Internal Convection: Correlations and Enhancements

How Heat Exchangers Work – Lesson 2

Laminar and Turbulent Pipe Flow Correlations

- In many applications the heat transfer coefficient must be known to predict the convection heat transfer.
- We will see how we can define some correlations for laminar and turbulent flows that let us estimate the Nusselt number, in order to obtain the heat transfer coefficient for the specific geometry.
- The underlying assumptions for these analyses are:
  - The tube is a circular pipe with constant cross-sectional area
  - The heat conduction in the axial direction is neglected and the heat propagates only radially
  - The thermal energy is advected only in the axial direction
  - The flow is steady, incompressible and with constant properties

Laminar Pipe Flow Correlations

- Let's start with analyzing a laminar flow in the fully developed region. Assume a control volume of fluid. We can write the thermal equilibrium of this element as:

$$q_r - \left(q_r + \frac{\partial q_r}{\partial r}dr\right) = d\dot{m}c_p\left[\left(T + \frac{\partial T}{\partial x}dx\right) - T\right]$$

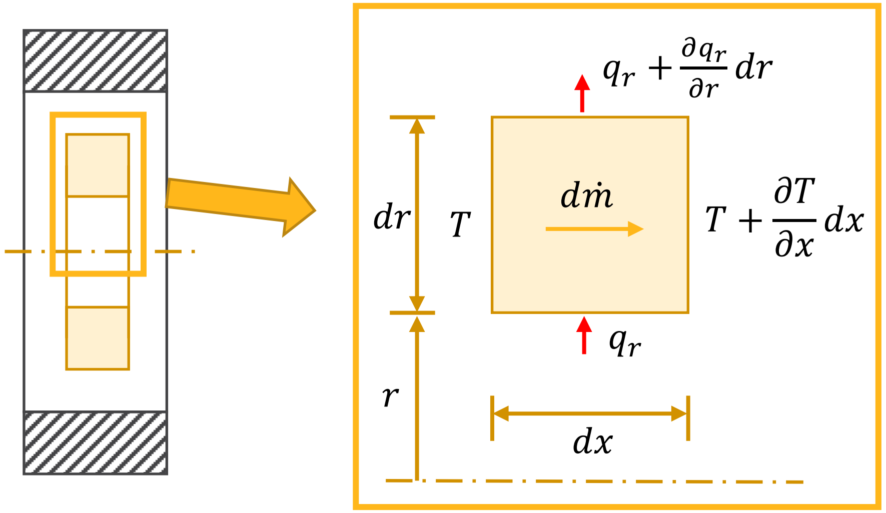
- The mass flow rate and the radial heat transfer can be expressed as:

$$d\dot{m} = 2\pi\rho u r dr$$

$$q_r = -2\pi r k\left(\frac{\partial T}{\partial r}\right)dx$$

- Combining the three expressions we obtain this final relation:

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$



- Depending on the wall conditions we would get different relations. Next, we will analyze the case of constant surface heat flux and constant surface temperature.

Laminar Pipe Flow Correlations (cont.)

- Let's first analyze the case of constant surface heat flux. In this case, the previous relation can be manipulated and integrated into this final equation:

$$T_m(x) - T_s(x) = -\frac{11}{48}\left(\frac{u_m r_0^2}{\alpha}\right)\frac{dT_m}{dx} = -\frac{11}{48}\frac{q_s'' D}{k}$$

- This directly gives us a relation for the heat transfer coefficient and for the Nusselt number:

$$h = \frac{48}{11}\frac{k}{D}$$

$$Nu_D = \frac{hD}{k} = 4.36$$

- Next, we can analyze the case of constant surface temperature. In this case the heat balance can be recast in this form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_m}{\alpha}\frac{dT_m}{dx}\left[1 - \left(\frac{r}{r_0}\right)^2\right]\frac{T_s - T}{T_s - T_m}$$

- The solution can be obtained numerically using iterative methods. The profile does not have an algebraic expression. However, the Nusselt number for this case can be derived:

$$Nu_D = 3.66$$

- In both cases, for fully developed laminar flows in a circular pipe with constant diameter, the Nusselt number is constant and independent on the Reynolds and Prandtl numbers. Once we have the Nusselt number we can estimate the heat transfer coefficient.

Laminar Pipe Flow Correlations (cont.)

- Let's now analyze the entrance region for laminar flows. In this case the thermal profile is not fully developed, and several of the assumptions made in the previous analysis are not valid anymore. The axial heat transfer cannot be neglected anymore, and the advection acts also in the radial direction.

- We can study two different situations:

  - The thermal entrance length problem, where only the velocity profile is assumed to be fully developed.

  - The combined entrance length problem, where both velocity and thermal profiles are developing.

- The following are suitable correlations that can be used to estimate the Nusselt number along the entrance length:

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 3.66 + \frac{0.0668\left(\frac{D}{L}\right)Re_D Pr}{1 + 0.04\left[\left(\frac{D}{L}\right)Re_D Pr\right]^{2/3}}$$

Thermal entrance length
&
Combined entrance length for $\mathrm{Pr} \gtrsim 5$

$$\overline{Nu}_D = 1.86\left(\frac{Re_D Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

Combined entrance length for
$0.6 \lesssim \mathrm{Pr} \lesssim 5$
$0.0044 \lesssim (\mu/\mu_s) \lesssim 9.75$

Note that the properties should be estimated at $\bar{T}_m = (T_{m,i} - T_{m,o})/2$ (except μ_s)

Turbulent Pipe Flow Correlations

- When dealing with turbulent flows, it is often impossible to derive simple algebraic expressions for the quantities of interest. Hence, most of the relations we typically use for turbulent flows are based on empirical correlations.
- There are numerous correlations that can be found in literature. However, for the scope of this course, we will analyze only the most commonly used. These relations apply both to constant surface temperature and constant heat flux cases.

Good for small properties variations

$$Nu_D = 0.023 Re_D^{4/5} \mathrm{Pr}^{0.4}$$

Use for heating $T_s > T_m$

$$Nu_D = 0.023 Re_D^{4/5} \mathrm{Pr}^{0.3}$$

Use for cooling $T_s < T_m$

Valid for:
$0.7 \lesssim \mathrm{Pr} \lesssim 160$
$Re_D \gtrsim 10{,}000$
$L/D \gtrsim 10$

Better for large properties variations

$$Nu_D = 0.027 Re_D^{4/5} \mathrm{Pr}^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

Fluid viscosity at wall temperature (μ_s)

Valid for:
$0.7 \lesssim \mathrm{Pr} \lesssim 16{,}700$
$\mathrm{Re}_D \gtrsim 10{,}000$
$L/D \gtrsim 10$

- These relations take into account the effect of viscosity variation on heat transfer due to temperature. Note that for these relations the properties should be estimated at T_m (except μ_s).

Ansys

Turbulent Pipe Flow Correlations (cont.)

- The following correlation is valid for a larger range of Reynolds numbers that includes also the transition region, but requires the friction factor f:

$$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(Pr^{2/3} - 1)(f/8)^{0.5}}$$

Valid for:
$3000 \lesssim Re_D \lesssim 5 \times 10^6$ $0.7 \lesssim \mathrm{Pr} \lesssim 2000$

- Even in this case the properties should be estimated at T_m. The friction factor can be obtained from the Moody Diagram.

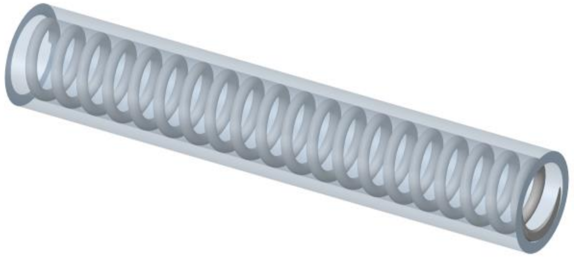
- In the entrance region, we can assume to have the same Nusselt number as in the fully developed region, since the entrance length is relatively short.

- However, in case of short tubes, the average Nusselt number would be a function of the inlet shape, Re, Pr and entry region class (thermal, combined).

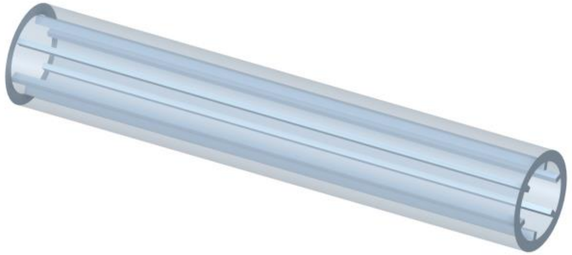
Heat Transfer Enhancements

- How can we enhance heat transfer along pipes? There are many different ways to do so:
  - Longitudinal fins or helical ribs can increase the inner surface area of the tube and improve the convection coefficient
  - Adding coil-spring wires inside the tube can enhance the turbulence inside the tube
  - Twisted tapes inside the tube make the flow swirl and add a tangential component to the fluid motion
- Some of these techniques are used, for example, to enhance heat transfer in solar collector pipes, in order to better heat up water flowing through the tubes.

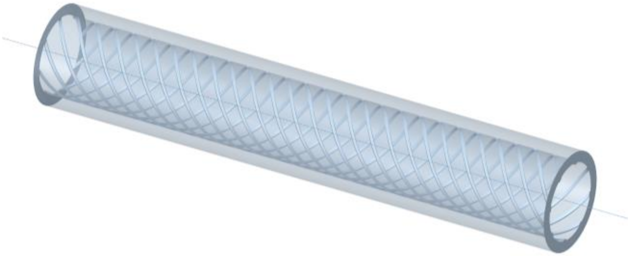
Solar collectors



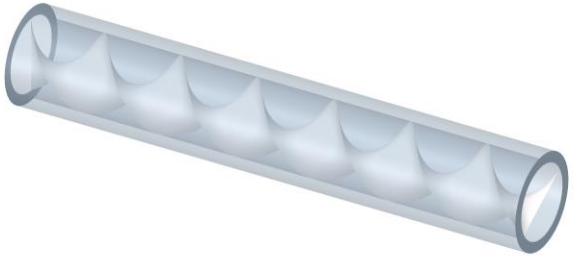
Coil-spring wire



Longitudinal fins



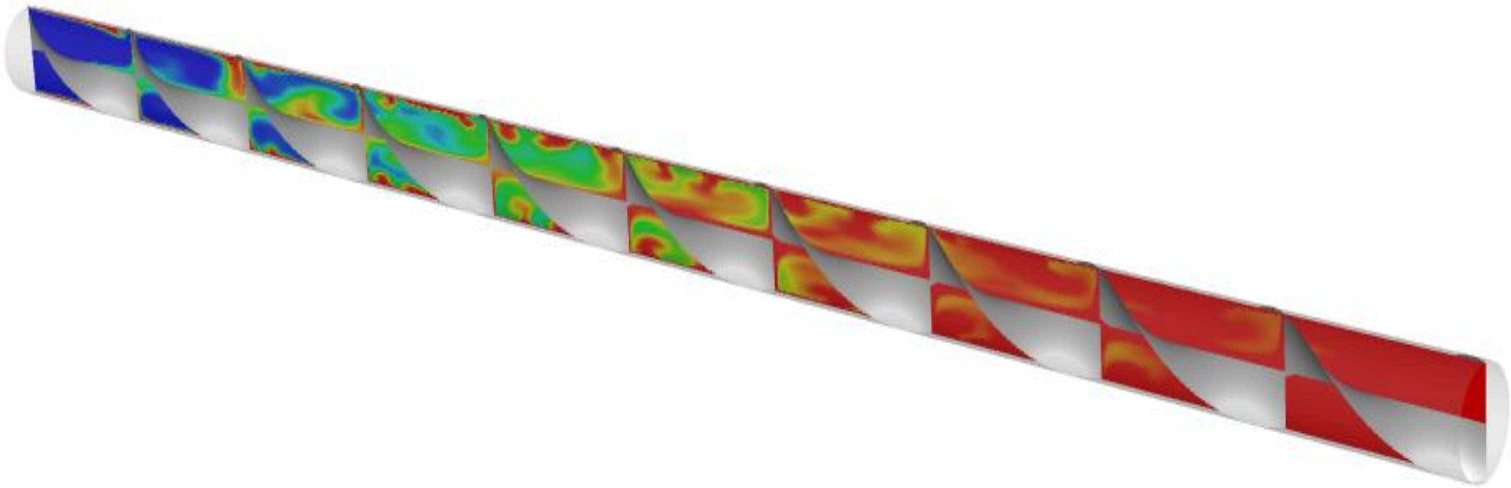
Helicoidal ribs



Twisted tape

Heat Transfer Enhancements (cont.)

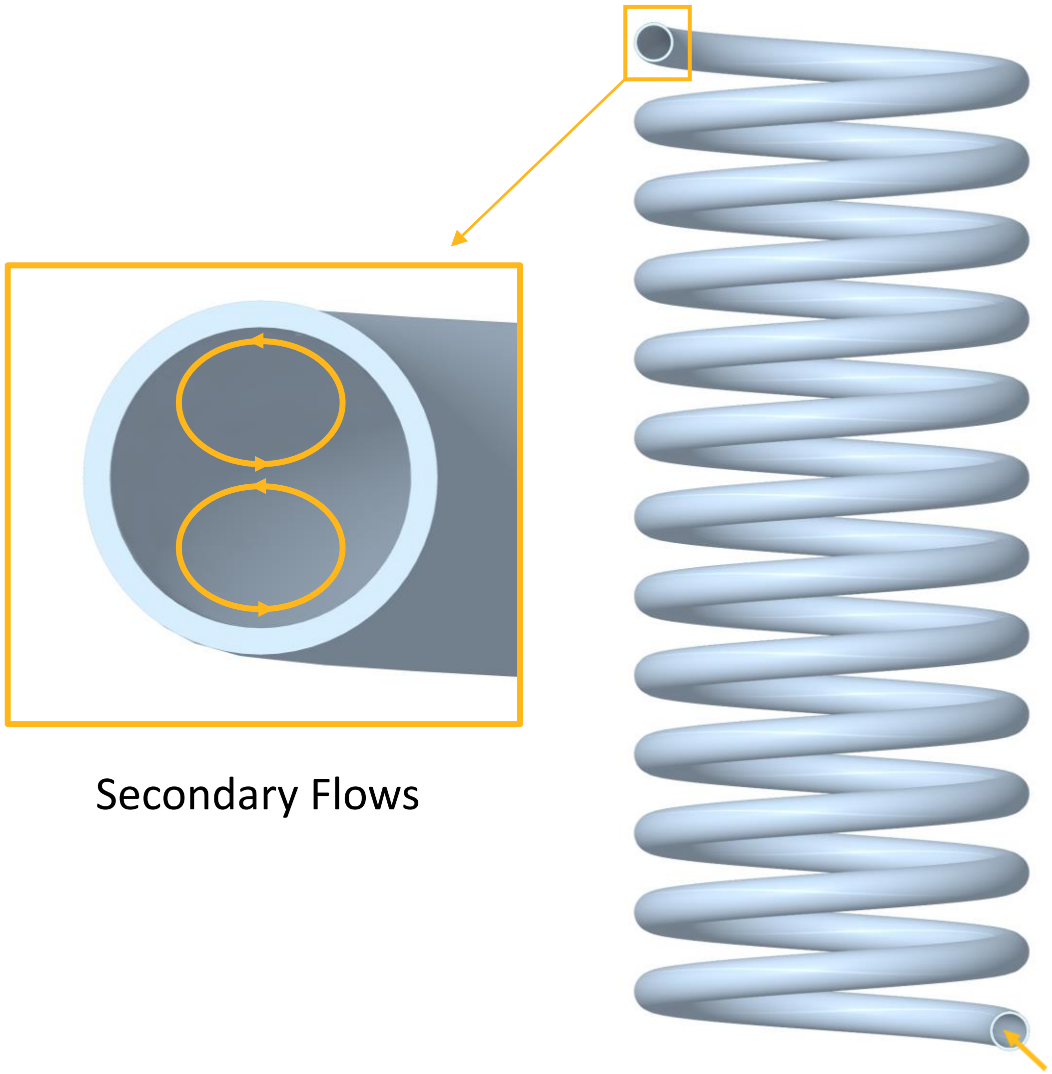
- All these design enhancements come at a cost. The pressure drop increases when using these designs, hence more power is needed from a pump to move the fluid though the pipes.
- Also, not all the heat transfer enhancers are equally appropriate. Designers must avoid any insert which can break up and possibly damage the pump.
- Depending on the flow rates, sturdier coil-spring wires or twisted tapes are preferred.



Temperature contours along a turbulator

Heat Transfer Enhancements (cont.)

- Another approach is to coil the tube without using any insert.
- The curved shape of the tube creates a longitudinal secondary flow, driven by two counter-rotating vortices. A tangential velocity component is added to the flow, which increases the fluid velocity near the wall. This enhances the heat transfer rate and reduces the entrance length.
- This design has another advantage: The secondary flow helps to mix the fluid, and the temperature across a section of the tube is more uniform.
- For this reason, we can find this design commonly used in chemical processes to produce pharmaceutical products.
- The secondary flows, however, also increase the pressure losses along the tube. These must be accounted for when designing the system.



Secondary Flows

Summary

- In this lesson we analyzed the heat transfer due to convection in internal flows.
- We defined the difference between hydrodynamic and thermally fully developed flows and found ways to estimate the respective entrance lengths.
- We learned how to perform a heat transfer analysis on internal flows for constant heat flux and constant surface temperature conditions. Then, we discussed useful correlations to estimate the Nusselt number at different operating conditions for laminar and turbulent flows, so that we can estimate the expected heat transfer coefficient.
- Last, we analyzed different designs to enhance the heat transfer in tubes and pipes.

Ansys