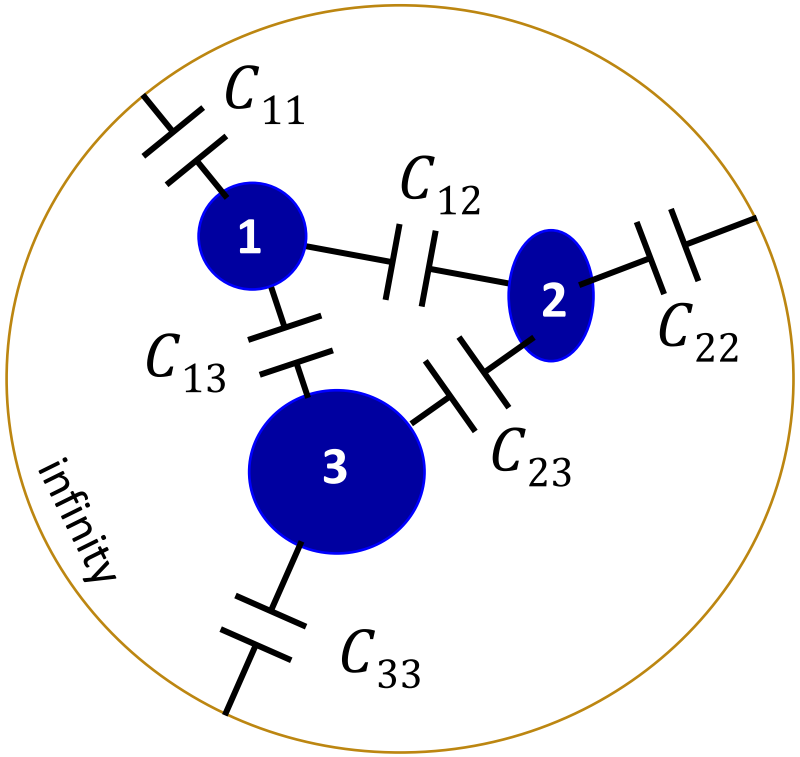
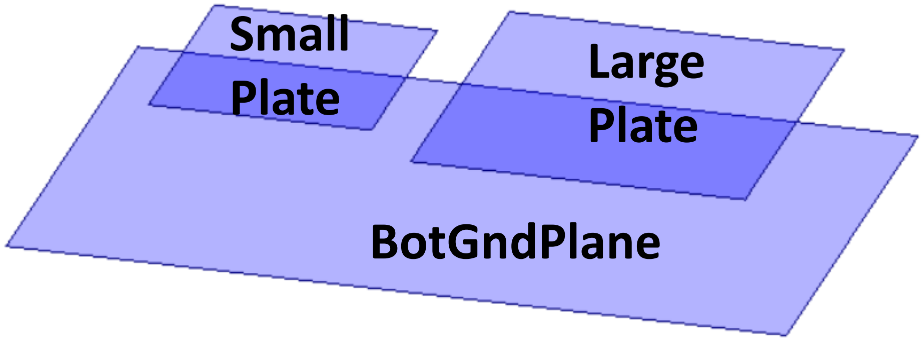
Module 4: Q3D Capacitance Matrix Reduction

Release 2020 R1

Maxwell Capacitance Matrix Reduction with 3-Conductors Example

The blue numbered ellipses are conducting objects in space.





This three-copper-plate Q3D example is used to simulate the various matrix reduction operations described here.

For additional discussion and reference, please see:

Circuit Matrix Reduction Operations
by J. Eric Bracken.

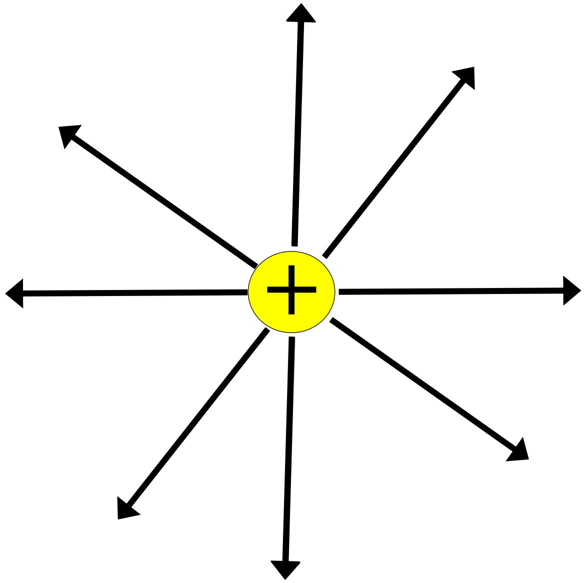
and

Matrix Reduction Operations In Q3D
By Greg Pitner

and the Q3D Extractor online Help

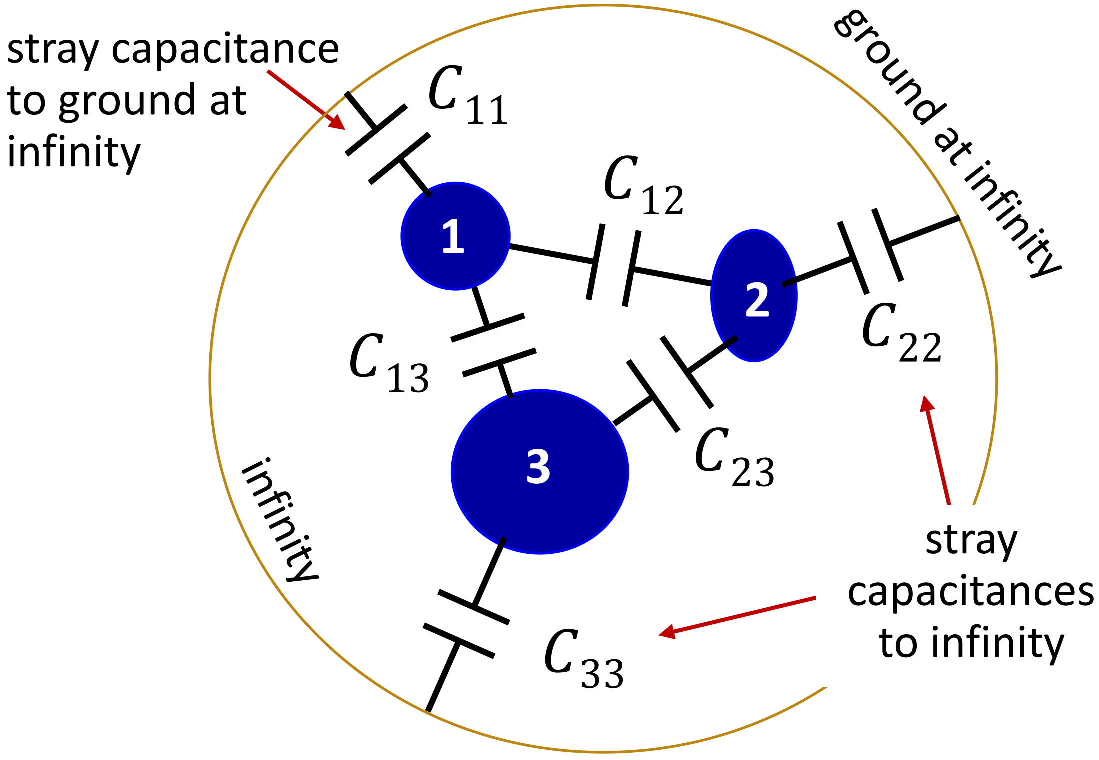
Ground at Infinity in Electrostatics

The notion of ground-to-infinity can be found in the physics treatment of field lines from an electrical charge.



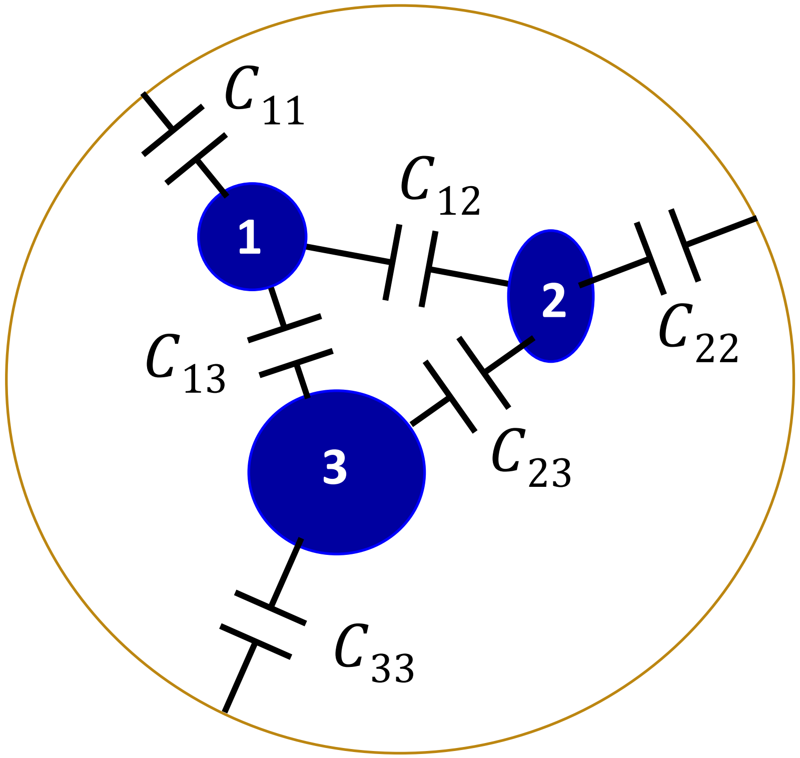
If the positive charge is isolated in space, one can consider the field lines to ground at infinity.

The blue numbered ellipses are conducting objects in space. The C_{NN} capacitors indicate stray capacitances to infinity, represented by the grey sphere.



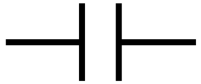
These blue numbered ellipses and the capacitors will be utilized in describing a capacitance matrix.

Three Conductors and Two-Terminal Capacitances



Charged conductors represented by their two-terminal mutual and self-capacitances. The outer circle represents ground at infinity.

$$Q = Cv$$



The charge on any one of the nearby conductors depends on the voltages on all the conductors. Using the equation $Q = Cv$, the total charge Q_k on the k^{th} conductor, with voltage v_k, can be expressed with two-terminal capacitances:

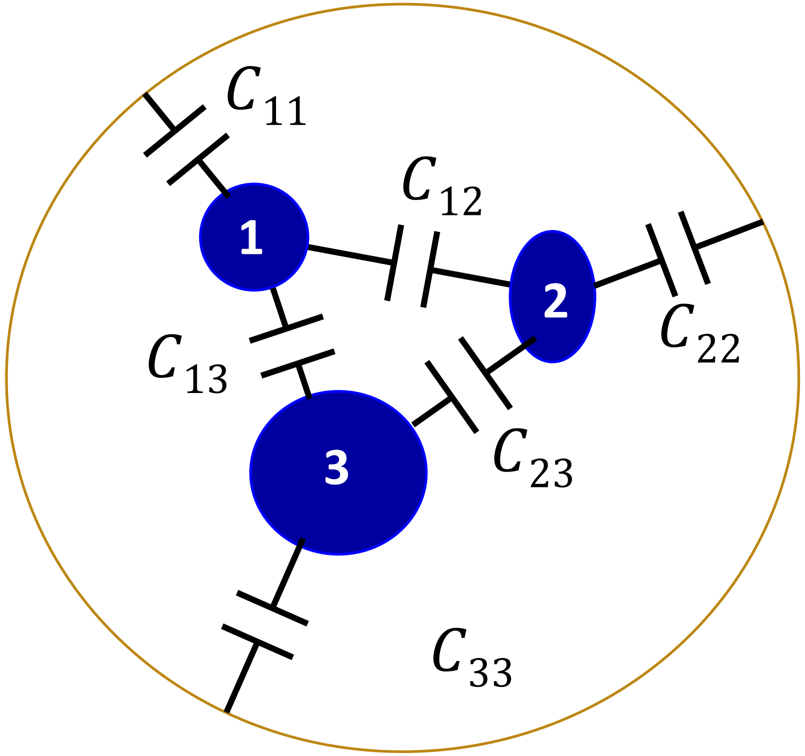
$$Q_1 = C_{11}v_1 + C_{12}(v_1 - v_2) + C_{13}(v_1 - v_3)$$

$$Q_2 = C_{12}(v_2 - v_1) + C_{22}v_2 + C_{23}(v_2 - v_3)$$

$$Q_3 = C_{13}(v_3 - v_1) + C_{23}(v_3 - v_2) + C_{33}v_3$$

The voltage across a given capacitor is the difference in voltages between two conductors. Each conductor has one capacitor connected to the ground at infinity; the voltage across that capacitor is the voltage of the conductor.

Rearrange Mutual and Self-Capacitance Terms by Voltage



Charged conductors represented by their two-terminal mutual capacitances. The outer circle represents ground at infinity.

$$Q_1 = C_{11}v_1 + C_{12}(v_1 - v_2) + C_{13}(v_1 - v_3)$$

$$Q_2 = C_{12}(v_2 - v_1) + C_{22}v_2 + C_{23}(v_2 - v_3)$$

$$Q_3 = C_{13}(v_3 - v_1) + C_{23}(v_3 - v_2) + C_{33}v_3$$

The equations for charge in terms of two-terminal mutual capacitance can be rearranged to group like voltages:

$$Q_1 = (C_{11} + C_{12} + C_{13})v_1 - C_{12}v_2 - C_{13}v_3$$

$$Q_2 = -C_{12}v_1 + (C_{21} + C_{22} + C_{23})v_2 - C_{23}v_3$$

$$Q_3 = -C_{13}v_1 - C_{23}v_2 + (C_{13} + C_{23} + C_{33})v_3$$

The Maxwell Capacitance Matrix

This new arrangement of the capacitance terms leads to the Maxwell capacitance matrix.

$$Q_1 = (C_{11} + C_{12} + C_{13})v_1 - C_{12}v_2 - C_{13}v_3$$

$$Q_2 = -C_{12}v_1 + (C_{12} + C_{22} + C_{23})v_2 - C_{23}v_3$$

$$Q_3 = -C_{13}v_1 - C_{23}v_2 + (C_{13} + C_{23} + C_{33})v_3$$

$$\begin{bmatrix} C_{11}^M & C_{12}^M & C_{13}^M \\ C_{12}^M & C_{22}^M & C_{23}^M \\ C_{13}^M & C_{23}^M & C_{33}^M \end{bmatrix} = \begin{bmatrix} C_{11} + C_{12} + C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{12} + C_{22} + C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{13} + C_{23} + C_{33} \end{bmatrix}$$

Maxwell capacitance matrix

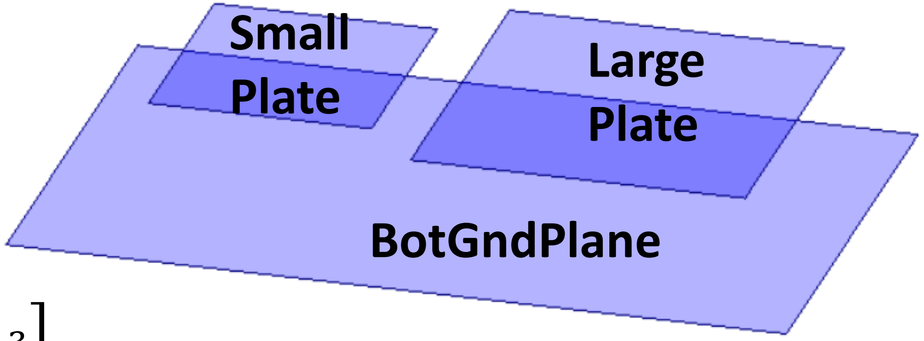
The Maxwell capacitance matrix describes the relationship between the total charge on each conductor and the conductor voltages (measured with respect to the ground at infinity.)

These are expressions for the Maxwell self and mutual capacitances in terms of the two-terminal capacitances. Two-terminal capacitances are used in circuit simulations.

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} C_{11}^M & C_{12}^M & C_{13}^M \\ C_{12}^M & C_{22}^M & C_{23}^M \\ C_{13}^M & C_{23}^M & C_{33}^M \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

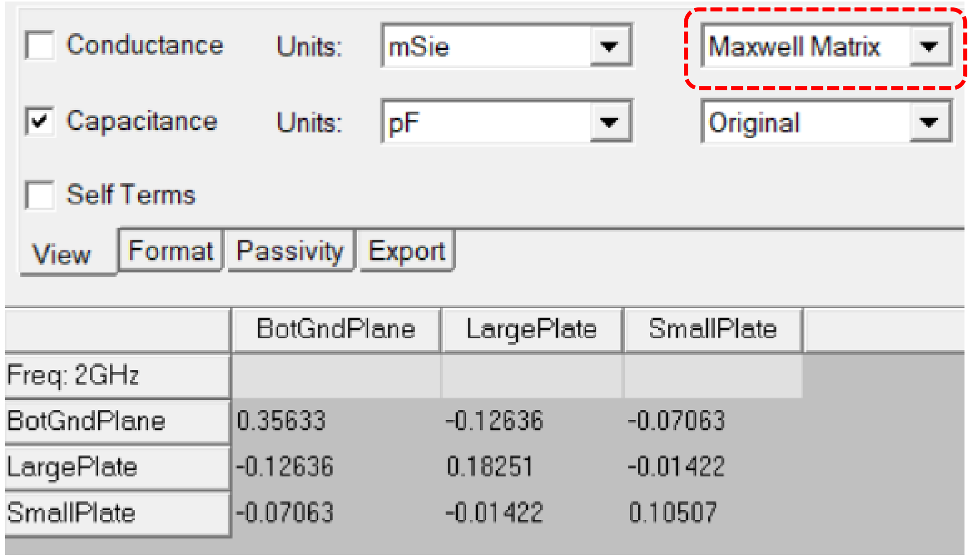
Maxwell and SPICE Capacitance Matrices

Q3D can display both the Maxwell capacitance matrix and a SPICE capacitance matrix. The SPICE capacitance matrix is the matrix of two-terminal mutual and self capacitors.

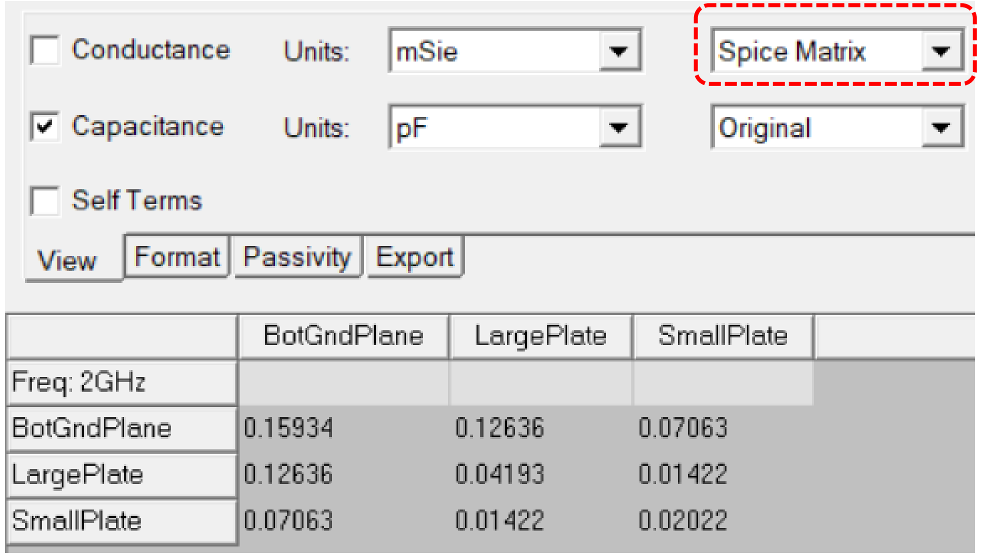


$$\begin{bmatrix} C_{11}^M & C_{12}^M & C_{13}^M \\ C_{12}^M & C_{22}^M & C_{23}^M \\ C_{13}^M & C_{23}^M & C_{33}^M \end{bmatrix}$$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$



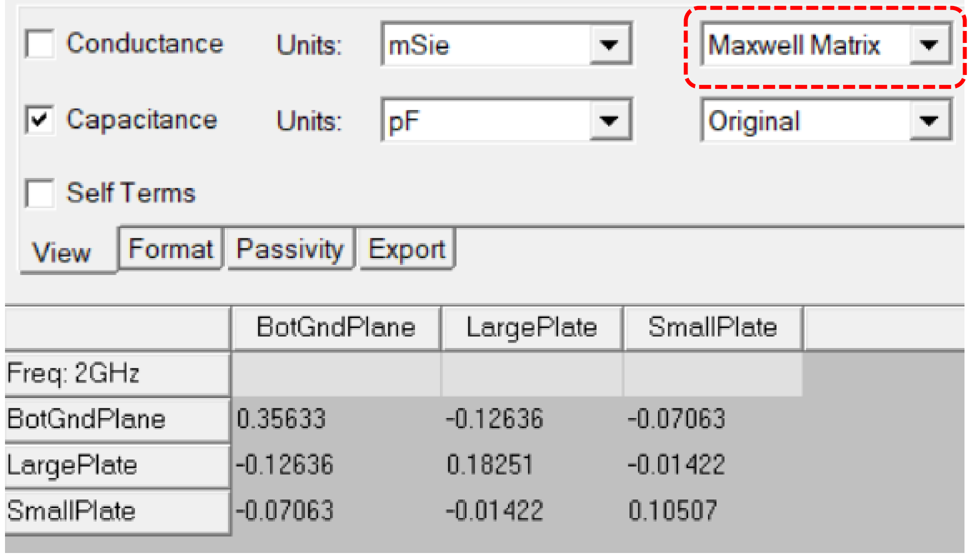
	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.35633	-0.12636	-0.07063
LargePlate	-0.12636	0.18251	-0.01422
SmallPlate	-0.07063	-0.01422	0.10507



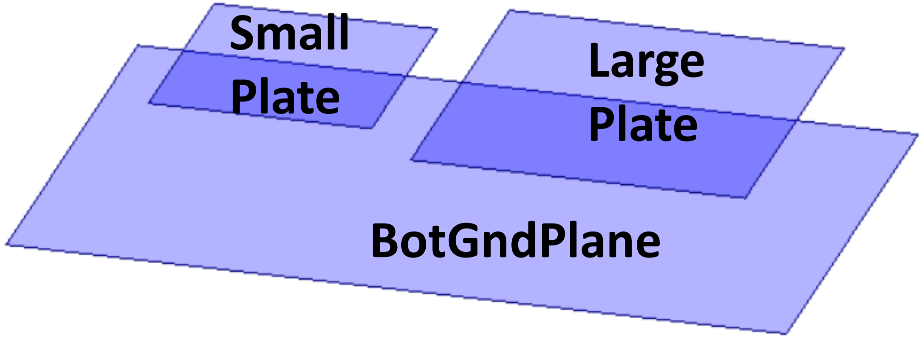
	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.15934	0.12636	0.07063
LargePlate	0.12636	0.04193	0.01422
SmallPlate	0.07063	0.01422	0.02022

Maxwell Capacitance Matrix Column Sums

In a given column of the Maxwell capacitance matrix, the terms sum to the C_{NN} term, minus the stray capacitance to infinity.



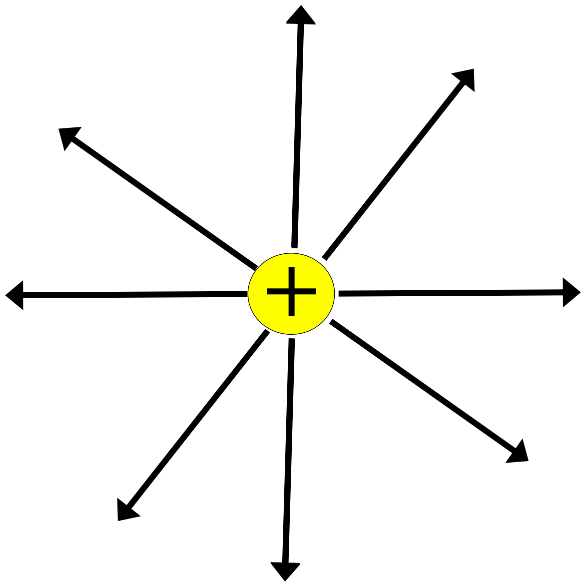
	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.35633	-0.12636	-0.07063
LargePlate	-0.12636	0.18251	-0.01422
SmallPlate	-0.07063	-0.01422	0.10507



The capacitance matrix default is ground to infinity.

$$\begin{bmatrix} C_{11}^M & C_{12}^M & C_{13}^M \\ C_{12}^M & C_{22}^M & C_{23}^M \\ C_{13}^M & C_{23}^M & C_{33}^M \end{bmatrix} = \begin{bmatrix} C_{11} + C_{12} + C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{12} + C_{22} + C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{13} + C_{23} + C_{33} \end{bmatrix}$$

Ground at Infinity in Q3D Online Help - ***Float at Infinity***



If the positive charge is isolated in space, one can consider the field lines to ground at infinity.

Formulation of the Capacitance/Conductance Solution

The electrostatic potential ϕ produced by a distribution of charges ρ on a surface S is given by the integral equation

(1)

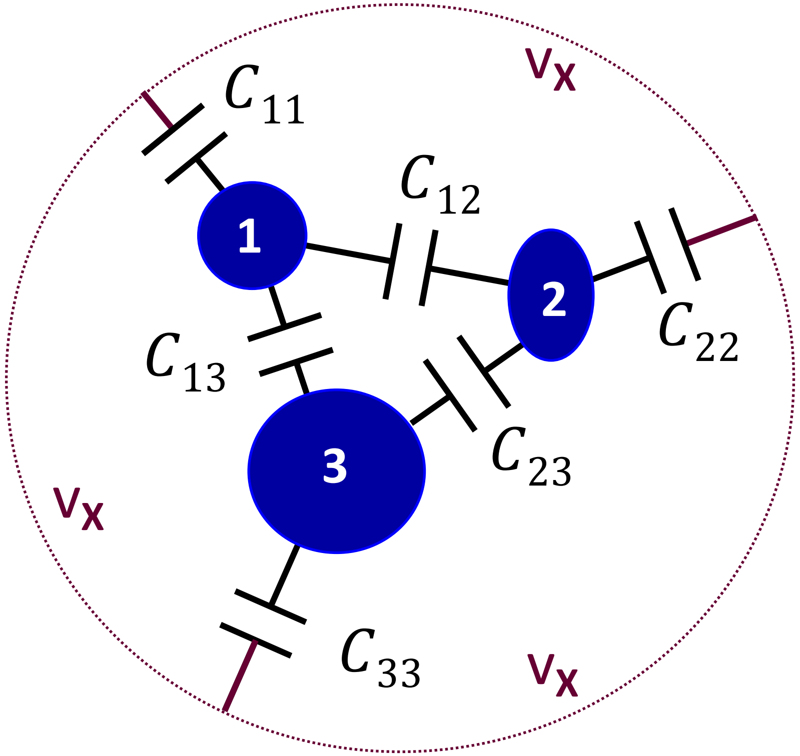
$$\phi(\vec{x}) = \int_S \frac{\rho(\vec{x}')}{4\pi\varepsilon\|\vec{x}-\vec{x}'\|} dS$$

Here the operator $\|\cdot\|$ above denotes the Euclidean length of a vector. The above equation is based upon an implicit assumption that the potential will go to zero as the distance between the source and observation points goes to infinity. This assumption can be relaxed by applying the **Floating at Infinity** matrix reduction operation as a post-processing step.

The assumption of ground-to-infinity is discussed in the Q3D Online Help, under ***Q3D Extractor Technical Notes***, in the section on the ***Capacitance/Conductance*** (CG) ***Solution Process.***

Ansys

Maxwell Capacitance - *Float at Infinity*



The node x voltage V_X is determined by the charge.

Float at Infinity disconnects the circle from ground at infinity; the C_{NN} capacitors still connect to one another at the sphere with unknown voltage V_X. The matrix is still the same dimension.

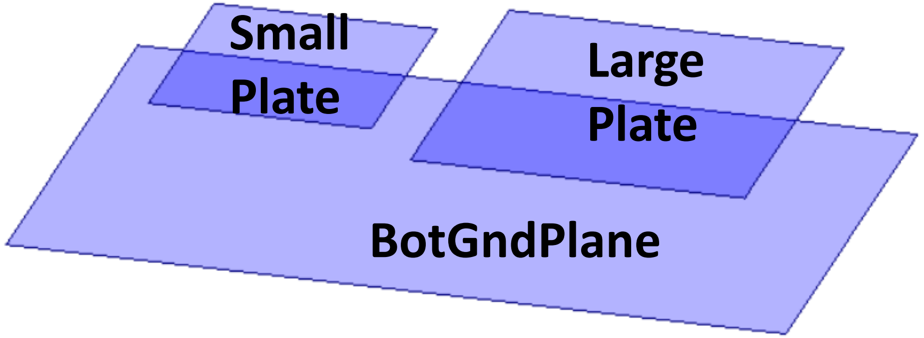
$$\begin{bmatrix} C_{11}^M & C_{12}^M & C_{13}^M \\ C_{12}^M & C_{22}^M & C_{23}^M \\ C_{13}^M & C_{23}^M & C_{33}^M \end{bmatrix} = \begin{bmatrix} C_{11}+C_{12}+C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{12}+C_{22}+C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{13}+C_{23}+C_{33} \end{bmatrix}$$

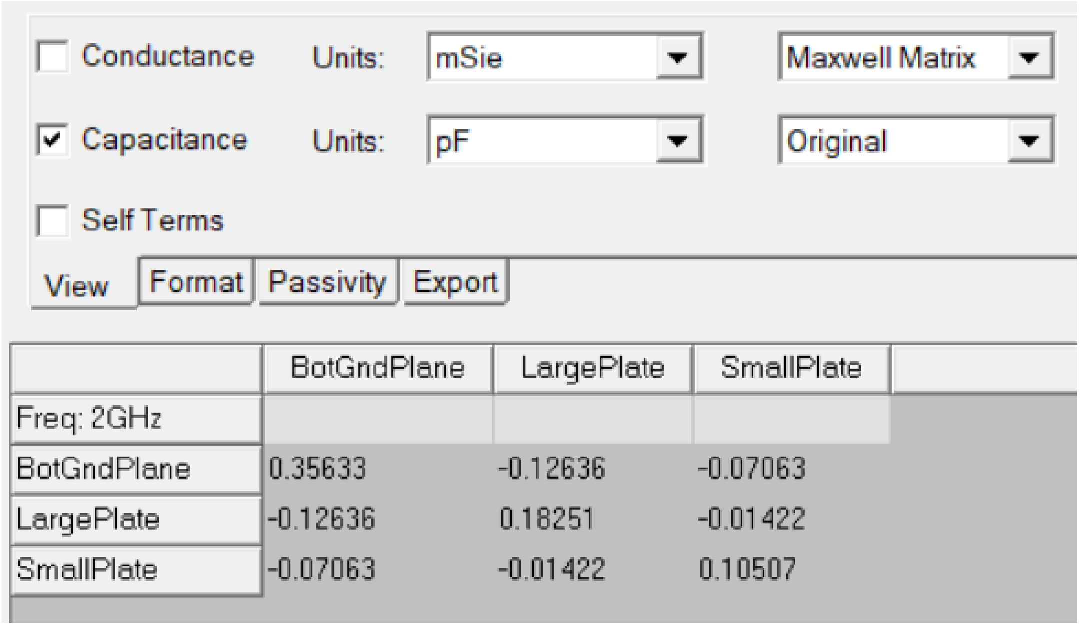
$$\begin{bmatrix} \frac{C_{11}(C_{22}+C_{33})}{C_{11}+C_{22}+C_{33}}+C_{12}+C_{13} & -\left(C_{12}+\frac{C_{11}C_{22}}{C_{11}+C_{22}+C_{33}}\right) & -\left(C_{13}+\frac{C_{11}C_{33}}{C_{11}+C_{22}+C_{33}}\right) \\ -\left(C_{12}+\frac{C_{11}C_{22}}{C_{11}+C_{22}+C_{33}}\right) & \frac{C_{22}(C_{11}+C_{33})}{C_{11}+C_{22}+C_{33}}+C_{12}+C_{23} & -\left(C_{23}+\frac{C_{22}C_{33}}{C_{11}+C_{22}+C_{33}}\right) \\ -\left(C_{13}+\frac{C_{11}C_{33}}{C_{11}+C_{22}+C_{33}}\right) & -\left(C_{23}+\frac{C_{22}C_{33}}{C_{11}+C_{22}+C_{33}}\right) & \frac{C_{33}(C_{11}+C_{22})}{C_{11}+C_{22}+C_{33}}+C_{13}+C_{23} \end{bmatrix}$$

In columns of the Maxwell capacitance matrix, with ***Float at Infinity Matrix Reduction*** operation applied, the non-diagonal terms sum to the diagonal term within a margin of error.

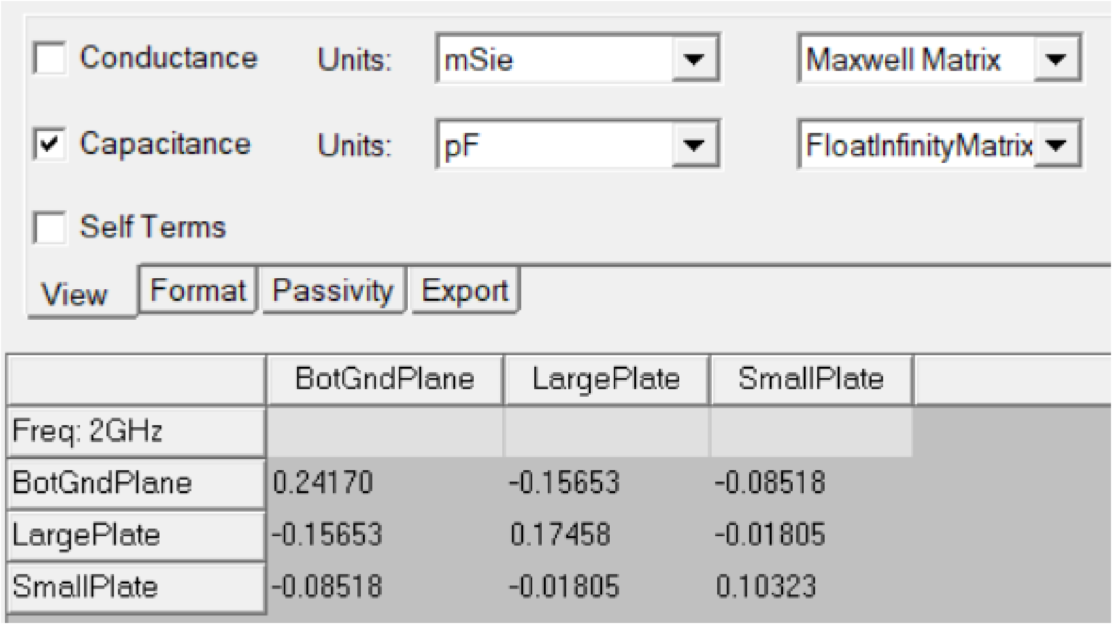
Maxwell Capacitance - *Float at Infinity*

Float at Infinity disconnects the ground at infinity; the C_{NN} capacitors still connect to one another at infinity with voltage V_X, a dependent variable determined by charge on the other conductors.





	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.35633	-0.12636	-0.07063
LargePlate	-0.12636	0.18251	-0.01422
SmallPlate	-0.07063	-0.01422	0.10507



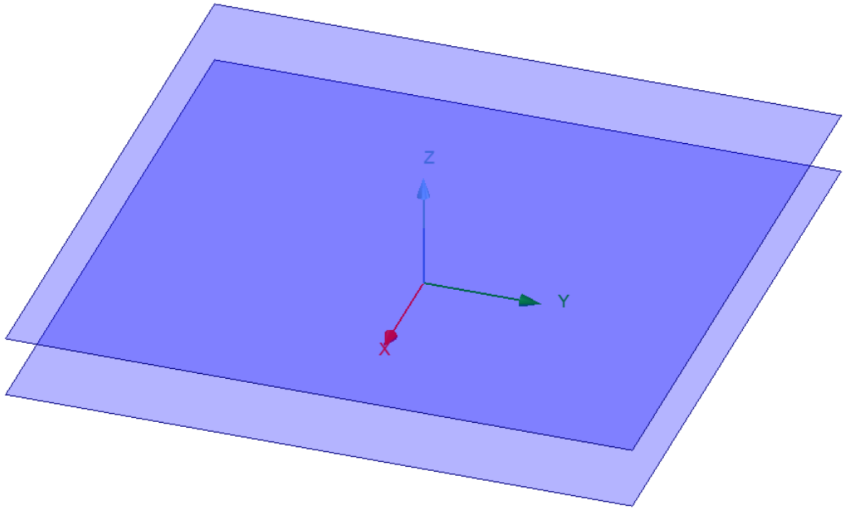
	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.24170	-0.15653	-0.08518
LargePlate	-0.15653	0.17458	-0.01805
SmallPlate	-0.08518	-0.01805	0.10323

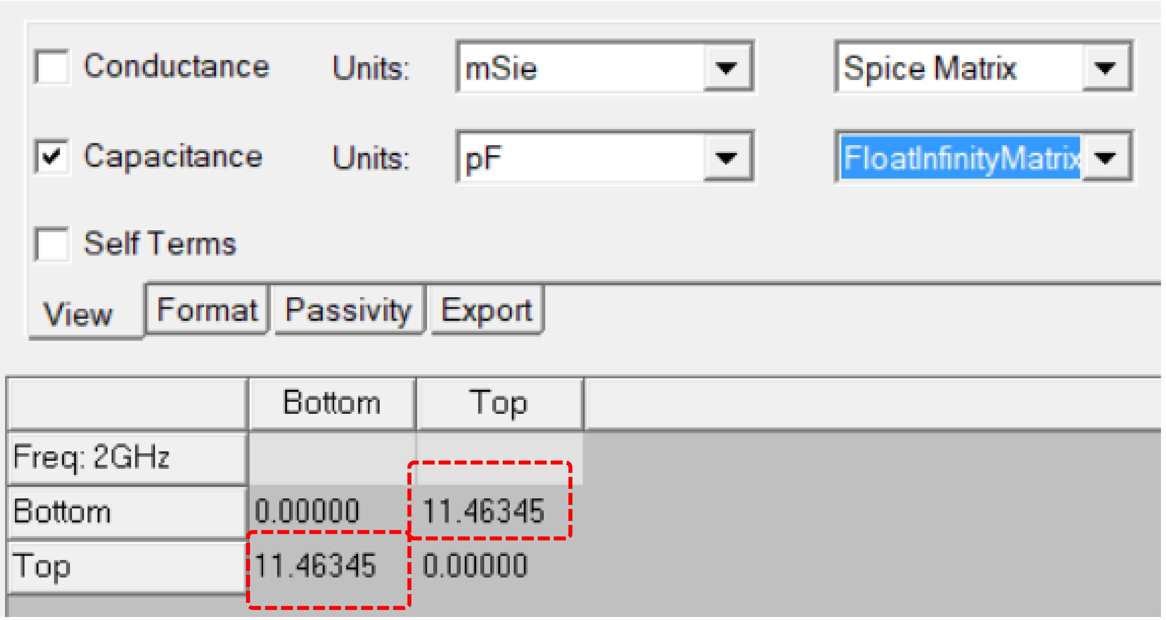
After the **Float at Infinity Matrix Reduction** operation is applied, the non-diagonal terms get larger and the diagonal terms, representing the C_{NN} self-capacitance terms, become smaller.

Ansys

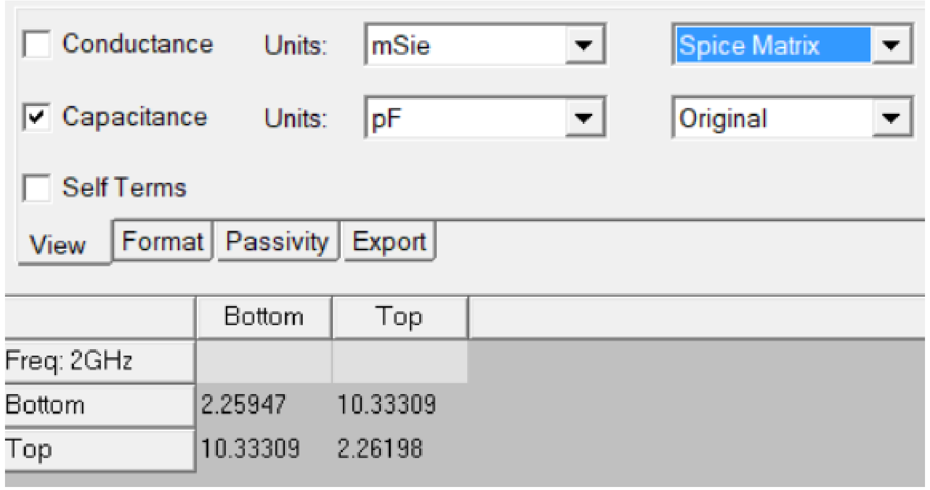
Parallel Plate Capacitance - ***Float at Infinity***

The best approximation of the capacitance of an ideal parallel plate capacitor would be to use ***Float at Infinity***. This disconnects everything except the mutual capacitance between the two plates.



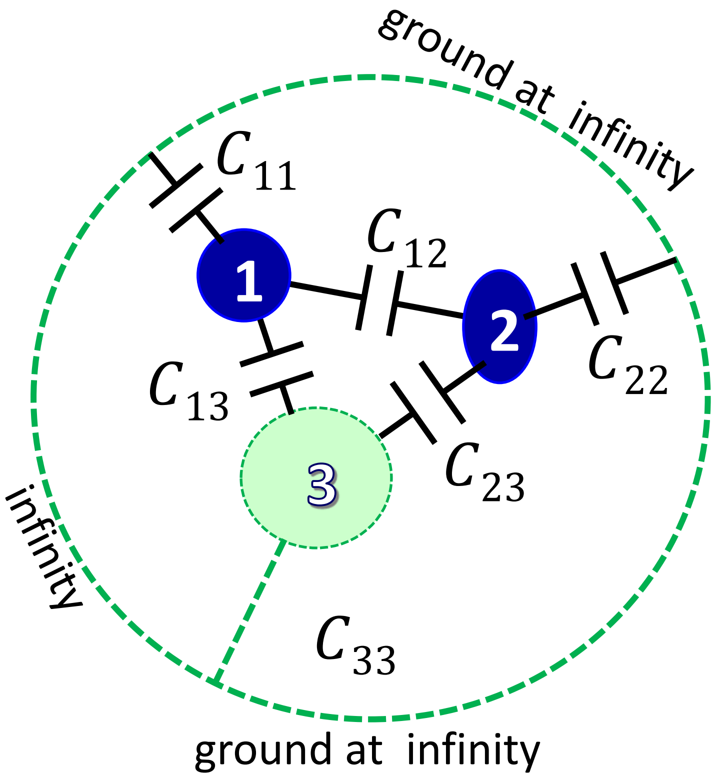


	Bottom	Top
Freq: 2GHz		
Bottom	0.00000	11.46345
Top	11.46345	0.00000



	Bottom	Top
Freq: 2GHz		
Bottom	2.25947	10.33309
Top	10.33309	2.26198

Capacitance *Matrix Reduction* - *Ground Net*



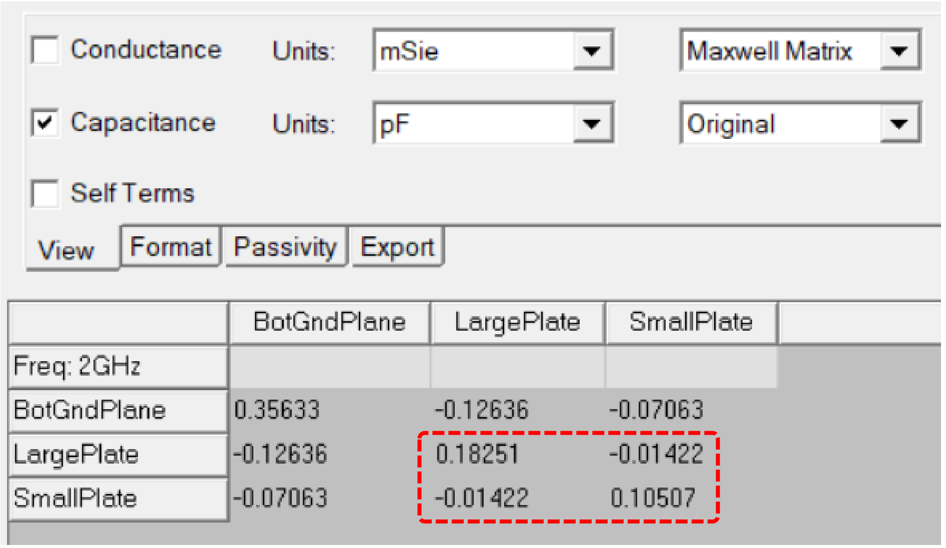
For a Maxwell capacitance matrix, the matrix operation ***Ground Net…*** on conductor 3 connects conductor 3 to the infinite ground, keeping the conductor 3 potential at ground. The capacitors C12 and C13 are now connected (grounded) to ground-at-infinity instead of to conductor 3.

$$\begin{bmatrix} C_{11}^M & C_{12}^M & C_{13}^M \\ C_{12}^M & C_{22}^M & C_{23}^M \\ C_{13}^M & C_{23}^M & C_{33}^M \end{bmatrix} = \begin{bmatrix} C_{11}+C_{12}+C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{12}+C_{22}+C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{13}+C_{23}+C_{33} \end{bmatrix}$$

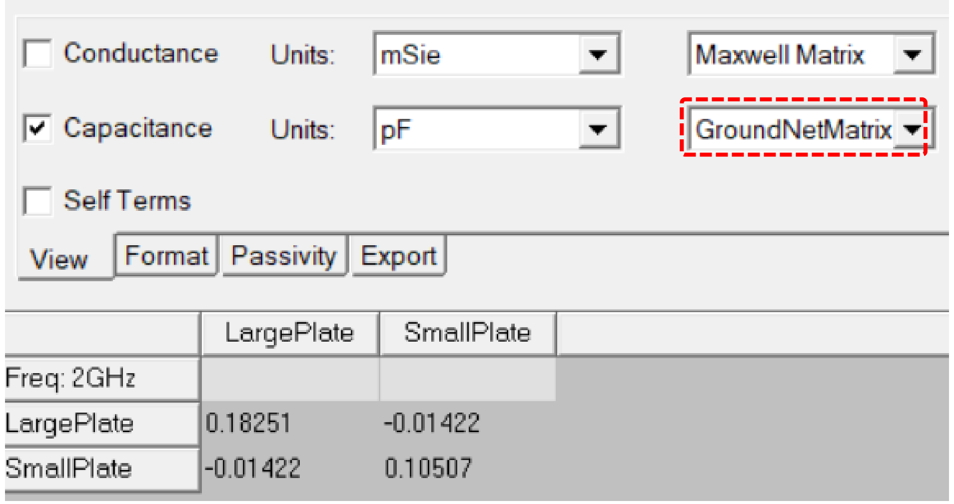
An entire row and column disappear from the capacitance matrix, giving us a 2 x 2 matrix, but the C_{13} and C_{23} capacitors do not go away.

$$\begin{bmatrix} C_{11}^M & C_{12}^M \\ C_{12}^M & C_{22}^M \end{bmatrix} = \begin{bmatrix} C_{11}+C_{12}+C_{13} & -C_{12} \\ -C_{12} & C_{12}+C_{22}+C_{23} \end{bmatrix}$$

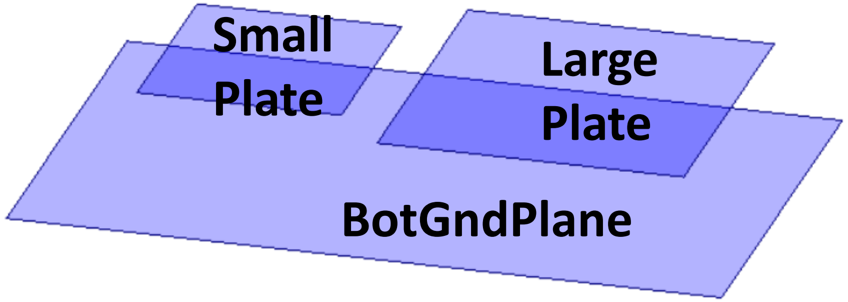
Capacitance *Matrix Reduction* - *Ground Net* Simulation Results



	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.35633	-0.12636	-0.07063
LargePlate	-0.12636	0.18251	-0.01422
SmallPlate	-0.07063	-0.01422	0.10507



	LargePlate	SmallPlate
Freq: 2GHz		
LargePlate	0.18251	-0.01422
SmallPlate	-0.01422	0.10507



$$\begin{bmatrix} C_{11}+C_{12}+C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{12}+C_{22}+C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{13}+C_{23}+C_{33} \end{bmatrix}$$

After performing the ***Reduce Matrix > Ground Net*** operation on the largest conductor BotGndPlane, an entire row and column disappear from the capacitance matrix, giving us a 2 x 2 matrix.

$$\begin{bmatrix} C_{11}+C_{12}+C_{13} & -C_{12} \\ -C_{12} & C_{12}+C_{22}+C_{23} \end{bmatrix}$$

Ansys

Return Path Performs *Float at Infinity* Then *Ground Net*

This drawing and technical background come from:

Circuit Matrix Reduction Operations

by J. Eric Bracken.

Whereas each conductor was originally connected to ground-at-infinity, the ***Reduce Matrix > Return Path*** operation performs a ***Float at Infinity*** operation as a first step.

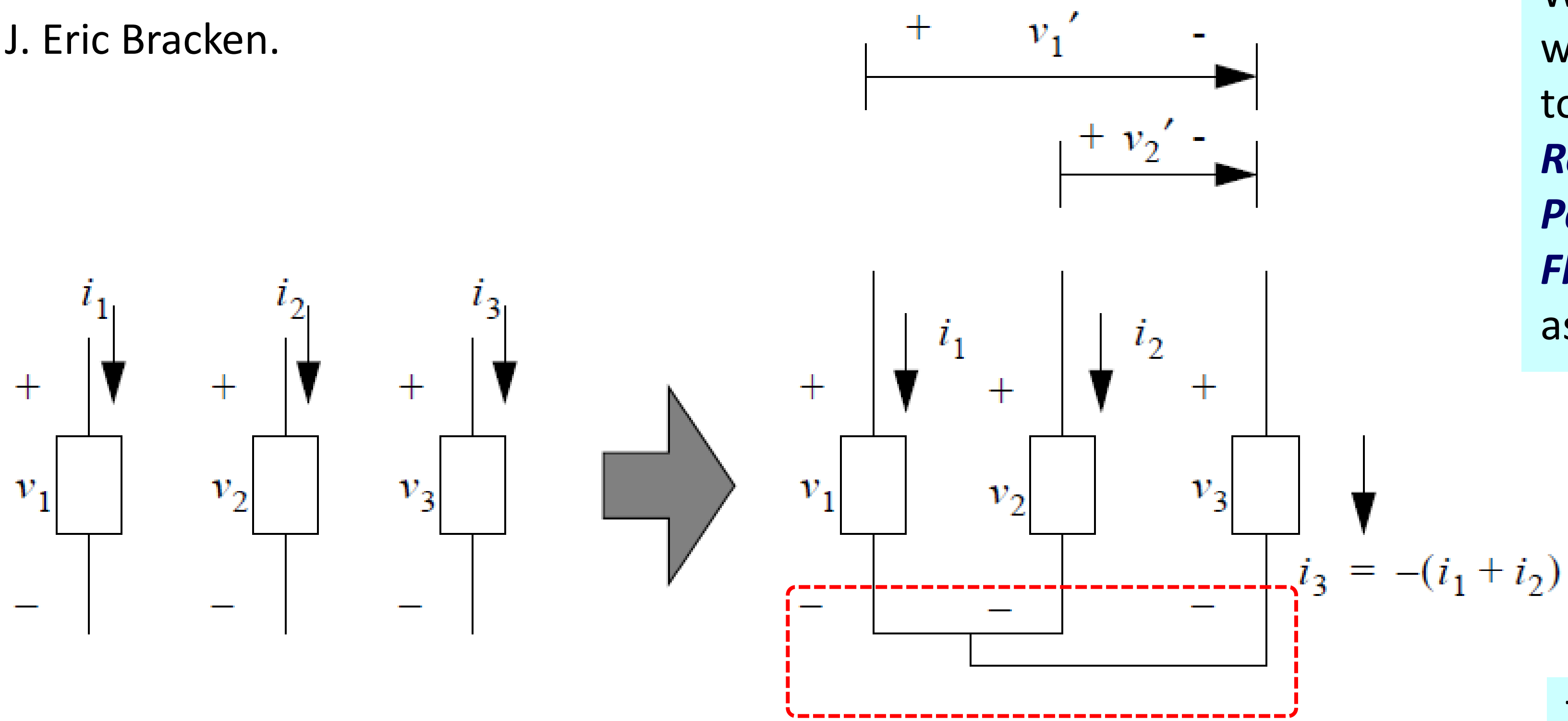
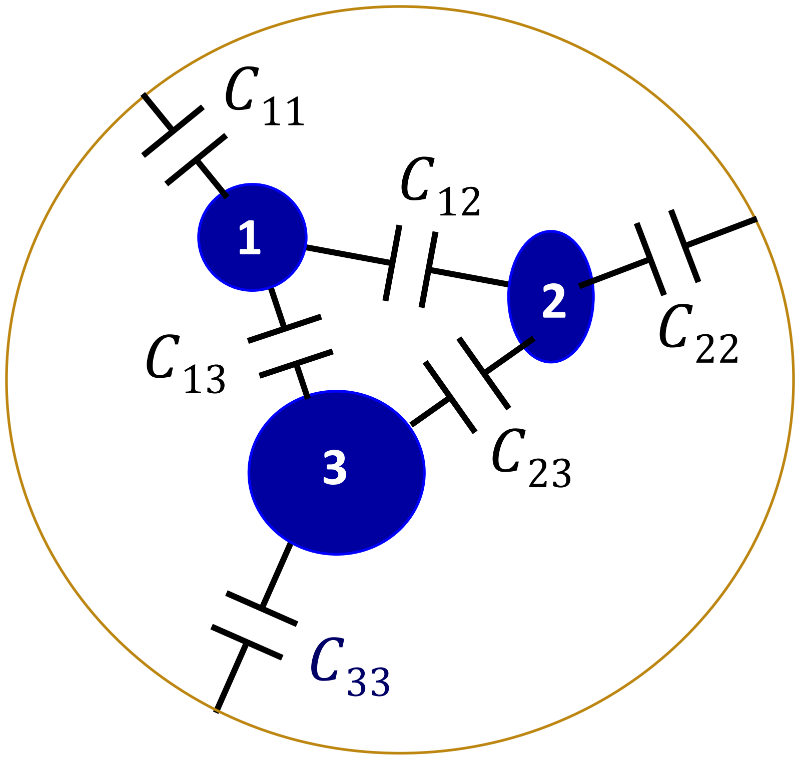


Figure 11 A return path reduction operation. In this case, conductor 3 is being taken as the return path for the other conductors. Notice that the negative reference node for defining the branch voltages has also been changed.

The ***Reduce Matrix > Return Path*** operation performs a ***Ground Net*** operation.

Capacitance *Matrix Reduction* - *Float Conductor*



Floating a conductor (***Float Net*** in Q3D) infinity removes the voltage source (which we rarely or never show) that puts 1 volt on each of the conductors in the example system. No capacitors are removed from the picture.

If we float conductor 3, there is no applied voltage to place charge on conductor 3 and the charge on conductor 3 equals zero. The voltage on conductor 3 results from the remaining mutual capacitances to conductors 1 and 2. V_3 becomes a dependent variable, and we get a 2 x 2 matrix.

1. When we float a conductor, the self-capacitances of the remaining conductors decrease.

2. The coupling capacitances between the remaining conductors will increase.

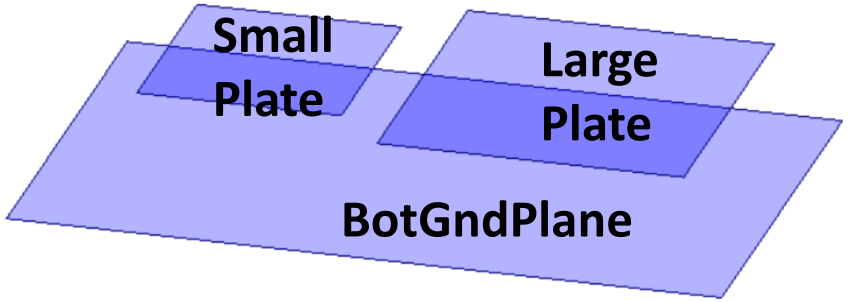
Not driving a conductor with voltage allows its capacitors to interact with the other capacitors from nodes that do have voltage impressed.

For additional discussion and reference, please see:

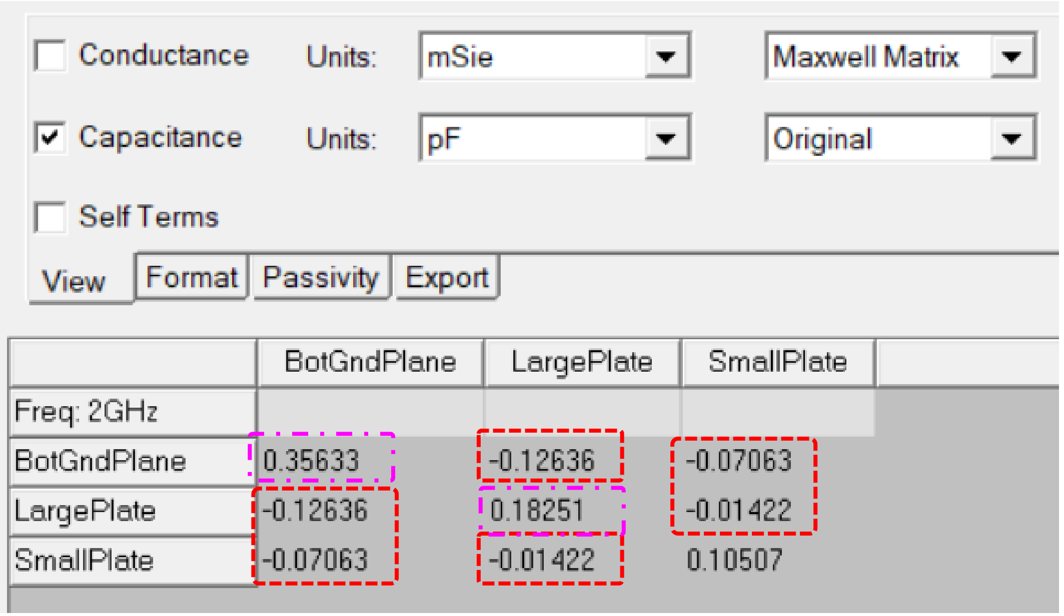
Circuit Matrix Reduction Operations

by J. Eric Bracken.

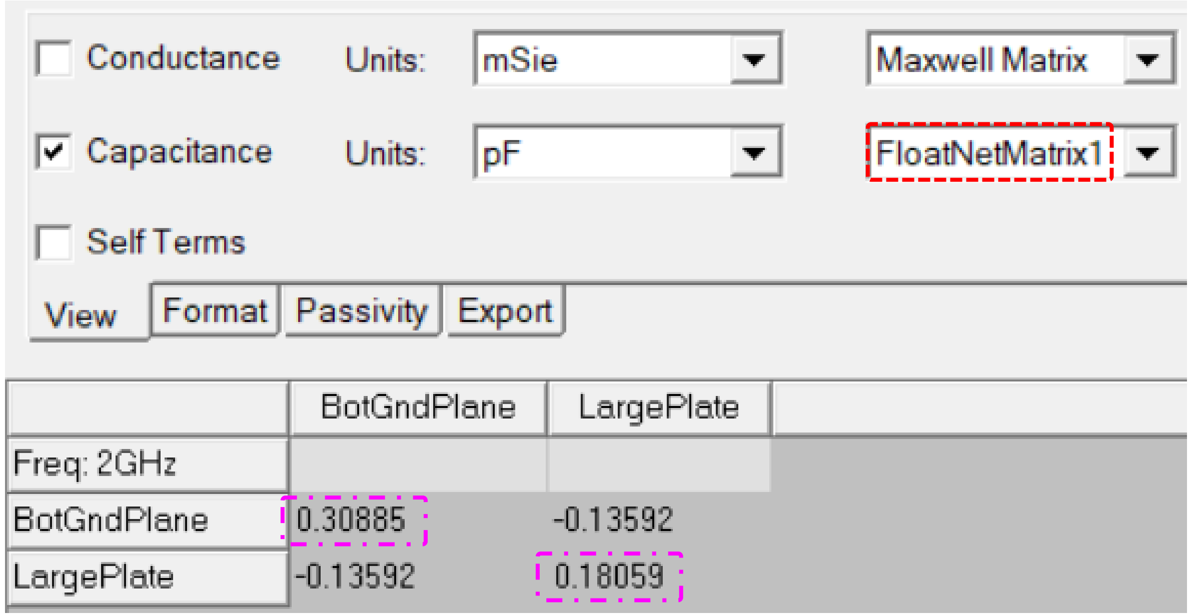
Capacitance Matrix - *Float Conductor* Simulation Results



The **SmallPlate** conductor was floated in this example.



	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.35633	-0.12636	-0.07063
LargePlate	-0.12636	0.18251	-0.01422
SmallPlate	-0.07063	-0.01422	0.10507



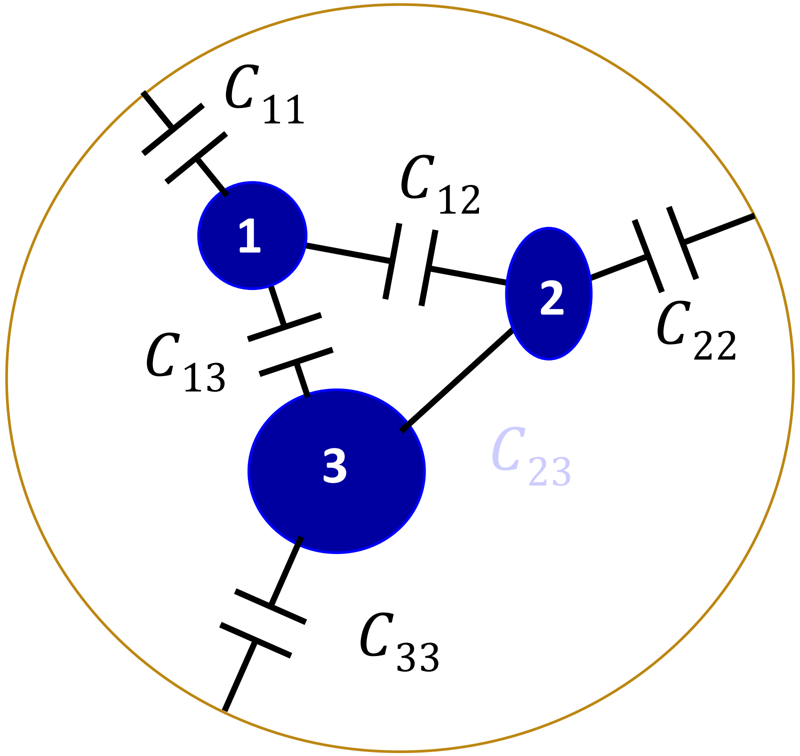
	BotGndPlane	LargePlate
Freq: 2GHz		
BotGndPlane	0.30885	-0.13592
LargePlate	-0.13592	0.18059

Coupling capacitances are smaller in this original capacitance matrix than the two -0.13592 values in the **FloatNet** matrix.

Self-capacitances are slightly smaller than in the original capacitance matrix.

Ansys

Capacitance *Matrix Reduction* - *Join in Parallel* - Same Voltage



In a conductance matrix, joining conductors in parallel means setting both conductors to the same voltage. This has the effect of short circuiting the capacitor between them and placing other capacitors in parallel.

Removing a capacitor yields a 2 x 2 matrix.

$$\begin{bmatrix} C_{11}+C_{12}+C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{12}+C_{22}+C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{13}+C_{23}+C_{33} \end{bmatrix}$$

We can see in the matrix how C11 and C13 are added in parallel. C33 and C23 are added to the lower right value where C23 has disappeared.

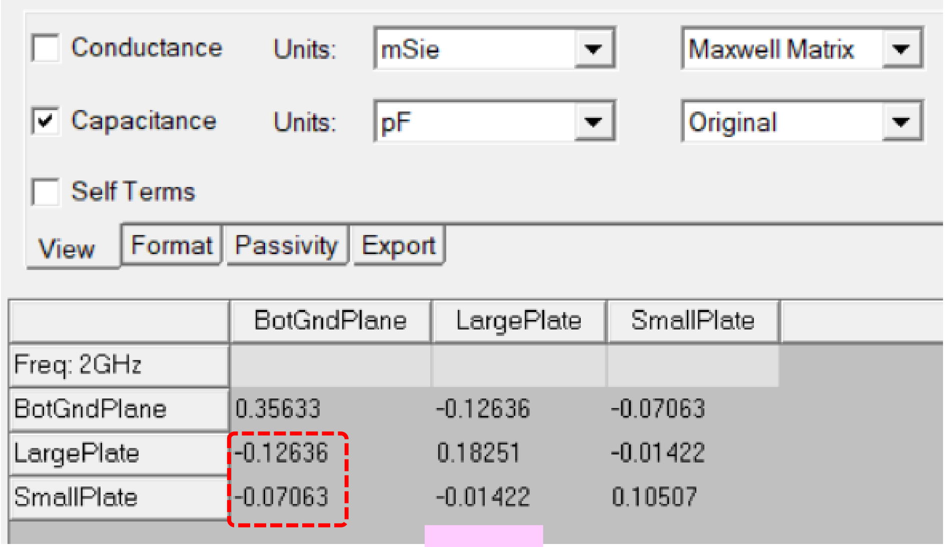
$$\begin{bmatrix} C_{11}^M & C_{12}^M \\ C_{12}^M & C_{22}^M \end{bmatrix} = \begin{bmatrix} C_{11}+C_{12}+C_{13} & -(C_{12}+C_{13}) \\ -(C_{12}+C_{13}) & C_{22}+C_{33}+C_{12}+C_{13} \end{bmatrix}$$

Join in Parallel Matrix Reduction does not require **Sources** and **Sinks** in the design.

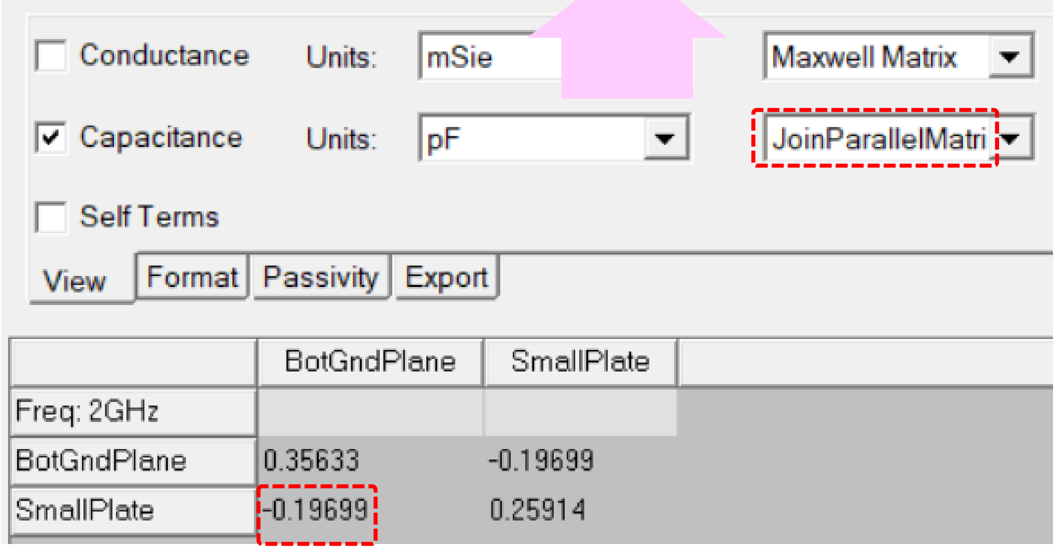
Join in Parallel Matrix Reduction Simulation Results

In this example, **LargePlate** and **SmallPlate** were joined in parallel.

$$\begin{bmatrix} C_{11}^M & C_{12}^M \\ C_{12}^M & C_{22}^M \end{bmatrix} = \begin{bmatrix} C_{11} + C_{12} + C_{13} & -(C_{12} + C_{13}) \\ -(C_{12} + C_{13}) & C_{22} + C_{33} + C_{12} + C_{13} \end{bmatrix}$$



	BotGndPlane	LargePlate	SmallPlate
Freq: 2GHz			
BotGndPlane	0.35633	-0.12636	-0.07063
LargePlate	-0.12636	0.18251	-0.01422
SmallPlate	-0.07063	-0.01422	0.10507



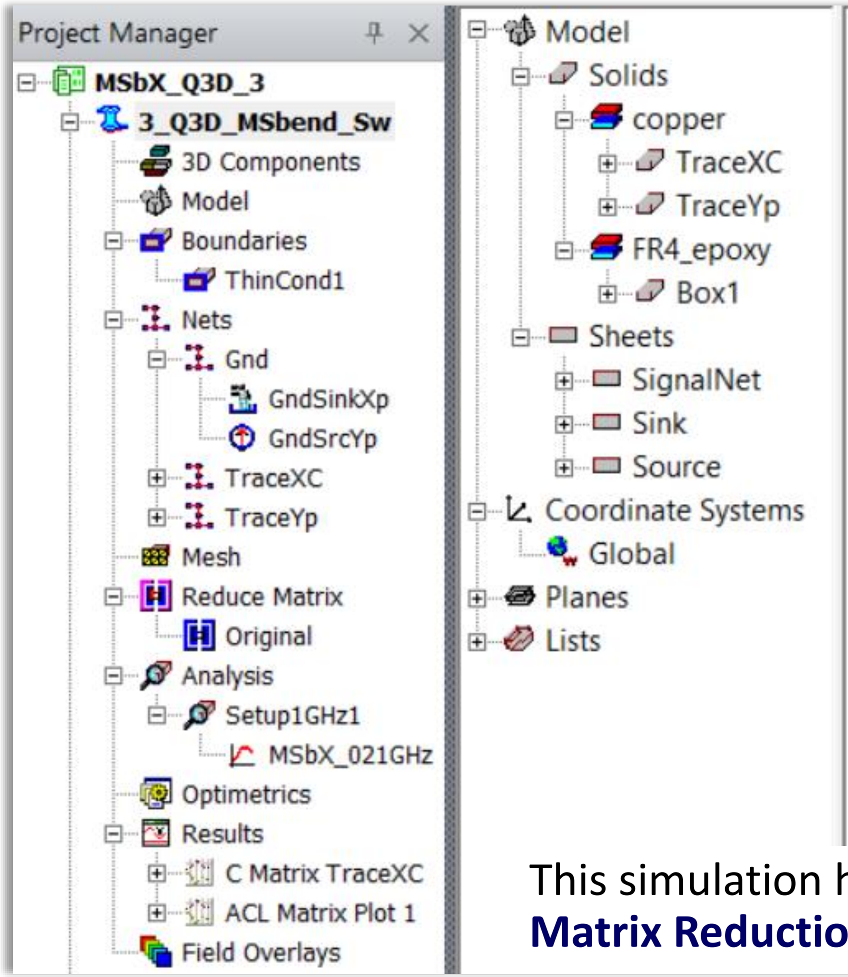
	BotGndPlane	SmallPlate
Freq: 2GHz		
BotGndPlane	0.35633	-0.19699
SmallPlate	-0.19699	0.25914

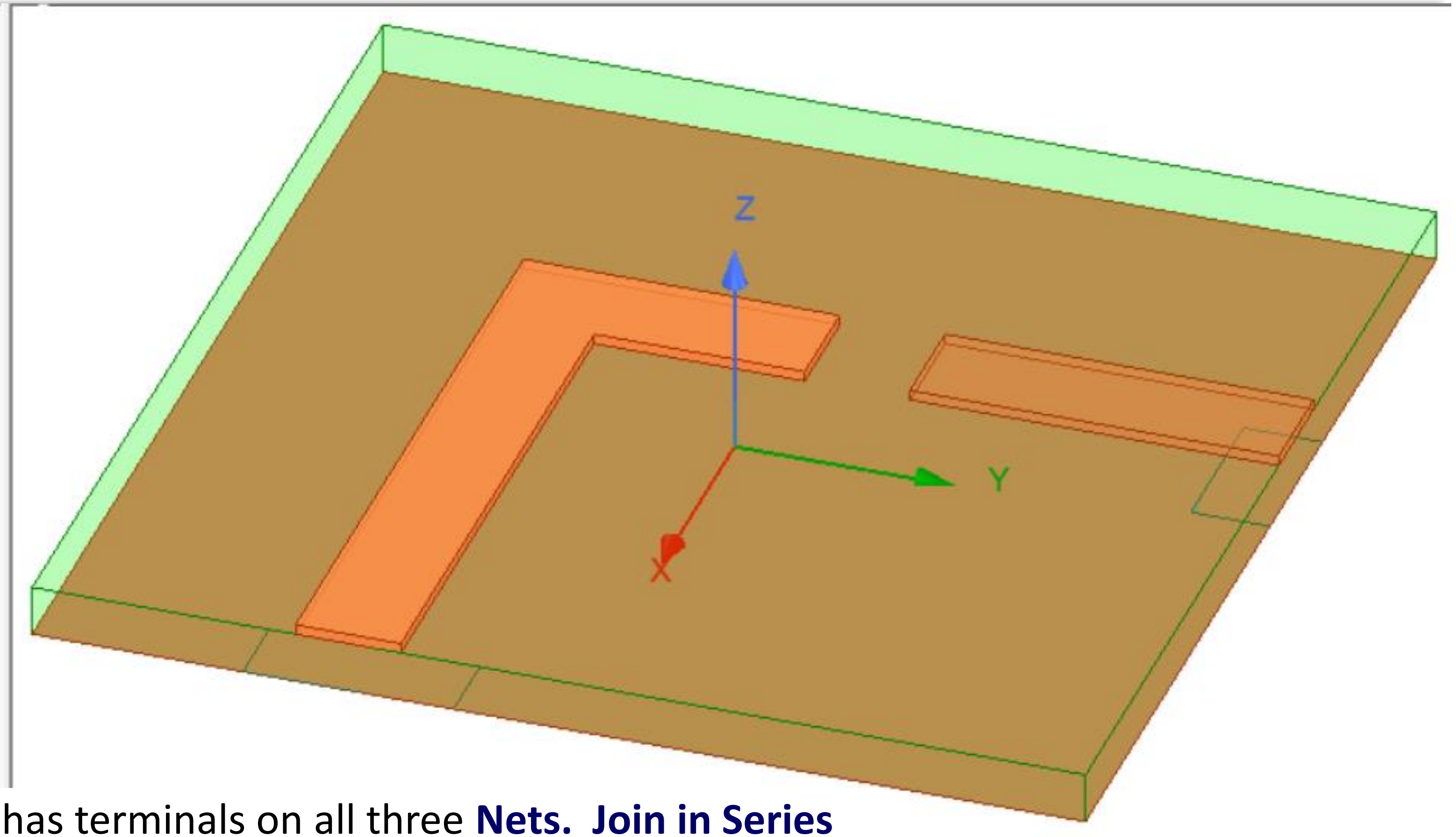
$$\begin{bmatrix} C_{11} + C_{12} + C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{12} + C_{22} + C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{13} + C_{23} + C_{33} \end{bmatrix}$$

The above capacitance -0.19699 for **SmallPlate** to **BotGndPlane** is about the same as the sum of **LargePlate** and **SmallPlate** to **BotGndPlane** in the ***Original*** matrix to the left.

Ansys

Join in Series Matrix Reduction Requires *Terminals*





This simulation has terminals on all three **Nets. Join in Series Matrix Reduction** requires **Source**s and **Sinks** be connected.

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End of Presentation