Forced Convection in Other Canonical External Flows

Forced Convection in External Flows – Lesson 4

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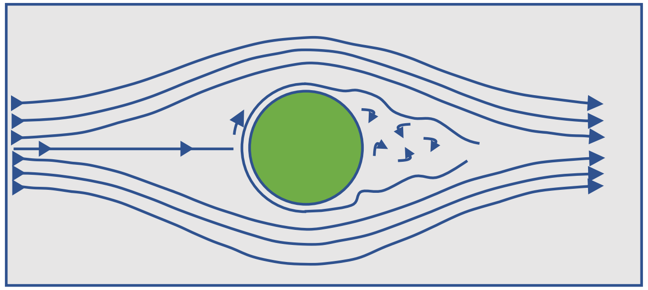
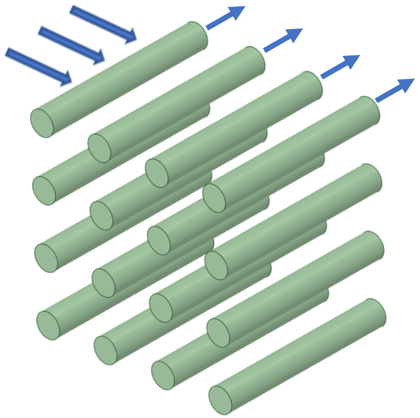
Intro

- In this lesson, we will analyze the forced convection heat transfer in some representative external flows.
- In cases such as fluid flow over a flat plate, flow over a cylinder, etc., the presence of a boundary layer has a strong impact on the heat transfer characteristics.
- The nature of the boundary layer, i.e., whether it is laminar or turbulent or transitional, can significantly alter the amount of heat transferred to/from the body.
- In this lesson, we will extend our discussion from the last lesson and analyze the heat transfer behavior for three different cases:

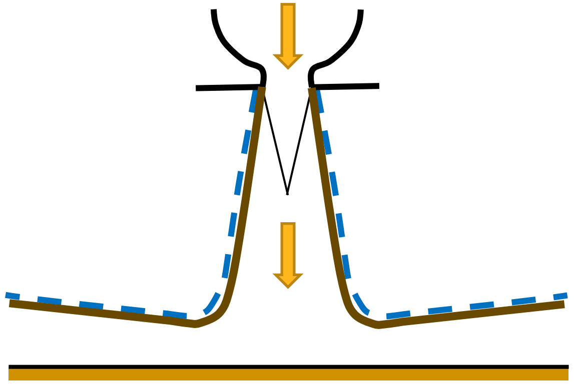
Cylinder in Cross-Flow



Flow across a Tube Bank

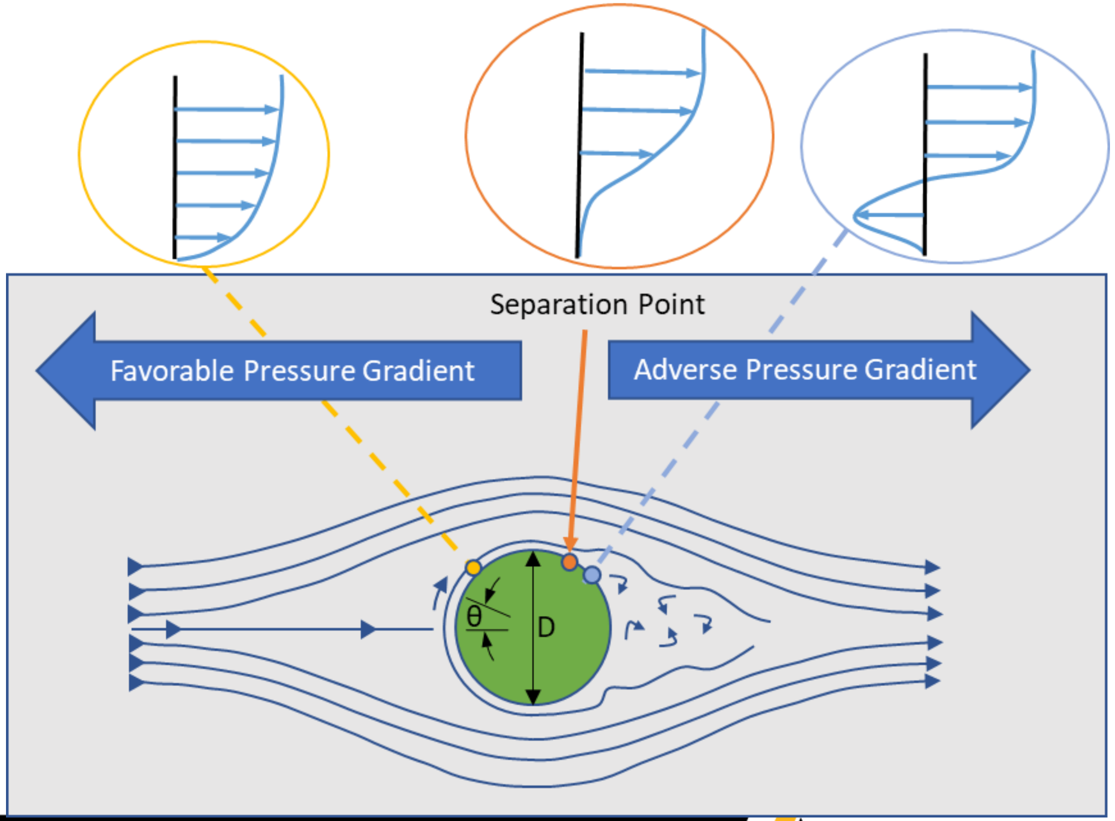


Impinging Jet Flow



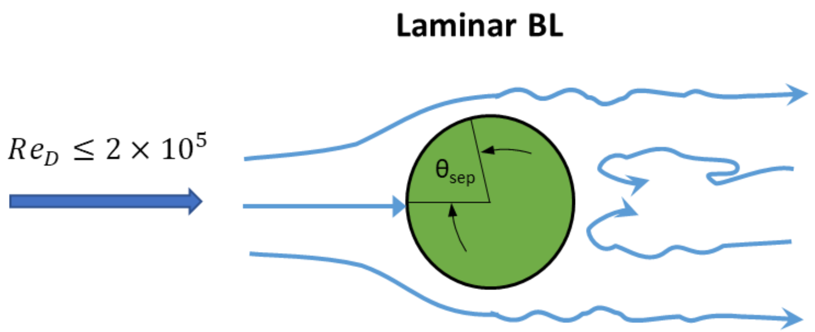
Cylinder in Cross-Flow

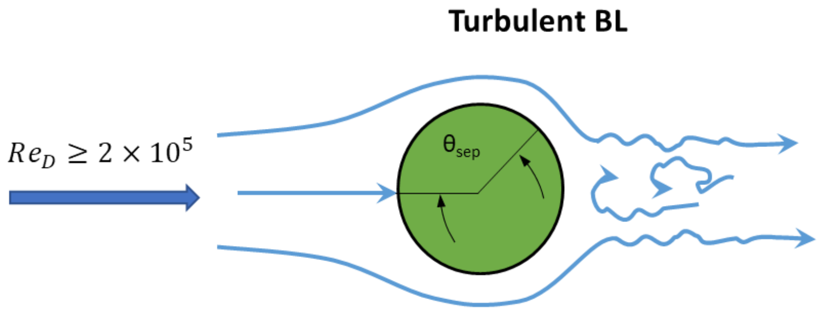
- The motion of a fluid normal to the axis of a cylinder is considered a cylinder in cross-flow. Because of ease of manufacturing, the circular cylinder is the most used tube shape in heat exchangers.
- Let us quickly recap some fundamental aspects of flow over a cylinder.
- The flow stagnates as it first encounters the cylinder. The forward stagnation point is so named because the flow is brought to a complete stop at this location. Consequently, the fluid pressure is maximum at this location.
- As the flow turns around the cylinder it accelerates, accompanied by the reduction in fluid pressure. i.e., $\frac{dp}{dx} < 0$ – favorable pressure gradient. The boundary layer also begins to grow around the cylinder.
- At a certain circumferential location, $\frac{dp}{dx} = 0$ and the flow reaches its maximum velocity. Beyond this point, because of an adverse pressure gradient, i.e., $\frac{dp}{dx} > 0$, the flow starts to decelerate.
- At a further circumferential location, the flow momentum is not strong enough to overcome the adverse pressure gradient. As a result, the boundary layer detaches from the surface of the cylinder. At this location, called the separation point, $\frac{du}{dy}\big|_{y=0}$ is 0.



Cylinder in Cross-Flow (cont.)

- The nature of the boundary layer strongly influences the location of the separation point.
- Let's define a Reynolds number based on the cylinder diameter (D) as $Re_D = \frac{\rho V D}{\mu}$.
- For $Re_D \lesssim 2 \cdot 10^5$, experiments show that the boundary layer is laminar, and the separation point is around $\theta = 80^0$.
- For $Re_D \gtrsim 2 \cdot 10^5$, the boundary layer is turbulent, and the separation point is around $\theta = 140^0$.
- Separation is delayed because the presence of high momentum fluid in the turbulent boundary layer helps to sustain the adverse pressure gradient over a longer circumferential distance.

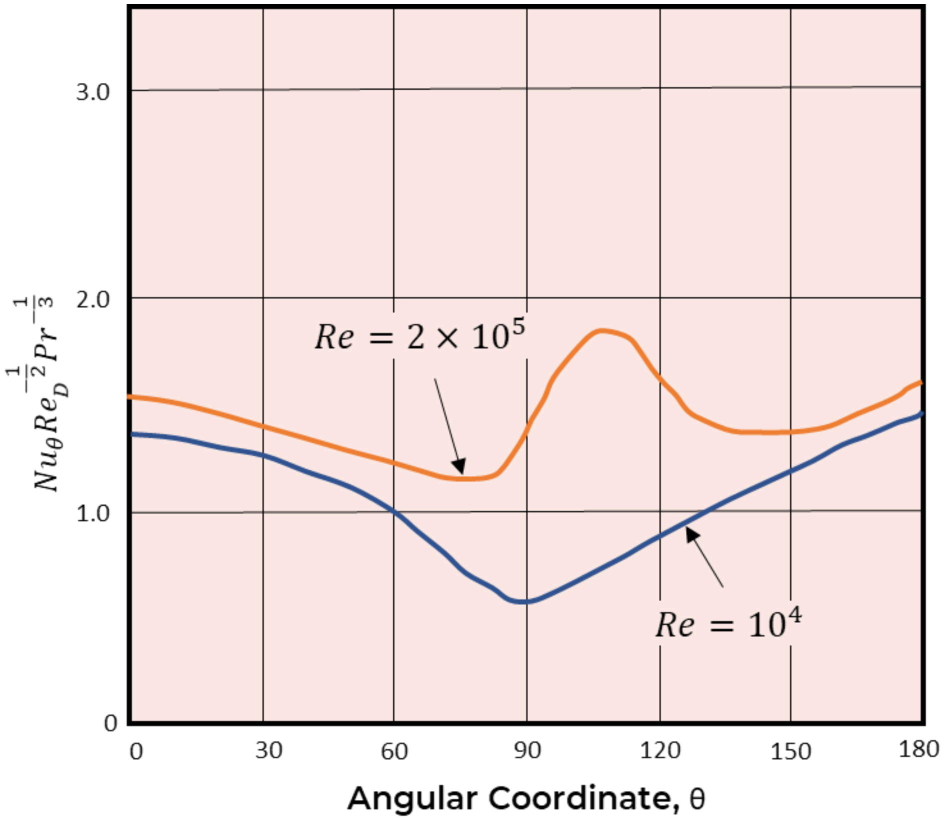




Cylinder in Cross-Flow (cont.)

- The convective heat transfer, represented by the local Nusselt number (Nu_θ), is strongly influenced by the nature of the boundary layer.
- For $Re_D \leq 10^5$, Nu_θ decreases with increasing circumferential distance, i.e., θ.
- At the separation point, i.e., $\theta \sim 80^0$, the local Nusselt number attains its minimum value. Due to enhanced mixing from vortices, Nu_θ increases beyond the stagnation point.
- For $Re_D \geq 10^5$, Nu_θ still shows a decreasing trend from the stagnation point. Between θ~80^0 and 100^0, because of the laminar-turbulent transition, a sharp increase is observed. Beyond $\theta = 100^0$, Nu_θ again decreases till the separation point, beyond which it increases again because of enhanced mixing.
- For $Re_D \leq 2000$ and $Pr \geq 0.6$, the following correlation can be used to calculate the local Nusselt number at the stagnation point:

$$Nu_D(\theta = 0) = 1.15 Re_D^{\frac{1}{2}} Pr^{\frac{1}{3}}$$



Cylinder in Cross Flow (cont.)

- Hilpert proposed the following empirical correlation ($\mathrm{Pr} \geq 0.7$) for the overall average conditions:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = CRe_D^m Pr^{\frac{1}{3}}$$

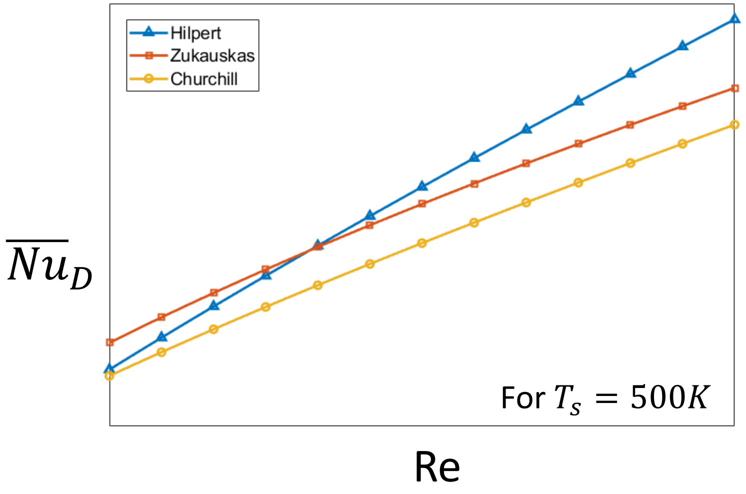
- Here C and m are constants that vary with Re_D.
- Zukauskas proposed a modified correlation of the following form:

$$\overline{Nu}_D = CRe_D^m Pr^n \left(\frac{Pr}{Pr_s}\right)^{\frac{1}{4}}$$

valid for

$$0.7 \leq Pr \leq 500$$

$$1 \leq Re_D \leq 10^6$$

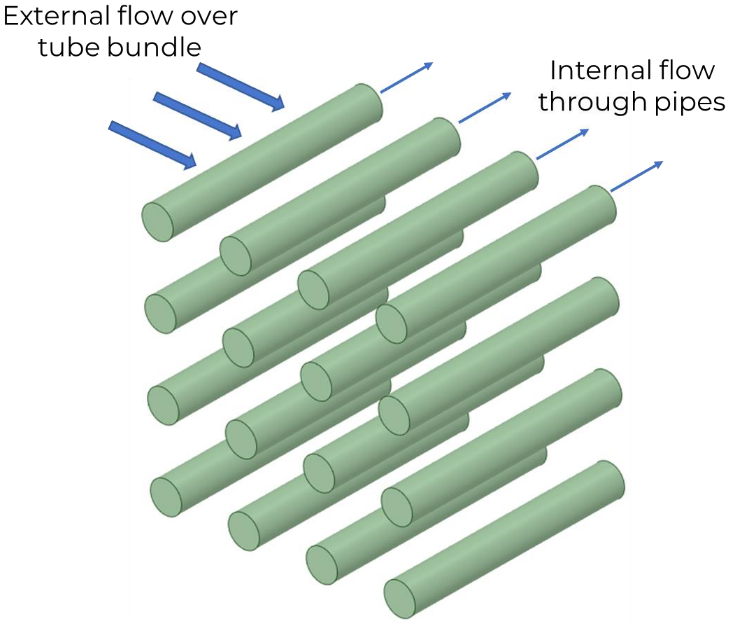


- Here, all quantities are evaluated at T_∞, expect Pr_s, which is evaluated at T_s. For $Pr \leq 10$, $n = 0.37$ and for $Pr \geq 10, n = 0.36$.
- A comprehensive correlation was proposed by Churchill and Bernstein which is valid for all Re_D and for $Pr > 0.2$.

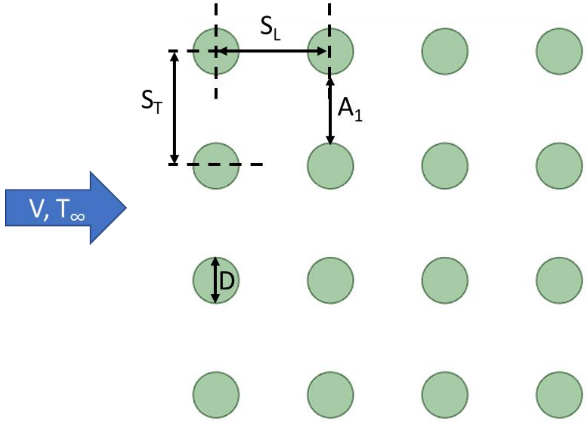
Please refer to Chapter-7 of Fundamentals of Heat and Mass Transfer by Incropera, DeWitt, Bergmann and Levine for a detail regarding the constants used in these correlations.

$$\overline{Nu}_D = 0.3 + \frac{0.62Re_D^{1/2}Pr^{1/3}}{\left(1+\left(\frac{0.4}{Pr}\right)^{2/3}\right)^{1/4}}\left(1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right)^{4/5}$$

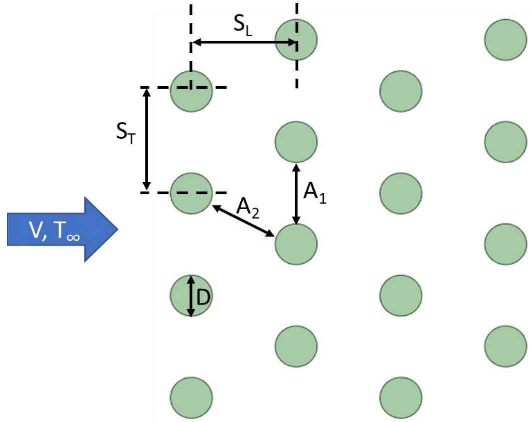
Flow Across a Tube Bank



- A tube bank is a set or collection of tubes carrying a hot/cold fluid. These are commonly seen in devices such as heat exchangers where a second hot/cold fluid is passed over the tube bank to add/remove heat from the fluid inside the tubes.
- The heat transfer to/from the tube bank is significantly impacted by the arrangement of the tube rows of the bank, which can take two major forms:
  - Aligned
  - Staggered
- If multiple rows are arranged such that the successive rows are only longitudinally displaced from the previous row, such a setup is called an aligned arrangement.



- If the successive rows are displaced both longitudinally and transversely relative to the previous row, such a setup is called a staggered setup.



Flow Across a Tube Bank (cont.)

- In both the arrangements, the flow around the first row of tubes is equivalent to that of a cross-flow over a singular cylinder, which we saw earlier.
  - If the cylinders are closer, the presence of the neighboring cylinders affects the separation pattern and therefore the heat transfer levels.
- In the aligned arrangement, the successive tube rows are in the turbulent wake of the previous row. Due to the turbulent nature of the fluid, for a specific range of the longitudinal distance, S_L, the downstream tube rows exhibit higher convection coefficients.
- This convection coefficient increases till about the fifth tube row, beyond which it remains relatively constant.
- For values of $S_T/S_L < 0.7$, the downstream rows are shielded from the flow by the upstream rows and, as a consequence, the heat transfer levels are dramatically reduced.
- This issue is not as pronounced in the staggered arrangement and, therefore, the downstream tube rows exhibit enhanced levels of heat transfer.

Flow Across a Tube Bank (cont.)

- To determine the average heat transfer coefficient across the entire tube bundle with number of rows ($N_L \geq 10$), Grimison proposed the following correlation based on film temperature:

$$\overline{Nu_D} = C_1 Re_{D,max}^{m}$$ valid for $Pr = 0.7$, $2000 \leq Re_D \leq 40{,}000$

- Here, $Re_{D,max} = \frac{\rho V_{max} D}{\mu}$, and C_1 and m are dependent on the ratio of $\frac{S_L}{D}, \frac{S_T}{D}$, and the type of tube arrangement.
- The above equation can be extended to any fluid using a factor $1.13 Pr^{\frac{1}{3}}$

$$\overline{Nu_D} = 1.13 C_1 Re_{D,max}^{m} Pr^{\frac{1}{3}}$$ valid for $Pr \geq 0.7$, $2000 \leq Re_D \leq 40{,}000$

- For $N_L < 10$, the above formula can be modified using a correlation factor:

$$\overline{Nu_D}\Big|_{N_L<10} = C_2 \overline{Nu_D}\Big|_{N_L \geq 10}$$

Please refer to Chapter-7 of Fundamentals of Heat and Mass Transfer by Incropera, DeWitt, Bergmann and Levine for a details regarding the constants used in these correlations.

Flow Across a Tube Bank (cont.)

- The $Re_{D,max}$ is based on the maximum fluid velocity in the bank. Based on conservation of mass for an incompressible fluid, for the aligned arrangement, $V_{\max}$ can be written as:

$$V_{max} = \frac{S_T}{S_T - D} V$$

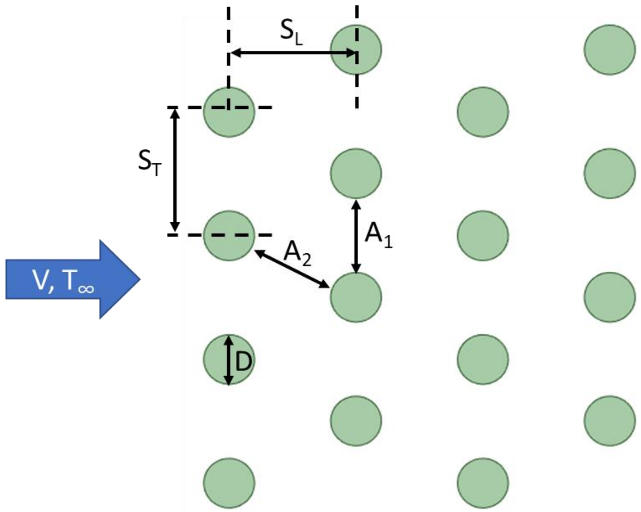
- In the staggered arrangement, the maximum velocity can occur either in the transverse ($A1$) or the longitudinal ($A2$) plane. The maximum velocity will occur in longitudinal ($A2$) plane if the following relation holds:

$$2(S_D - D) < (S_T - D)$$

- The $V_{\max}$ formula for the aligned arrangement can be used if the maximum occurs in $A1$; if not, the following relation can be used to compute $V_{\max}$:

$$V_{max} = \frac{S_T}{2(S_T - D)} V$$

- Zukauskas proposed a correlation where all properties, except Pr_s, are based on the arithmetic mean of inlet and outlet temperature:



$$\overline{Nu_D} = C Re_{D,max}^{m} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{\frac{1}{4}}$$

valid for

$$N_L \geq 20$$

$$0.7 \leq Pr \leq 500$$

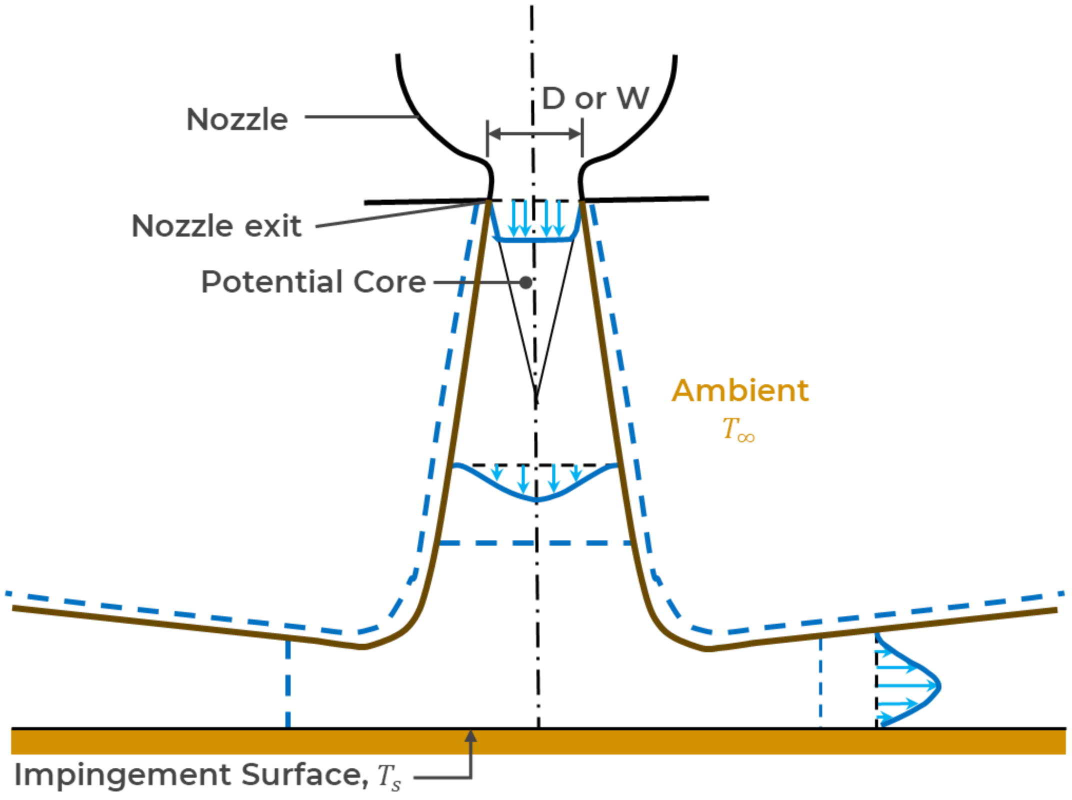
$$1000 \leq Re_{D,max} \leq 2 \cdot 10^6$$

This modified correlation accounts for significant temperature differences between the inlet and outlet temperature.

Please refer to Chapter 7 of Fundamentals of Heat and Mass Transfer by Incropera, DeWitt, Bergmann and Levine for details regarding the constants used in these correlations.

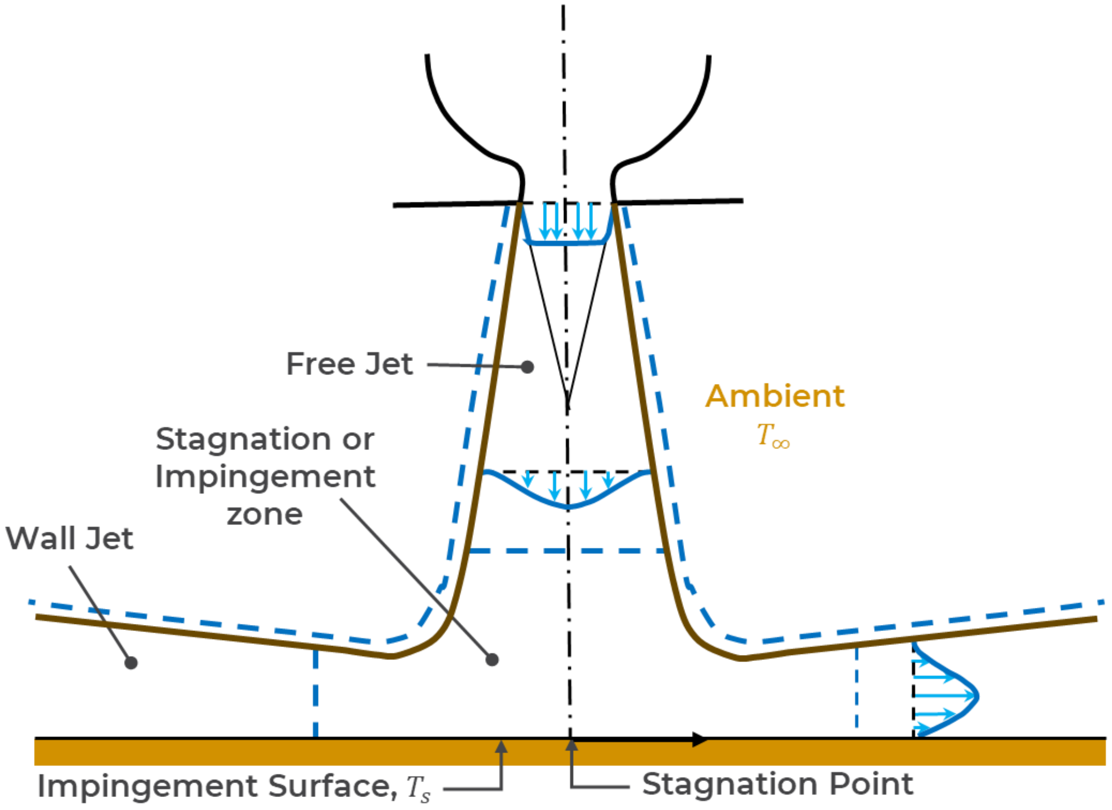
Impinging Jet flow

- A fast-moving flow impinging on a solid surface is referred to as an impinging jet flow.
- Such flow configurations are used to achieve enhanced heat transfer rates in applications such as glass tempering, metal annealing, deicing of aircraft, etc.
- A typical configuration of an impinging jet is shown the figure on the right.
  - A nozzle, which can either be circular/round or rectangular/slot, is used to discharge the jet.
  - The fluid exits the nozzle with a uniform velocity distribution. As it interacts with and entrains the ambient fluid, the jet starts spreading laterally.
  - The potential core or the inviscid central region of the jet starts to contract and eventually collapses. Beyond the core collapse region, the velocity profile across the jet is entirely non-uniform.



Impinging Jet flow (cont.)

- The entire flow field of an impinging jet can be broadly classified into three zones:
  - Free jet zone – In this region, the impact of the impingement surface is not felt, and the jet develops freely.
  - Stagnation/impingement zone – This region is significantly influenced by the presence of the impingement surface.
    - The fluid flow is decelerated in the wall-normal direction, and closer to the plate the flow is forced to change direction.
    - A stagnation point with zero velocity exists where the jet hits the surface. The linear momentum of the jet is transformed into a horizontal acceleration.
  - Wall jet zone – The region in which stagnation flow transforms into a wall jet flow
    - The accelerating horizontal flow in the stagnation zone starts entraining fluid from the ambient.
    - As a result, the flow starts spreading radially outward and takes the form of a wall jet.



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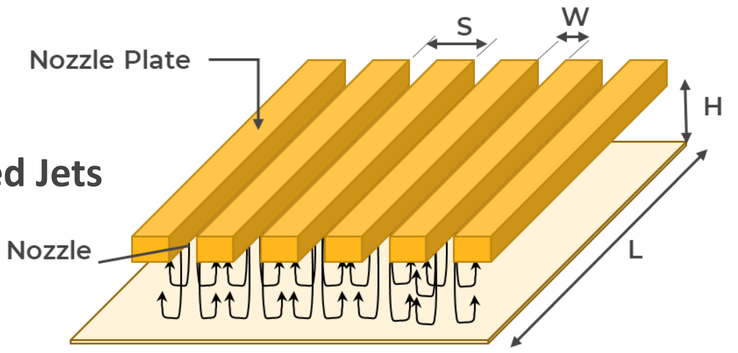
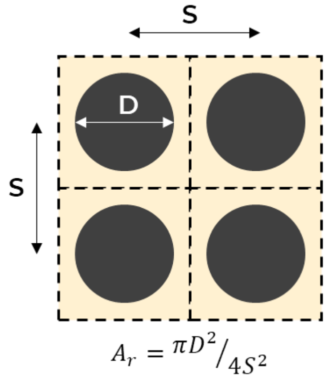
Impinging Jet flow (cont.)

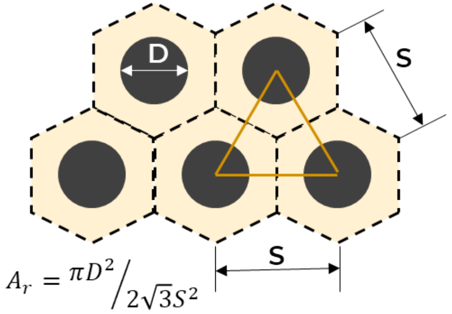
- Most configurations of impinging jets used for heating/cooling applications have multiple jet arrays.
- Due to the interaction of adjoining jets, secondary stagnation zones can also be formed.
- Once the fluid from the jet impacts and extracts/imparts the heat from/to the surface and can no longer be used for heat transfer purposes, this fluid is called spent gas. The temperature of the spent gas is in between that of the nozzle exit and the surface. The overall heat transfer from/to the surface strongly depends on how the spent gas is vented from the system.
- For the configurations shown here, if the spent gas is allowed to escape through the vents between the nozzles, equivalent local and average values of convection coefficients exist in close proximity (dotted region) near the nozzle.
- For single/isolated nozzle, the heat transfer coefficients are dependent on the lateral distance away from the nozzle.

Array of Slotted Jets

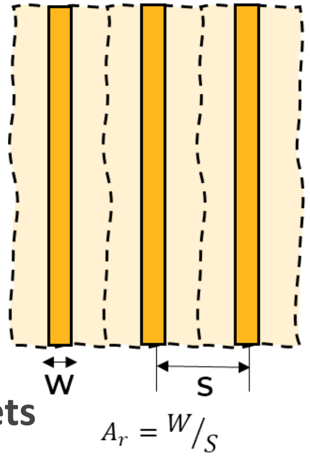




Aligned Round Jets



Staggered Round Jets



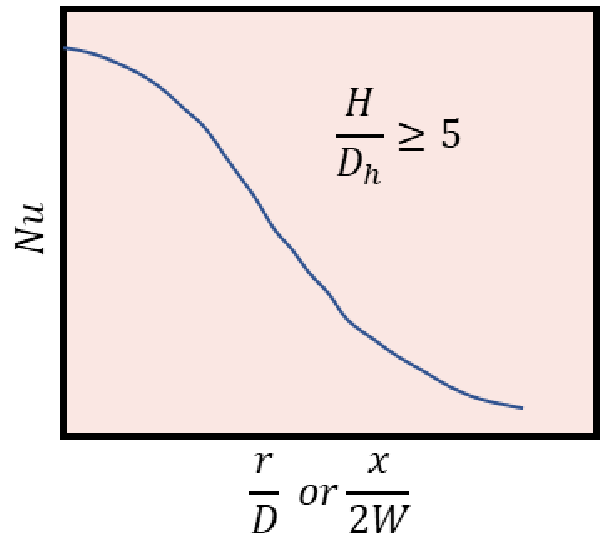
Top view of Slotted Jets

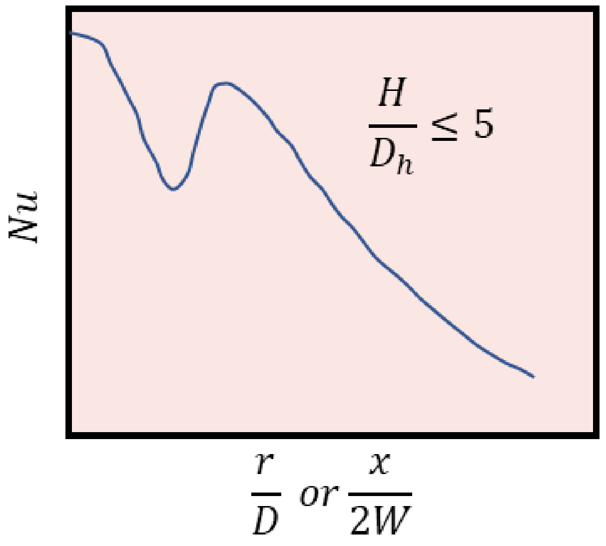
Impinging Jet flow (cont.)

- According to Newton's law of cooling, for a surface temperature of T_s and nozzle exit temperature of T_e, the heat flux can be written as:

$$q'' = h(T_s - T_e)$$

- For a single round nozzle, $Nu = \frac{hD}{k}$ and for a single slot nozzle, $Nu = \frac{h(2W)}{k}$, where W is the width of the slot.
- When the distance between the nozzle exit and the surface is large, i.e., $H/D_h \geq 5$, heat transfer is maximum at the stagnation point and decreases with increasing distance.
- For smaller separations, the Nu number distribution exhibits a second maximum. This is due to the rise in turbulence levels associated with the transformation of the accelerating stagnation region flow to a wall jet.
  - Additional maxima are also possible due to phenomena like vortex formation, laminar-to-turbulent wall jet transition, etc.
- For jet arrays, interaction of adjoining jets also results in secondary maxima. For such configurations, the heat transfer distribution is also 2D, i.e., variations of heat transfer coefficient can be observed in both directions parallel to the surface.





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Impinging Jet flow (cont.)

- To obtain average Nu numbers, the local values are integrated over an appropriate surface area.
- For a single round jet, Martin proposed the following correlation:

$$\frac{\overline{Nu}}{Pr^{0.42}} = G\left(A_r, \frac{H}{D}\right)\left[2Re^{\frac{1}{2}}(1 + 0.005Re^{0.55})^{\frac{1}{2}}\right]$$

where

$$G = \frac{2A_r^{\frac{1}{2}}\left(1 - 2.2A_r^{\frac{1}{2}}\right)}{1 + 0.2\left(\frac{H}{D} - 6\right)A_r^{\frac{1}{2}}}$$

Valid for

$0.004 \le A_r \le 0.04$

$2 \le H/D \le 12$

$2000 \le Re \le 400{,}000$

- For an array of round jets,

$$\frac{\overline{Nu}}{Pr^{0.42}} = 0.5K\left(A_r, \frac{H}{D}\right)G\left(A_r, \frac{H}{D}\right)Re^{\frac{2}{3}}$$

where

$$K = \left[1 + \left(\frac{H/D}{0.6\,/\,(A_r^{\frac{1}{2}})}\right)^{6}\right]^{-0.05}$$

Valid for

$0.004 \le A_r \le 0.04$

$2 \le H/D \le 12$

$2000 \le Re \le 100{,}000$

- For a slot jet, the following correlation can be used to obtain averaged Nu number:

$$\frac{\overline{Nu}}{Pr^{0.42}} = \frac{3.06}{0.5/A_r + H/W + 2.78}Re^{m}$$

where

$$m = 0.695 - \left[\left(\frac{1}{4A_r}\right) + \left(\frac{H}{2W}\right)^{1.33} + 3.06\right]^{-1}$$

Valid for

$0.025 \le A_r \le 0.125$

$2 \le H/W \le 10$

$3000 \le Re \le 90{,}000$

- For an array of slot jets,

$$\frac{\overline{Nu}}{Pr^{0.42}} = \frac{2}{3}A_{r,o}^{\frac{3}{4}}\left(\frac{2Re}{A_r/A_{r,o} + A_{r,o}/A_r}\right)^{\frac{2}{3}}$$

where

$$A_{r,o} = \left[60 + 4\left(\frac{H}{2W} - 2\right)^{2}\right]^{-1/2}$$

Valid for

$0.008 \le A_r \le 2.5A_{r,o}$

$2 \le H/W \le 80$

$1500 \le Re \le 40000$

Please refer to Chapter 7 of Fundamentals of Heat and Mass Transfer by Incropera, DeWitt, Bergmann and Levine for details regarding the constants used in these correlations.

Summary

- In this lesson, our focus was on convection heat transfer characteristics in several canonical external flows.
- Specifically, we analyzed in detail three different flow configurations:
  - Cross-flow over a cylinder
  - Cross-flow over a tube bank
  - Impinging jet flow

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