Theory of Radiation

The Source of Electromagnetic Radiation

Developed by Kathryn L. Smith, PhD

Ansys

Sources

The material presented herein is from the following sources:

"Elements of Electromagnetics," by Matthew N.O Sadiku, 5th ed. (2010)
"Engineering Electromagnetics," by Nathan Ida, 3rd ed. (2015)
"Microwave Engineering," by David Pozar, 4th ed. (2012)
"Antenna Theory," by Constantine A. Balanis, 4th ed. (2016)

The Source of Electromagnetic Radiation

Electromagnetic radiation is embedded in Maxwell's equations and the constitutive relations. For the vacuum condition case, these equations are given below.

Maxwell's Equations in a vacuum:	
Faraday's Law $\nabla \times \bar{E} = -\frac{d\bar{B}}{dt}$	Ampere's Law $\nabla \times \bar{H} = \frac{d\bar{D}}{dt} + \bar{J}$
Gauss's Law $\nabla \cdot \bar{D} = \rho$	Solenoidal Law $\nabla \cdot \bar{B} = 0$

Constitutive Relations in a vacuum:

$\bar{D} = \varepsilon_0 \bar{E}$

$\varepsilon_0 = 8.854\ x\ 10^{-12}$ farads/meter

$\bar{B} = \mu_0 \bar{H}$

$\mu_0 = 4\pi\ x\ 10^{-7}$ henrys/meter

$\bar{E}$ = electric field in volts per meter
$\bar{D}$ = electric flux density in Coulombs per meter squared

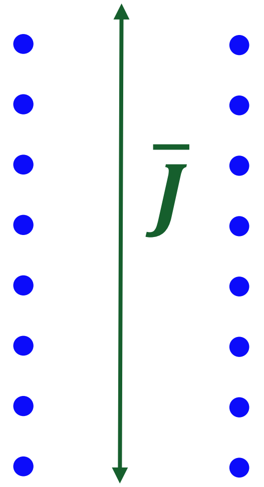
$\bar{J}$ = electric current density in Amperes per square meter

$\bar{H}$ = magnetic field in Amperes per meter
$\bar{B}$ = magnetic flux density in Webers per meter squared

The Source of Electromagnetic Radiation

Let's look at this in a couple of cases…

Case I: consider an infinite line current of time-constant density: $\bar{J} \propto C$



By Ampere's Law, a constant current $\bar{J}$ leads to a constant component of $\bar{H}$

$$\nabla \times \bar{H} = \frac{d\bar{D}}{dt} + \bar{J}$$

but by Faraday's Law, a constant magnetic field $\bar{H}$ does not contribute to $\bar{E}$

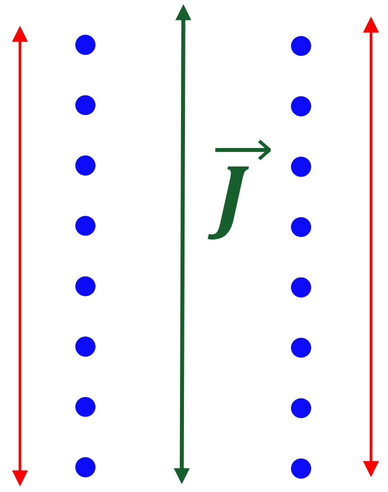
$$\nabla \times \bar{E} = -\frac{d\bar{B}}{dt}$$

so for a constant current, this is where the process stops.

The Source of Electromagnetic Radiation

Let's look at this in a couple of cases…

Case II: suppose that you have an infinite line current of constant density that varies according to a **linear** function: $\bar{J} \propto Ct$



By Ampere's Law, a linearly time-varying current $\bar{J}$ leads to a linearly time-variant component of $\bar{H}$

$$\nabla \times \bar{H} = \frac{d\bar{D}}{dt} + \bar{J}$$

then by Faraday's Law, this linearly time-variant magnetic field $\bar{H}$ leads to a constant electric field $\bar{E}$

$$\nabla \times \bar{E} = -\frac{d\bar{B}}{dt}$$

by Ampere's law again, this constant electric field contributes nothing to the magnetic field.

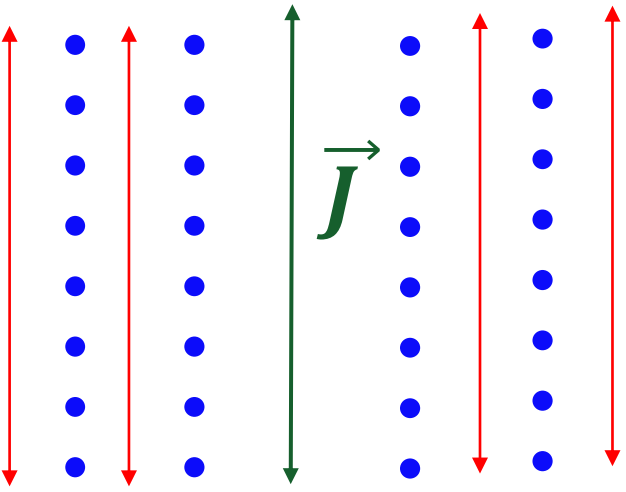
$$\nabla \times \bar{H} = \frac{d\bar{D}}{dt}$$

so for a linearly time-variant current, this is where the process stops.

The Source of Electromagnetic Radiation

Let's look at this in a couple of cases…

Case III: suppose that you have an infinite line current of constant density that varies according to a **sinusoidal** function: $\bar{J} = \mathrm{A}\sin(\omega t) + \mathrm{B}\cos(\omega t)$



By Ampere's Law, a sinusoidally time-varying current $\bar{J}$ leads to a sinusoidally time-variant component of $\bar{H}$

$$\nabla \times \bar{H} = \frac{d\bar{D}}{dt} + \bar{J}$$

then by Faraday's Law, this sinusoidally time-variant magnetic field $\bar{H}$ leads to a sinusoidally time-variant electric field $\bar{E}$

$$\nabla \times \bar{E} = -\frac{d\bar{B}}{dt}$$

by Ampere's law again, this sinusoidally time-variant electric field leads to a sinusoidally time-variant magnetic field $\bar{H}$.

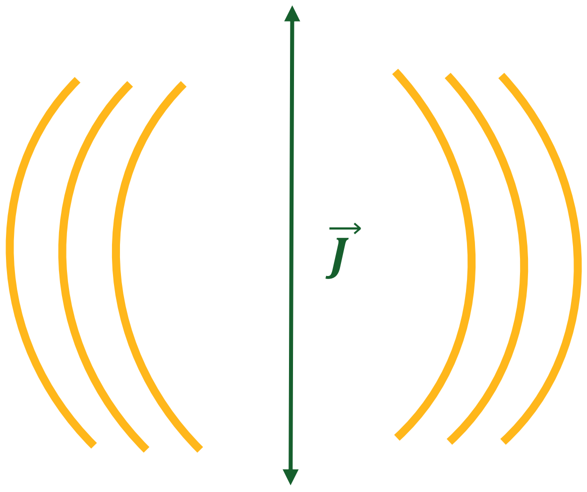
$$\nabla \times \bar{H} = \frac{d\bar{D}}{dt}$$

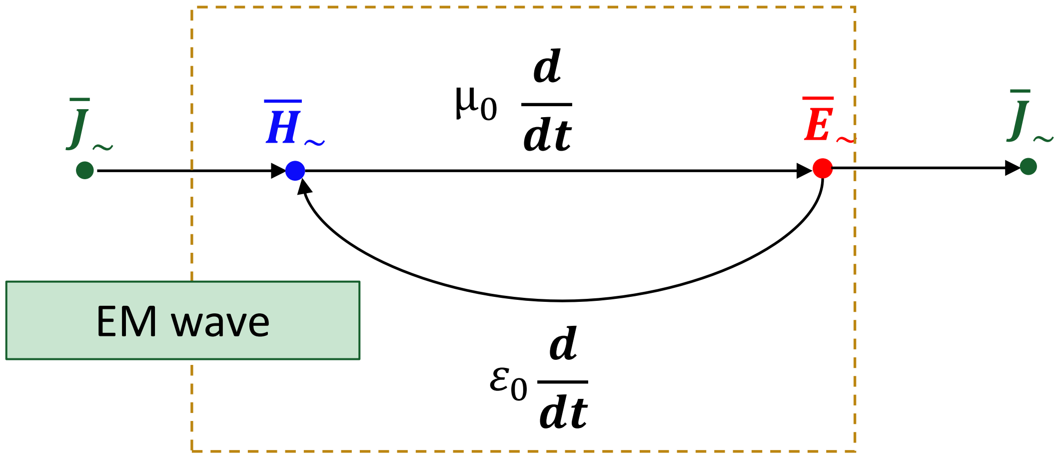
and for a sinusoidally time-variant current, these last two steps repeat (theoretically) infinitely.

The Source of Electromagnetic Radiation

So, conceptually speaking, a sinusoidal current leads to a magnetic field, which leads to an electric field, which leads to a magnetic field, which leads to an electric field…

$$\nabla \times \bar{E} = -\frac{d\bar{B}}{dt} \qquad \nabla \times \bar{H} = \frac{d\bar{D}}{dt} + \bar{J}$$





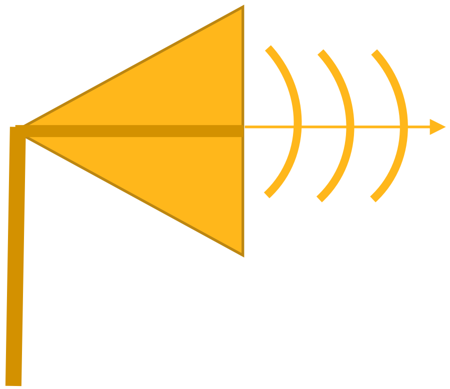
This leads to the **self-propagation** of the electromagnetic wave, which, critically, is sourced by a **sinusoidally varying** current source.

This process may also occur in reverse, when an electromagnetic wave impinges upon a conductor and **creates** surface currents there.

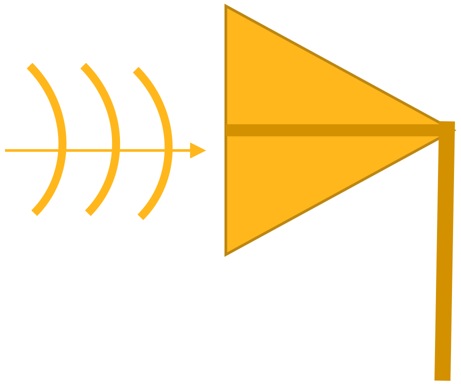
The Source of Electromagnetic Radiation

By engineering conductors that support specific current patterns, we can both create and receive propagating electromagnetic (EM) waves.

→ Antennas



Antenna in transmit mode



Antenna in receive mode

Ansys