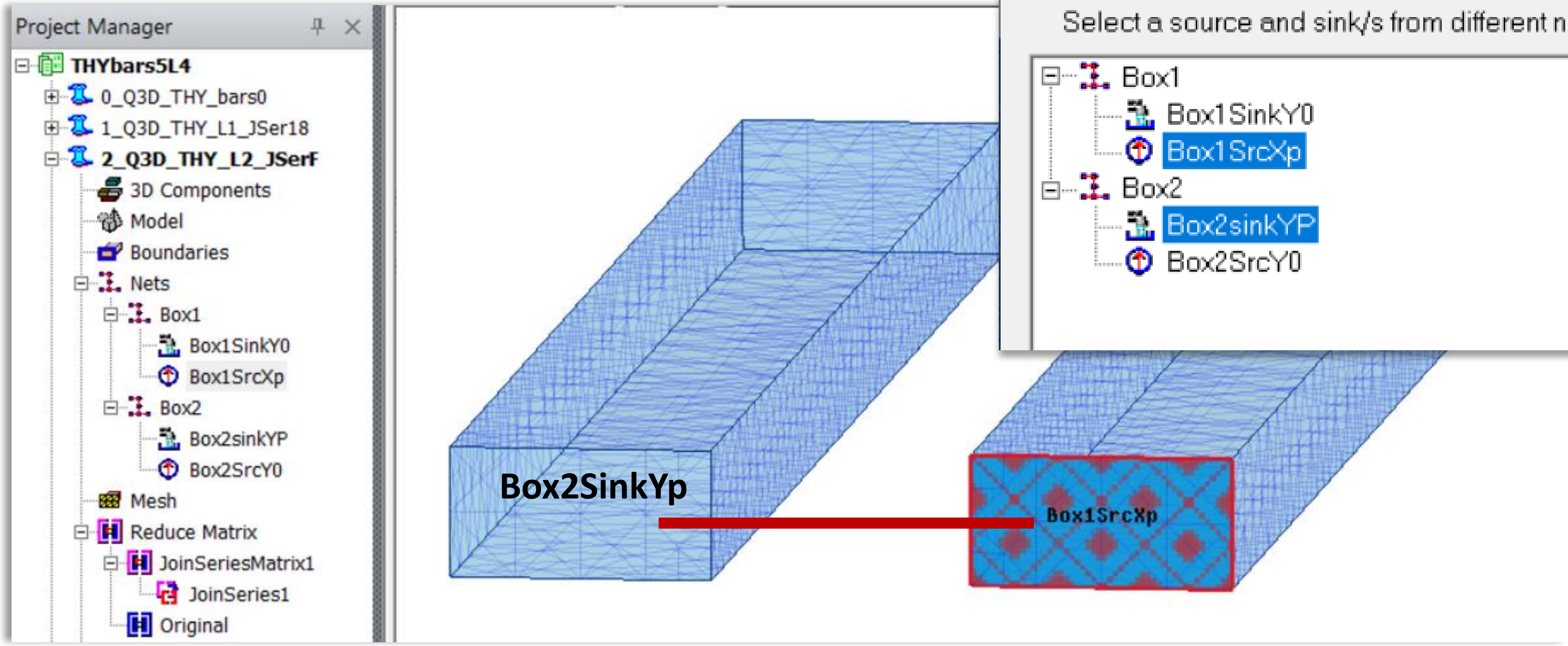
The ***Join in Series*** inductance value @100 MHz is **10.07 nH**

The ***Join in Series*** inductance value @100 MHz is **4.16 nH**

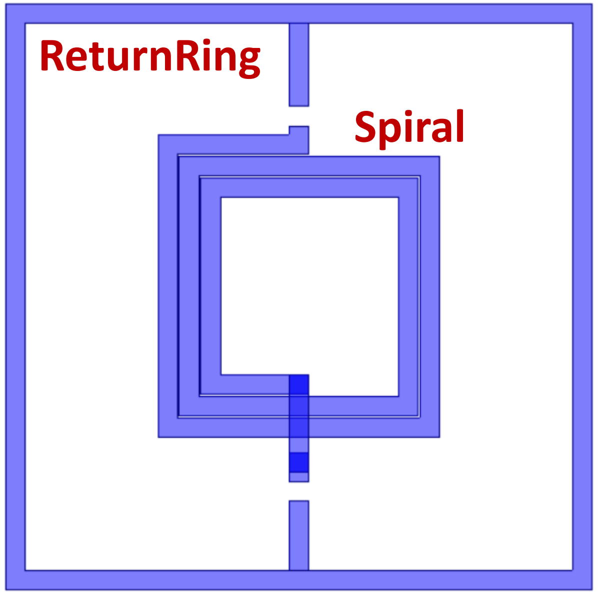
Join in Series Matrix Reduction Requires Terminals

Sources and ***Sinks*** specify the connection in ***Join in Series Matrix Reduction***. Here **Box1SrcXp** connects to **Box2sinkYP** as shown with the **dark red** line.

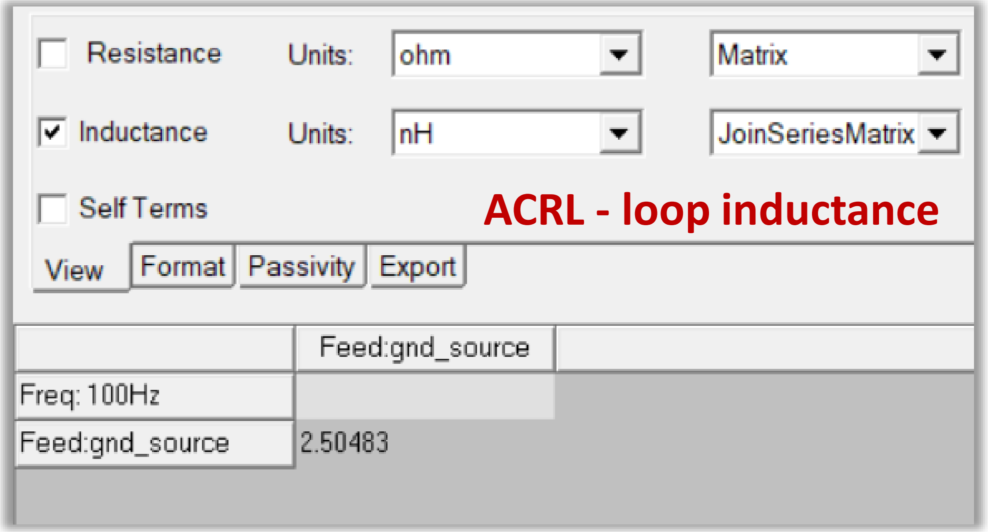


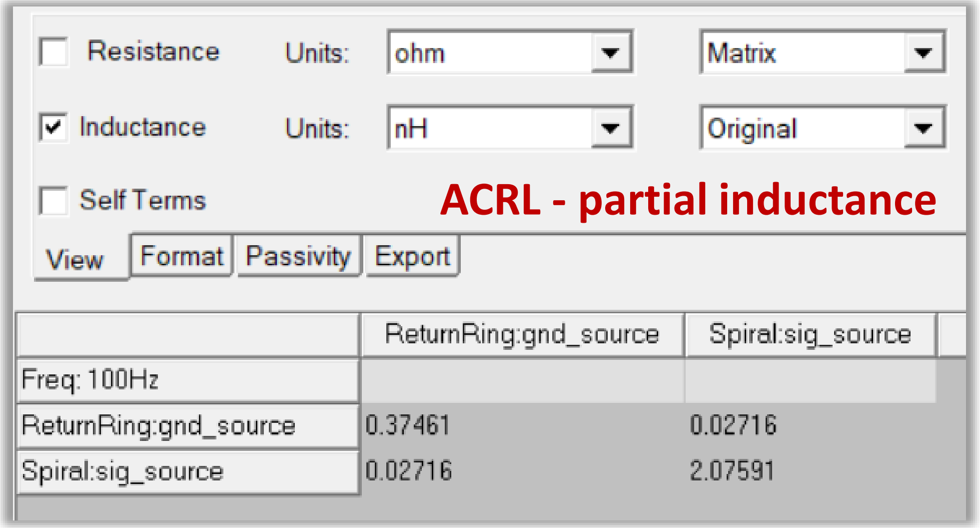


Q3D Partial and Loop Inductance with *Join in Series…*



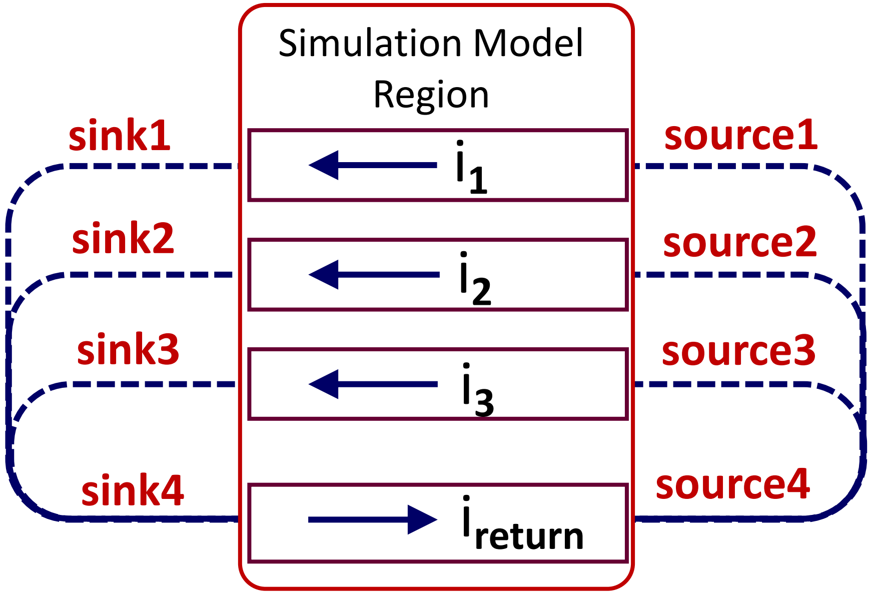
In the Q3D Solution Data for ACRL simulation at 100 Hz, the ***JoinSeriesMatrix***, from the ***Join in Series … Matrix Reduction***, includes the inductance of both the ground return path ***Net* ReturnRing** and the ***Net* Spiral**.

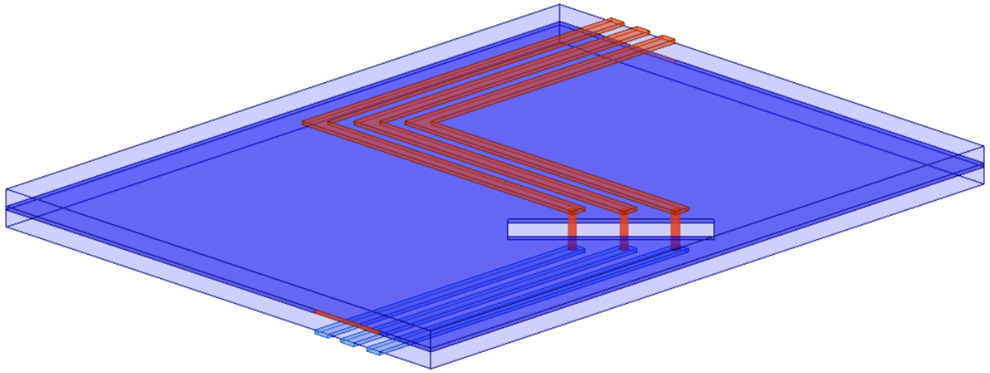




Q3D Calculates Return Path Using *Matrix Reduction*

- Conductors can be connected in series, using ***Matrix Reduction*** - ***JoinSeries***…, to create a current loop.
- A return current path may be chosen from any ***Net*** in the Q3D simulation model. The return current path might be the outer ring(s) of a spiral inductor.
- The return current path might be a ground plane below traces in a printed circuit board.
- Using ***Matrix Reduction*** - ***Return Path***… a ground plane can be specified as the return path for several different conductors.
- ***Matrix Reduction*** - ***Return Path***… allows us to specify a net to serve as a return path for current. We can then approximate the loop inductance of a closed conduction path. Specifying a return path does not simulate the effect of the full current loop because parts of the loop are not explicitly modeled.





Return Path Performs *Float at Infinity* Then *Ground Net*

This drawing and technical background come from:

Circuit Matrix Reduction Operations

by J. Eric Bracken.

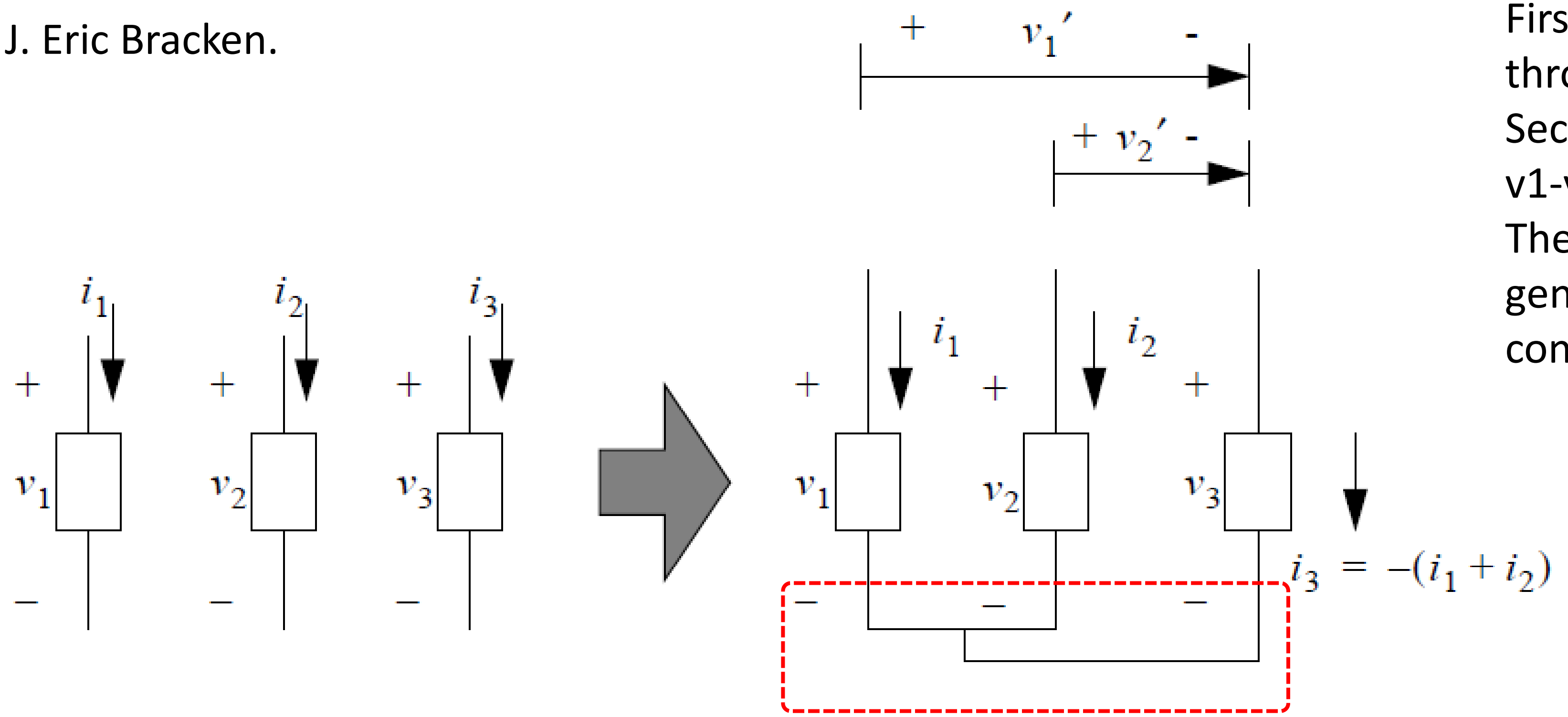
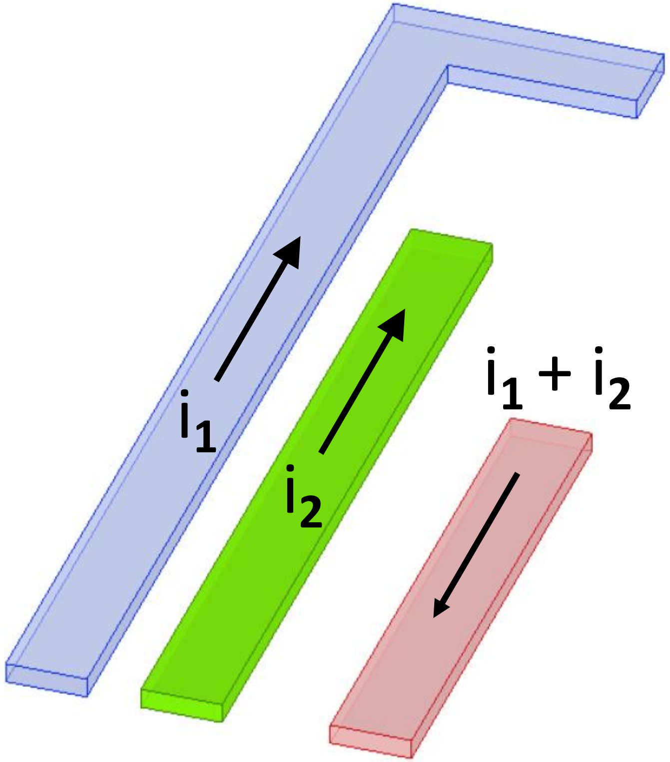


Figure 11 A return path reduction operation. In this case, conductor 3 is being taken as the return path for the other conductors. Notice that the negative reference node for defining the branch voltages has also been changed.

First, currents i1, i2 return through conductor 3.
Second, the loop voltage is v1-v3, v2-v3.
Then it is a straightforward generalization to multiple conductors.

Inductance *Matrix Reduction* - *Return Path*



I3 becomes -(i1+i2)

- First, currents i1, i2 return through conductor 3 ➔ set i3 = -i1-i2
- Second, the loop voltage is v1-v3, v2-v3➔ Phi (loop1) = Phi1-Phi3
- Phi (loop2) = Phi2-Phi3

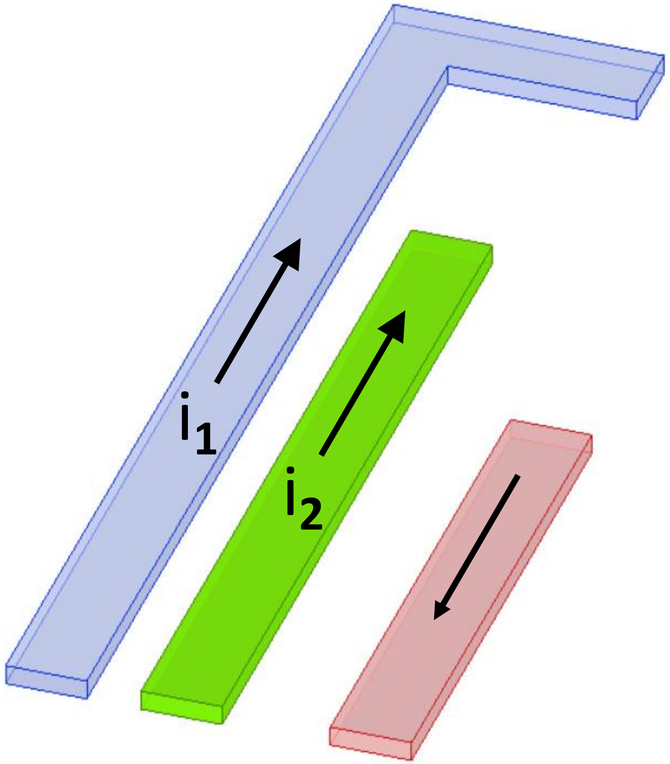
$$\varphi_1 = L_{11}i_1 + L_{12}i_2 + L_{13}i_3$$
$$\varphi_2 = L_{12}i_1 + L_{22}i_2 + L_{23}i_3$$
$$\varphi_3 = L_{13}i_1 + L_{23}i_2 + L_{33}i_3$$

- With KVL, KCL and some algebra will give this (see references)

$$L = \begin{bmatrix} L_{11} - 2L_{11} + L_{33} & L_{12} - L_{13} - L_{23} + L_{33} \\ L_{12} - L_{13} - L_{23} + L_{33} & L_{22} - 2L_{23} + L_{33} \end{bmatrix}$$

Return Path Net… where conductor 3 is chosen as the return path.

Inductance *Matrix Reduction* - *Return Path* versus *Ground Net*



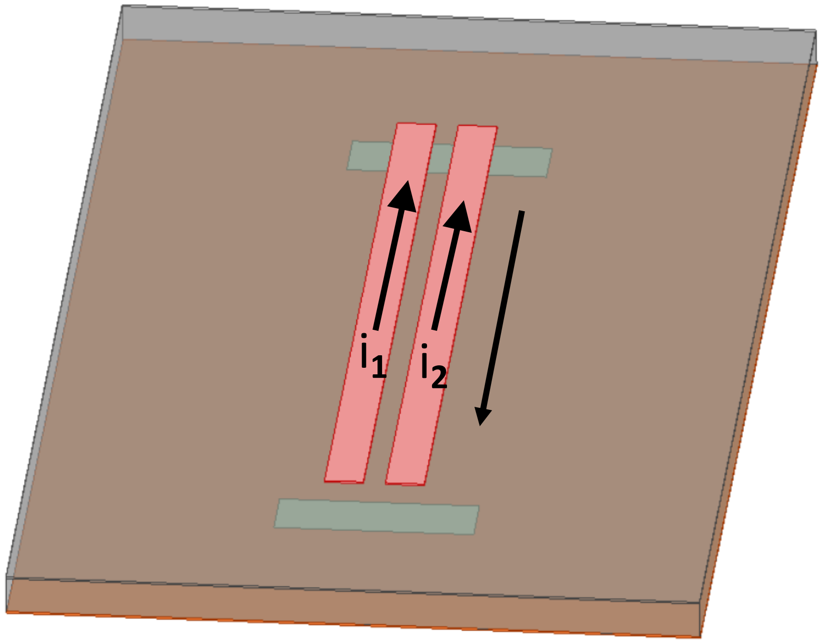
Ground Net… where conductor 3 is chosen as the ground net.

$$L=\begin{bmatrix} L_{11}-\frac{L_{13}^2}{L_{33}} & L_{13}-\frac{L_{13}L_{23}}{L_{33}} \\ L_{12}-\frac{L_{13}L_{23}}{L_{33}} & L_{22}-\frac{L_{23}^2}{L_{33}} \end{bmatrix}$$

Return Path … where conductor 3 is chosen as the return path.

$$L=\begin{bmatrix} L_{11}-2L_{13}+L_{33} & L_{12}-L_{13}-L_{23}+L_{33} \\ L_{12}-L_{13}-L_{23}+L_{33} & L_{22}-2L_{23}+L_{33} \end{bmatrix}$$

Large Ground Plane - *Return Path* versus *Ground Net*



Ground Net… where conductor 3 is chosen as the ground net.

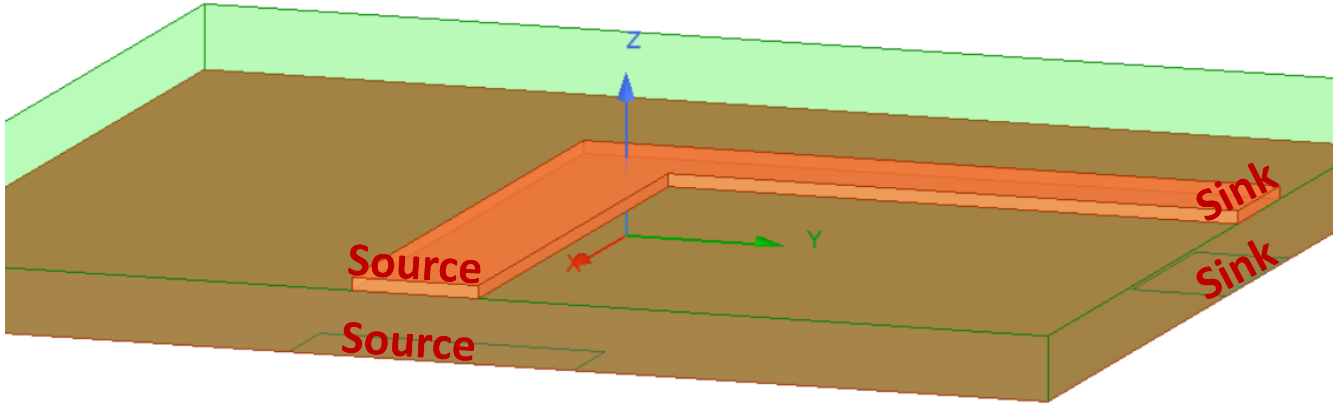
$$L=\begin{bmatrix} L_{11}-\frac{L_{13}^2}{L_{33}} & L_{13}-\frac{L_{13}L_{23}}{L_{33}} \\ L_{12}-\frac{L_{13}L_{23}}{L_{33}} & L_{22}-\frac{L_{23}^2}{L_{33}} \end{bmatrix}$$

Return Path … where conductor 3 is chosen as the return path.

$$L=\begin{bmatrix} L_{11}-2L_{13}+L_{33} & L_{12}-L_{13}-L_{23}+L_{33} \\ L_{12}-L_{13}-L_{23}+L_{33} & L_{22}-2L_{23}+L_{33} \end{bmatrix}$$

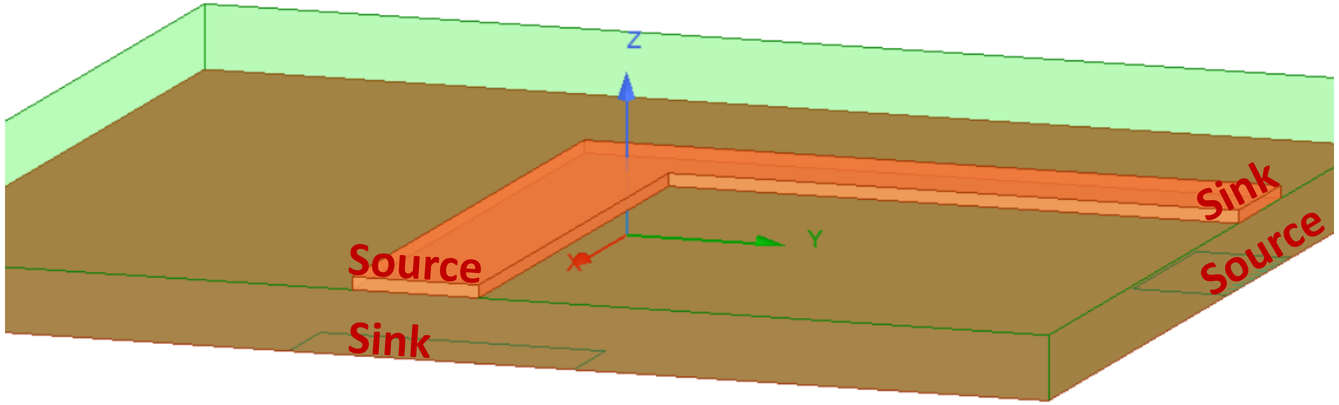
A trace above a large ground plane gives a similar result between ***Ground Net*** and ***Return Path***. The self-inductance of the ground (conductor 3) is equivalent to the mutual inductance between the ground and the trace. L13 ~=L33. (In both above matrices, the upper left terms reduce to L11 - L13.)

Loop Inductance with ***Matrix Reduction Return Path*** or ***Join in Series***



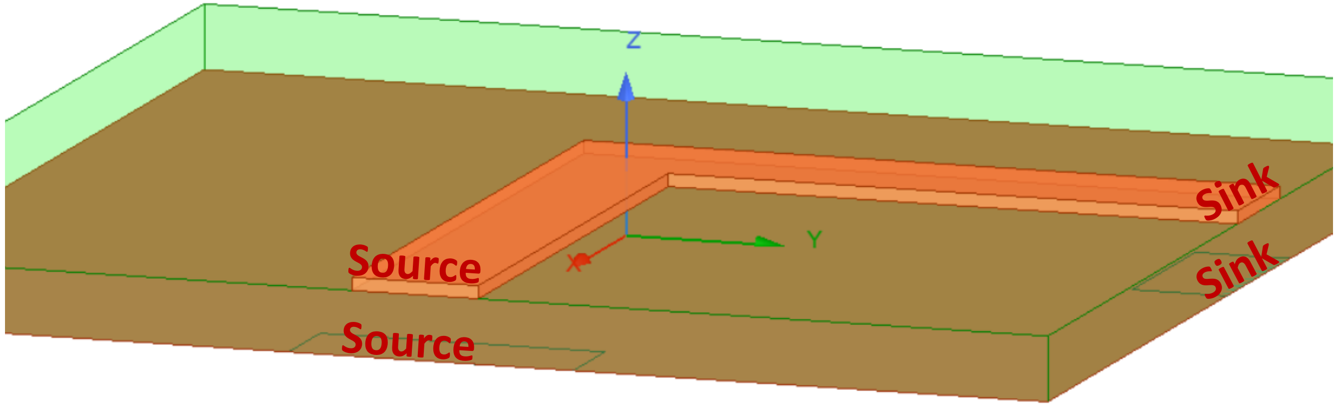
Q3D can simulate the microstrip signal trace net with a source on one end and a sink on the other end to define a current flow. Q3D simulates the partial inductance of this microstrip trace.

The bottom ground plane can be defined as a net and a return current path can be simulated with ***Matrix Reduction Return Path.*** This simulates a loop inductance.

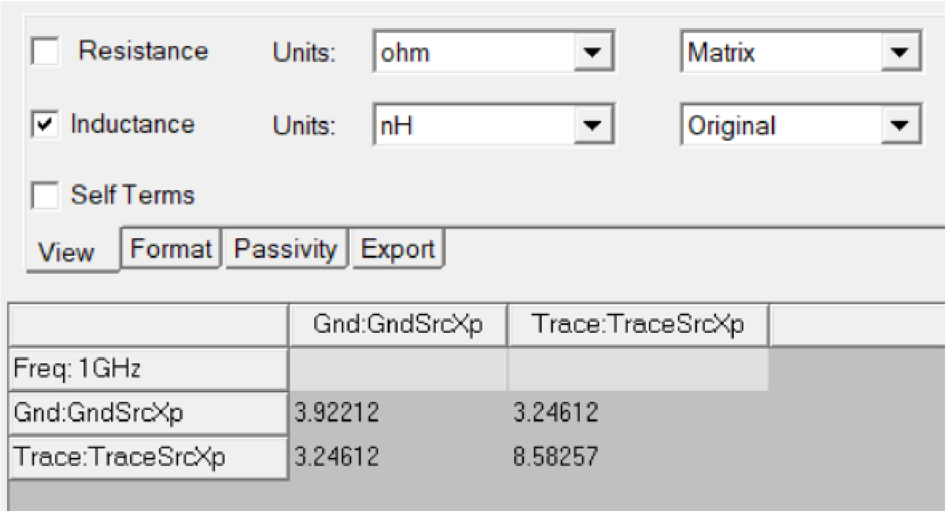


Alternatively, ***Matrix Reduction Join in Series*** can connect the trace and the ground on one side, calculating the loop inductance.

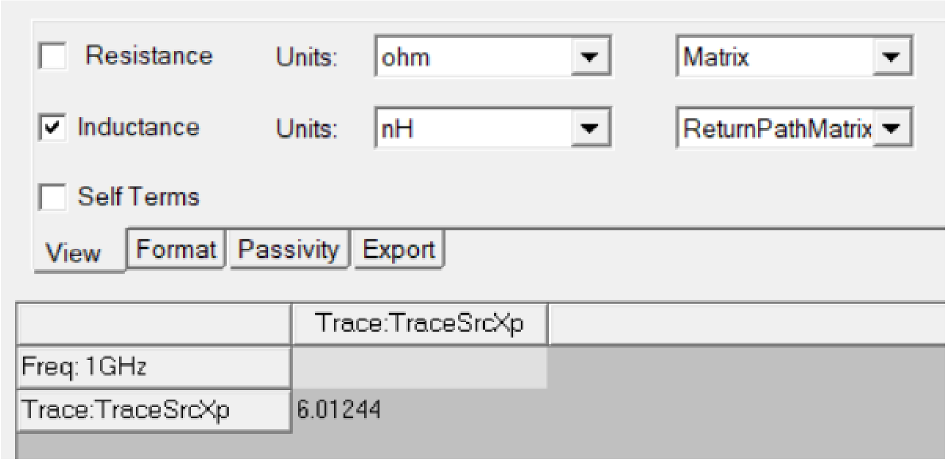
Inductance with *Matrix Reduction Return Path*



Matrix Reduction Return Path is applied to the ground plane, forcing the trace current exiting the ***Sink*** on the right into the ground plane ***Sink*** on the right side of the ground plane.

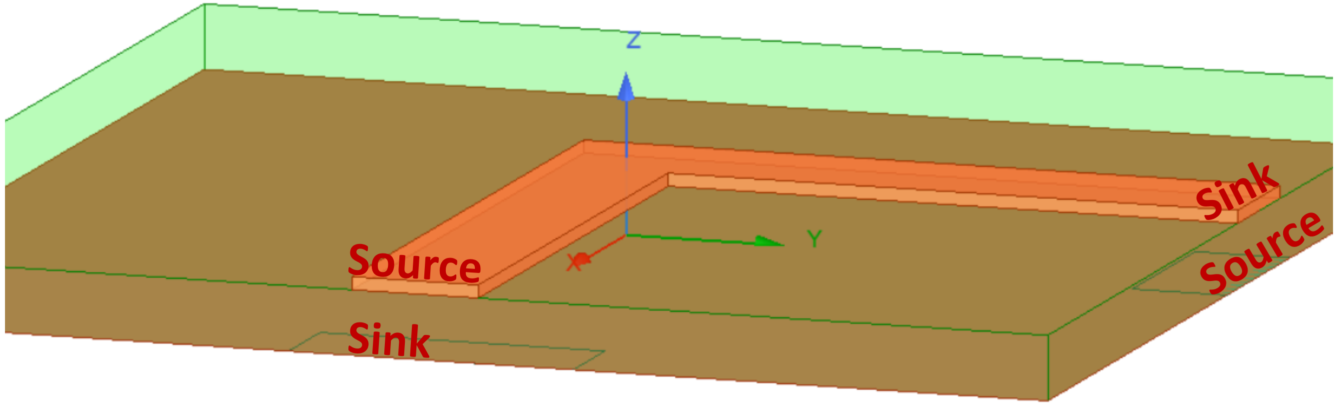


	Gnd:GndSrcXp	Trace:TraceSrcXp
Freq: 1GHz		
Gnd:GndSrcXp	3.92212	3.24612
Trace:TraceSrcXp	3.24612	8.58257

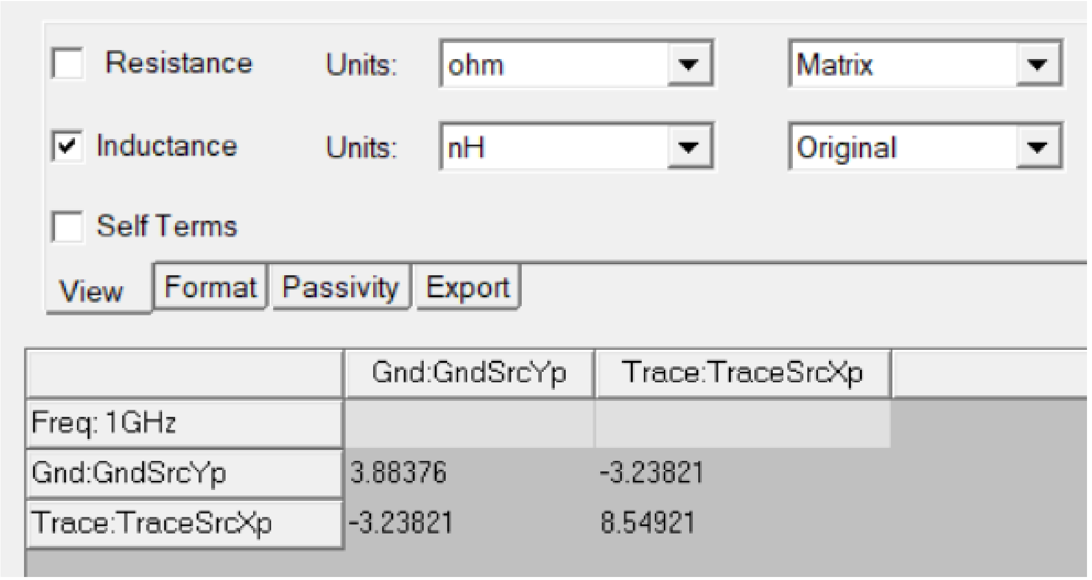


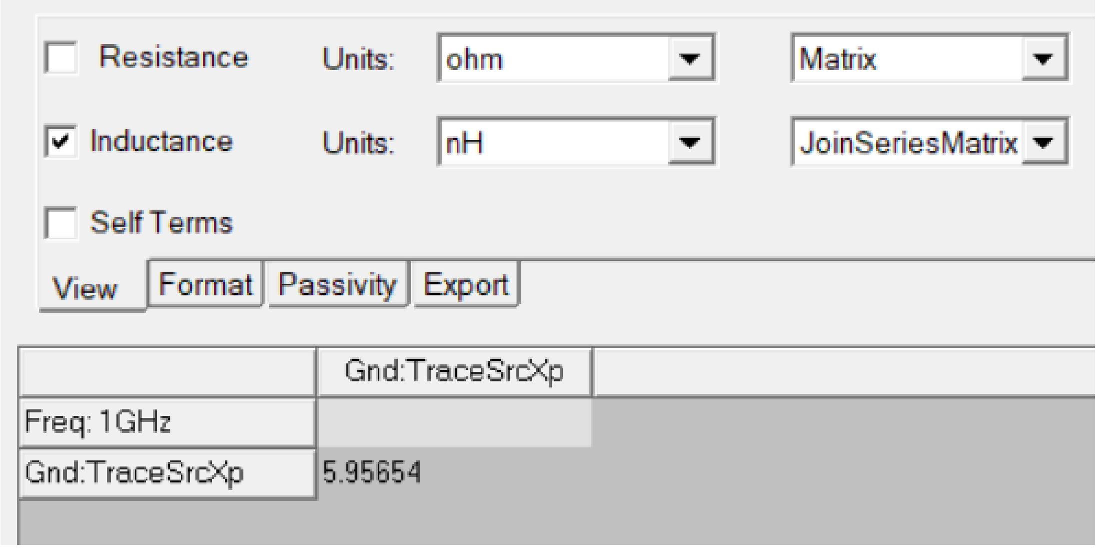
	Trace:TraceSrcXp
Freq: 1GHz	
Trace:TraceSrcXp	6.01244

Loop Inductance with ***Matrix Reduction Join in Series***



Matrix Reduction Join in Series connects the right-hand trace ***Sink*** to the right-hand ground ***Source***. The current direction on the ground plane is the opposite the current direction on the trace. The mutual inductance terms subtract from the self inductance terms to get the loop inductance value found with ***Join in Series*** on the right.





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End of Presentation