Natural Convection in Enclosures

Natural Convection – Lesson 5

Natural Convection in Enclosures

- In many engineering applications, heat is transferred between a fluid in an enclosed space and enclosure surfaces at different temperatures. For example – furnaces, ovens, cooling towers and even rooms where the walls are at different temperatures.
- Heat transfer due to natural convection is quite complex for such applications. As a first step in this direction, we will cover three canonical enclosures in this lesson:
  - Rectangular Cavities
  - Concentric Cylinders
  - Concentric Spheres

Natural Convection in Rectangular Cavities

- The rectangular cavity setup shown on the right has been a part of many extensive research studies, both theoretical and experimental.

- In this setup, two opposing walls are maintained at different temperatures – hot (T_1) and cold (T_2). The other surrounding walls are assumed to be adiabatic.

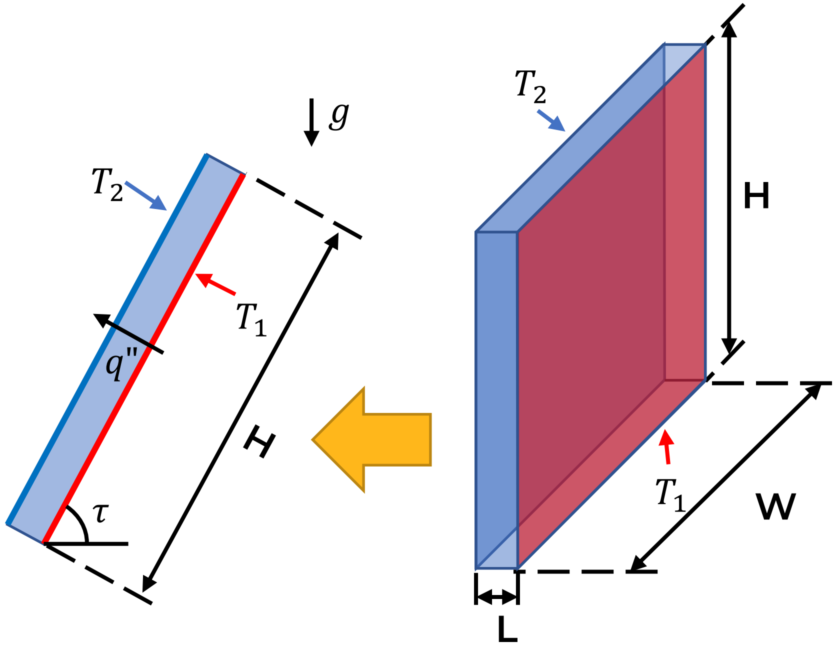
- Natural convection in a rectangular cavity depends on:
  - Aspect ratios, H/L and W/L
  - Tilt angle (τ) between hot and cold surfaces with the horizontal plane
    - $\tau = 0°$: Horizontal cavity with bottom heated
    - $\tau = 90°$: Vertical cavity with side heated

- The heat flux transferred across the cavity is given as:

$$q'' = h(T_1 - T_2)$$

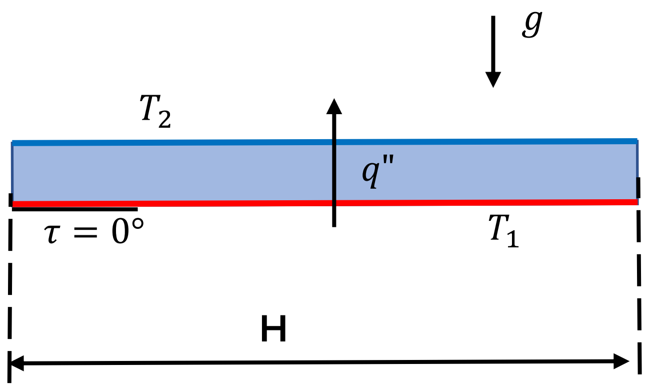
- The fluid flow behavior depends on the Rayleigh number of the flow.

$$Ra = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu}$$



Horizontal Heated Rectangular Cavities

- For a rectangular cavity with $H/L, W/L \gg 1$ at Rayleigh numbers lower than a critical value $\left(Ra_{L,c} = 1708\right)$, viscous forces are larger than the buoyancy and no advection is possible inside the cavity. Heat transfer between the top and bottom surfaces of the cavity is primarily because of conduction and radiation.

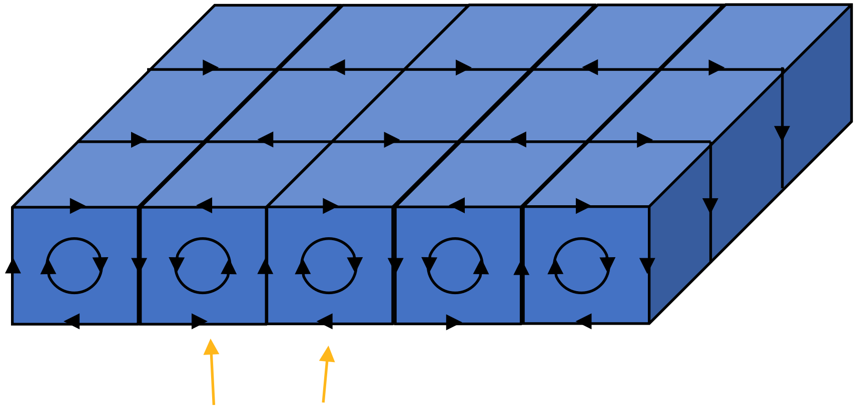


- The Nusselt number in such cases is ~ 1.
- For Rayleigh numbers greater than the critical value $\left(Ra_L > Ra_{L,c}\right)$, advection inside the cavity becomes prominent and heat transfer is because of conduction and convection (and radiation).

Horizontal Heated Rectangular Cavities (cont.)

- For $Ra_{L,c} < Ra_L \lesssim 5 \times 10^4$, the fluid motion forms a uniform pattern of regularly spaced roll cells.
- Beyond $Ra_L > 5 \times 10^4$, the fluid flow becomes turbulent. Globe and Dropkin obtained the following correlation for the average Nusselt number:

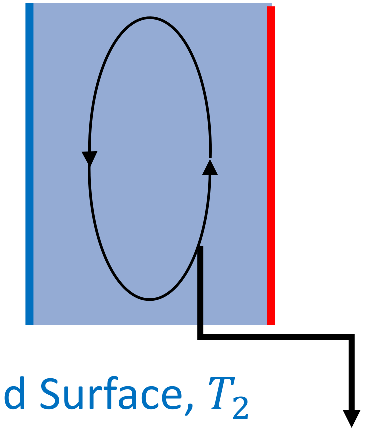
$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.069 Ra_L^{1/3} Pr^{0.074} \qquad 3 \times 10^5 \lesssim Ra_L \lesssim 7 \times 10^9$$



- Here the properties are estimated using $\bar{T} = (T_1 + T_2)/2$. The correlation can be applied if L/H is small, so that the side wall effect can be neglected.
- When the temperature of the top surface is greater than the bottom one ($\tau = 180^{\circ}$), the heat transfer is exclusively because of conduction (assuming no radiation), independent of the Rayleigh number.

Vertical Heated Rectangular Cavities (cont.)

- In vertically heated cavities (tilt angle $\tau = 90°$), the side walls are maintained hot and cold. The horizontal top and bottom walls of the cavity are assumed to be adiabatic.
- For small Rayleigh numbers ($Ra_L \lesssim 10^3$), the buoyancy forces are small and are not able to overcome the viscous effects. The primary mode of heat transfer is conduction (through the walls and the fluid) and the Nusselt number is ~ 1.
- As Ra_L increases, a recirculating fluid motion (or cellular flow) is set from the hot to the cold side. The bulk flow in the core is relatively stagnant and the fluid flow is concentrated in the thin boundary layers near the hot and cold sides of the enclosure.
- This flow will eventually transition into turbulence with increasing Rayleigh number (Ra_L).



Vertical Heated Rectangular Cavities (cont.)

Small Aspect Ratios

$$\overline{Nu}_L = 0.22\left(\left(\frac{Pr}{0.2+Pr}\right)Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4}$$

$$2 \lesssim \frac{H}{L} \lesssim 10$$
$$Pr \lesssim 10^5$$
$$10^3 \lesssim Ra_L \lesssim 10^{10}$$

$$\overline{Nu}_L = 0.18\left(\left(\frac{Pr}{0.2+Pr}\right)Ra_L\right)^{0.29}$$

$$1 \lesssim \frac{H}{L} \lesssim 2$$
$$10^{-3} \lesssim Pr \lesssim 10^5$$
$$10^3 \lesssim \frac{Ra_L Pr}{0.2+Pr}$$

Large Aspect Ratios

$$\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$

$$10 \lesssim \frac{H}{L} \lesssim 40$$
$$1 \lesssim Pr \lesssim 2\times 10^4$$
$$10^4 \lesssim Ra_L \lesssim 10^7$$

$$\overline{Nu}_L = 0.046 Ra_L^{1/3}$$

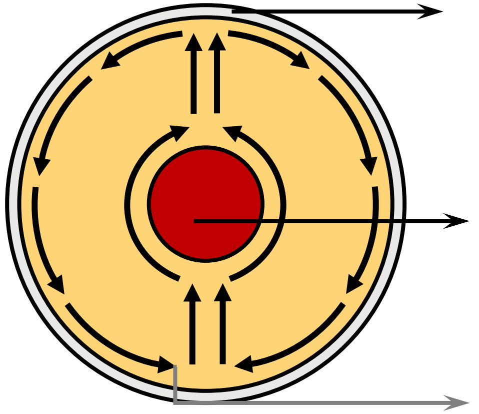
$$1 \lesssim \frac{H}{L} \lesssim 40$$
$$1 \lesssim Pr \lesssim 20$$
$$10^6 \lesssim Ra_L \lesssim 10^9$$

Concentric Cylinders

- For concentric cylinders, we are interested in estimating the natural convective heat transfer in the annular region between the inner and outer cylinders.

- If the inner cylinder is hotter than the outer cylinder $(T_i > T_o)$, a recirculating fluid motion is created such that the fluid rises along the inner cylinder walls and falls along the outer cylinder walls.

- This flow is reversed if $T_o > T_i$, as shown in the schematic here.



- To estimate the heat transfer between the cylinders, we use the conduction heat transfer theory to obtain the following relationship for concentric cylinders:

$$q = \frac{2\pi L k_{eff}(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)}$$

$$0.7 \lesssim Pr \lesssim 6000$$
$$Ra_L \lesssim 10^7$$

Concentric Cylinders (cont.)

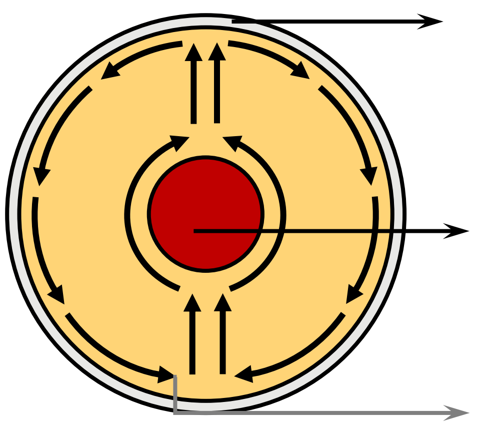
- The thermal conductivity, k, is replaced by an effective thermal conductivity value, k_{eff}, such that the stationary fictitious fluid will transfer the same heat as the moving fluid. The following correlation can be used to estimate k_{eff}:

$$k_{eff} = 0.386k\left[\left(\frac{Pr}{0.861 + Pr}\right)^{1/4} Ra_c^{1/4}\right]$$

- If the relation provides a value of $k_{eff}/k < 1$, then $k_{eff} = k$ should be used instead, since q cannot be lower than $q_{conduction}$ limit.
- The Rayleigh number used in the correlation is estimated using the following length scale.

$$L_c = \frac{2\left(ln\left(\frac{r_o}{r_i}\right)\right)^{4/3}}{\left(r_i^{-3/5} + r_o^{-3/5}\right)^{5/3}}$$

- The properties should be estimated at $T_m = (T_i + T_o)/2$.



Concentric Spheres

- Following an analysis like that used for the concentric cylinders, the total heat transfer rate between two concentric spheres is given by:

$$q = \frac{4\pi k_{eff}(T_i - T_o)}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}$$

$$0.7 \lesssim Pr \lesssim 4000$$
$$Ra_L \lesssim 10^4$$

- Here, the value of k_{eff} is given by the following correlation:

$$k_{eff} = 0.74k\left(\frac{Pr}{0.861 + Pr}\right)^{1/4} Ra_s^{1/4}$$

If the relation provides a value of $k_{eff}/k < 1$, then $k_{eff} = k$ should be used instead, since q cannot be lower than $q_{conduction}$ limit.

- The length scale used to estimate the Rayleigh number is given by:

$$L_c = \frac{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)^{4/3}}{2^{1/3}\left(r_i^{-7/5} + r_o^{-7/5}\right)^{5/3}}$$

- The properties should be estimated at $T_m = (T_i + T_o)/2$.

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Summary

- We learned about natural convection heat transfer in enclosures by understanding three canonical problems – rectangular cavity, concentric cylinders and concentric spheres.
- For rectangular cavities, the heat transfer due to natural convection is dependent on the tilt angle (τ) and the flow Rayleigh Number (Ra_L).
- For slender Horizontal Rectangular Cavities:
  - $\tau = 0°$ & If $Ra_L < 1708$ - No natural convection. Conduction and Radiation (if any) are primary modes of heat transfer.
  - $\tau = 0°$ & $1708 < Ra_L \lesssim 5 \times 10^4$ - Natural Convection with regularly counter-rotating roll cells.
  - $\tau = 0°$ & $Ra_L \gtrsim 5 \times 10^4$ - Natural Convection with turbulent flow.
  - $\tau = 180°$ & Any Ra_L - No natural convection. Conduction and Radiation (if any) are primary modes of heat transfer.
- For Vertical Rectangular Cavities, natural convection starts beyond a critical Rayleigh Number of 10^3. As Ra_L increases, the fluid motion exhibits an interesting cellular recirculating flow. As Ra_L continues to rise, the fluid flow quickly becomes turbulent.
- We studied the various Nusselt Number correlations to estimate natural convection heat transfer for different scenarios of rectangular cavities.
- For both Concentric Cylindrical and Spherical enclosures, we used the conduction heat transfer theory to estimate the natural convection heat transfer by modifying the effective thermal conductivity of the fluid in the gap.

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