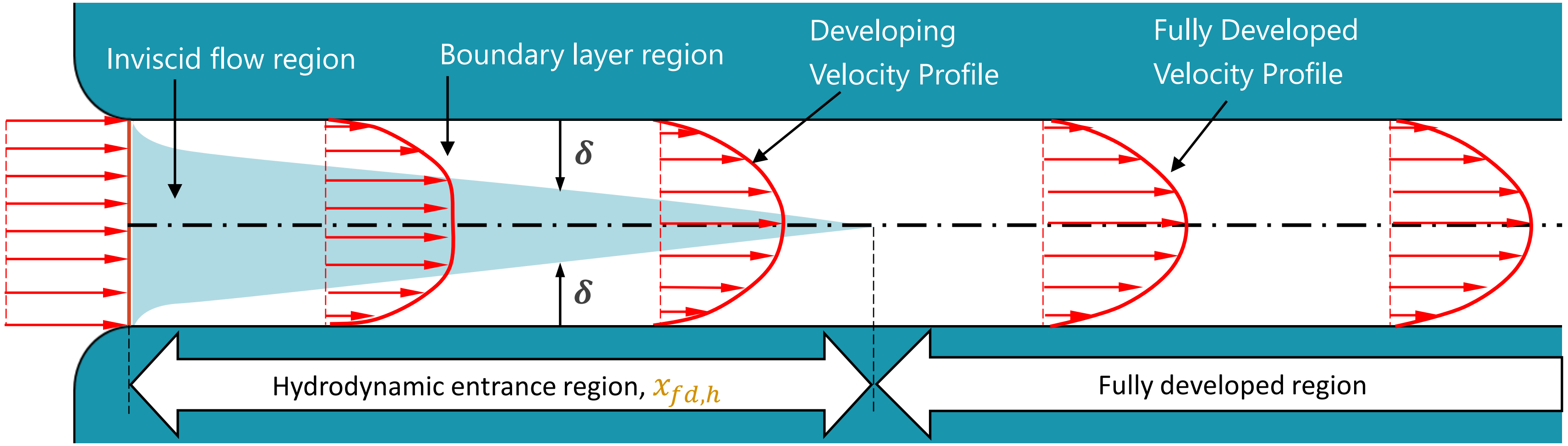
Basics of Internal Forced Convection

How Heat Exchangers Work – Lesson 1

Velocity Field in Internal Flows

- As a fluid enters a pipe, a boundary layer along the wall starts developing at the entrance. The layer is thin near the entrance and viscous effects are restricted to the near-wall region.
- The boundary layer grows as the fluid flows downstream, until eventually the layer edge reaches the pipe centerline, and the flow becomes fully developed.
- A fully developed velocity profile does not change as the fluid flows downstream.
- The length over which the flow evolves into the fully developed state is called the hydrodynamic entrance length ($x_{fd,h}$).



Velocity Field in Internal Flows (cont.)

- The velocity profile shape and the entrance length depend on whether the flow is laminar or turbulent.

- We can use the Reynolds number for a circular pipe to define whether the flow is laminar or turbulent and introduce simple relations to estimate the hydrodynamic entrance length.

$$Re_D = \frac{\rho V_\infty D}{\mu}$$

Laminar flow

$$Re_D \lesssim 2300$$

Hyd. Entrance Length

$$\left(\frac{x_{fd,h}}{D}\right) \approx 0.05 Re_D$$

Turbulent flow

$$Re_D \gtrsim 4000$$

Hyd. Entrance Length

$$\left(\frac{x_{fd,h}}{D}\right) \approx 4.4 Re_D^{1/6}$$

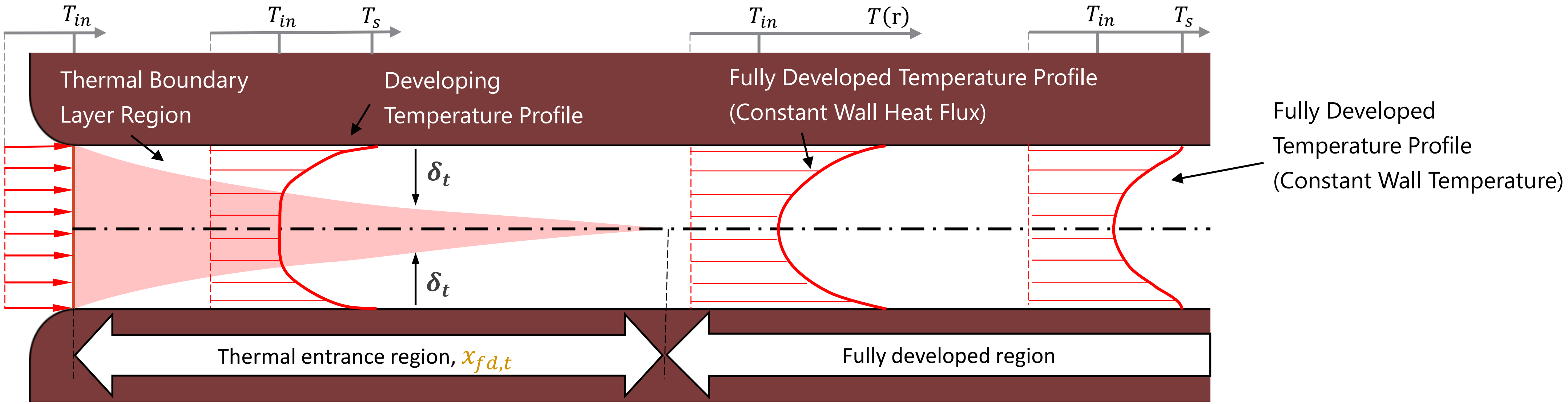
- In this course we will assume fully developed turbulent flow as $x/D > 10$, since past that length the entrance effects can be neglected for practical engineering applications.

Thermal Field in Internal Flows

- Assume that the fluid enters the pipe at a temperature lower than the pipe walls. In this case, a thermal boundary layer would start to form along the wall due to convection.

- As the fluid moves downstream, the thermal boundary layer grows. If the surface of the wall is at constant temperature or it is maintained at a constant heat flux, then fluid eventually becomes thermally fully developed. The length over which the flow evolves into the thermally fully developed state is called the thermal entrance length ($x_{fd,t}$).

- The shape of fully developed thermal profile depends on the wall conditions. The temperature difference between the fluid temperature and the entrance temperature varies in the axial direction.



Thermal Field in Internal Flows (cont.)

- Similarly, as was done for the velocity field, we can use the Reynolds number for a circular pipe to the hydrodynamic entrance length.

Thermal Entrance Length

$$\left(\frac{x_{fd,t}}{D}\right) \approx 0.05 Re_D Pr$$

Laminar flow

Thermal Entrance Length

$$\left(\frac{x_{fd,t}}{D}\right) \approx 10$$

Turbulent flow

- For laminar flows, the thermal entrance length can be longer or shorter than the hydrodynamic entrance length. Indeed, depending on the Prandtl number the thermal boundary layer can grow slower or faster than the hydrodynamics boundary layer.

$$Pr > 1 \longrightarrow x_{fd,t} > x_{fd,h}$$

$$Pr < 1 \longrightarrow x_{fd,t} < x_{fd,h}$$

- On the other hand, for turbulent flows the length is nearly independent of the Prandtl number and we can assume it to be $x_{fd,t}/D \approx 10$. This is because the turbulence enhances the transport of heat, causing the velocity and thermal layers to be about the same thickness.

Concepts of Mean Velocity and Mean Temperature

- Since the velocity and temperature vary along the pipe's cross section, we need to define a mean velocity (u_m) and a mean temperature (T_m).

- The mean velocity is the velocity that would provide the mass flow rate of a tube, given the tube's cross - sectional area and fluid density.

$$\dot{m} = \rho u_m A_C \longrightarrow u_m = \frac{\dot{m}}{\rho A_C}$$

- For an incompressible flow in a circular pipe, we can obtain the following expression for the mean velocity:

$$u_m = \frac{\int_{A_C} \rho u(r,x) dA_C}{\rho A_C} = \frac{2}{r_0} \int_0^{r_0} u(x,r)\, rdr$$

Concepts of Mean Velocity and Mean Temperature (cont.)

- The mean, or bulk, temperature is defined such that the product $\dot{m}c_pT_m$ is equal to the thermal energy transported by the fluid through a cross section of the duct.

Advected Thermal Energy

$$\dot{m}c_pT_m = \int_{A_C} \rho u c_p T dA_C$$

$$T_m = \frac{\int_{A_C} \rho u c_p T dA_C}{\dot{m}c_p}$$

- For an incompressible flow in a circular pipe, we can obtain the following expression for the mean temperature:

$$T_m = \frac{2}{u_m r_0^2}\int_0^{r_0} uTrdr$$

General Heat Transfer Analysis in Internal Flows

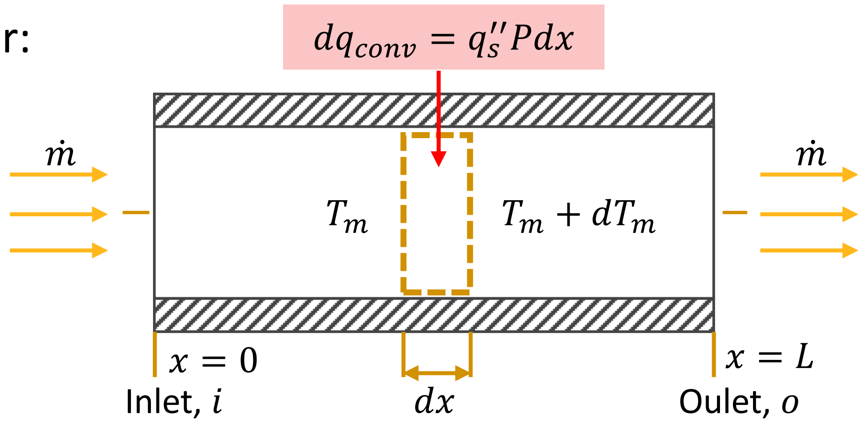
- For an incompressible fluid or an ideal gas subjected to small pressure variations, we can make an energy balance to obtain an expression for the mean temperature as function of the axial location $T_m(x)$.
- Let's first analyze the heat transfer for a fluid moving through a finite tube. Neglecting the effect of viscous dissipation and conduction in the tube's axial direction, we can define the total convection heat transfer as the difference between the thermal energy advected that enters and exits the tube:

$$q_{conv} = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right)$$

- This equation does not depend on the flow conditions or the wall thermal conditions.

General Heat Transfer Analysis in Internal Flows (cont.)

- Now let's analyze a simple control volume inside the tube. We can quickly obtain the following differential expression for the convection heat transfer:



$$dq_{conv} = \dot{m}c_p[(T_m + dT_m) - T_m] = \dot{m}c_p dT_m$$

- We can define the heat transfer in terms of the heat flux through the tube surface as:

Heat flux Perimeter

$$dq_{conv} = q_s''Pdx$$

- Substituting terms yields this differential expression for the mean temperature:

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h(T_s - T_m)$$

If $T_s > T_m$ the heat would move from the tube to the fluid and T_m would increase along the tube. If $T_s < T_m$ the opposite would happen.

General Heat Transfer Analysis in Internal Flows (cont.)

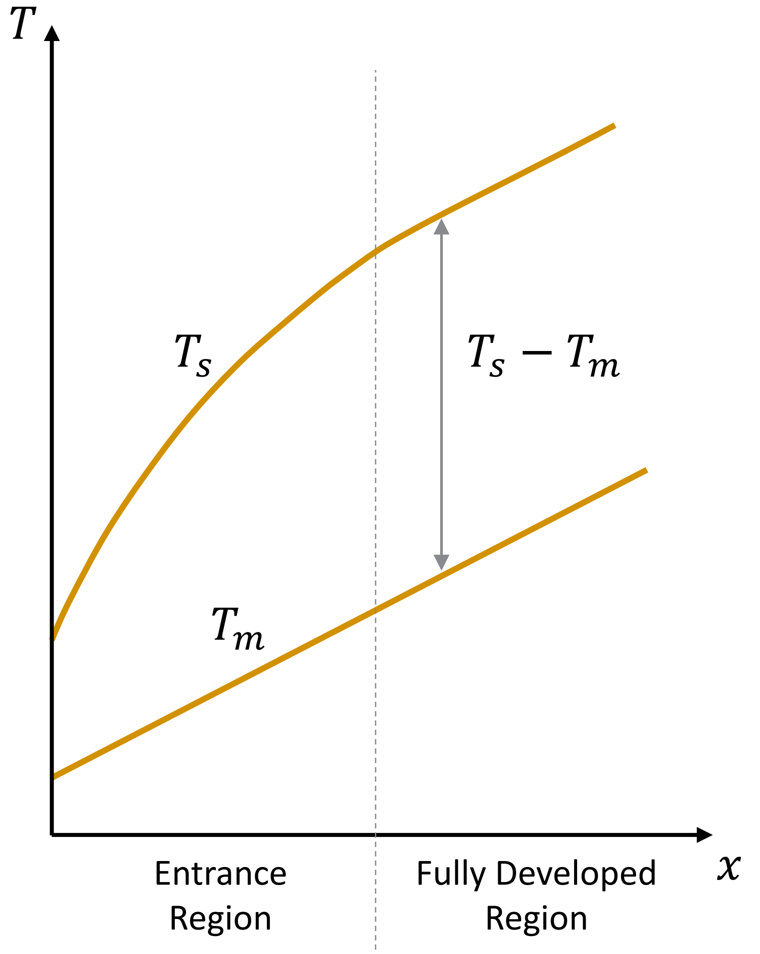
- We can find solutions of the equation we just derived for the mean temperature using specific wall conditions:
  - Constant surface heat flux
  - Constant surface temperature
- For the following analyses we will assume that the cross-sectional perimeter is constant along the tube.
- Let's start with the first case. Since the heat flux q_s'' is constant we can immediately estimate the total convection heat transfer as:

$$\boxed{q_{conv} = q_s'' P L}$$

- Also, the term on the right-hand side for the mean temperature relation is not a function of x anymore. In this case the mean temperature varies linearly along the tube.

$$\boxed{\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = const.} \longrightarrow \boxed{T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x}$$

- It is interesting to note that when the flow becomes fully developed, the heat transfer coefficient stays constant. Hence, the difference between the wall surface temperature and the mean temperature remains constant too.



General Heat Transfer Analysis in Internal Flows (cont.)

- The second case in the analysis is the constant surface temperature (T_s) at the wall. In this case it is convenient to recast the original equation in terms of $\Delta \mathrm{T} = T_s - T_m$:

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h\Delta T$$

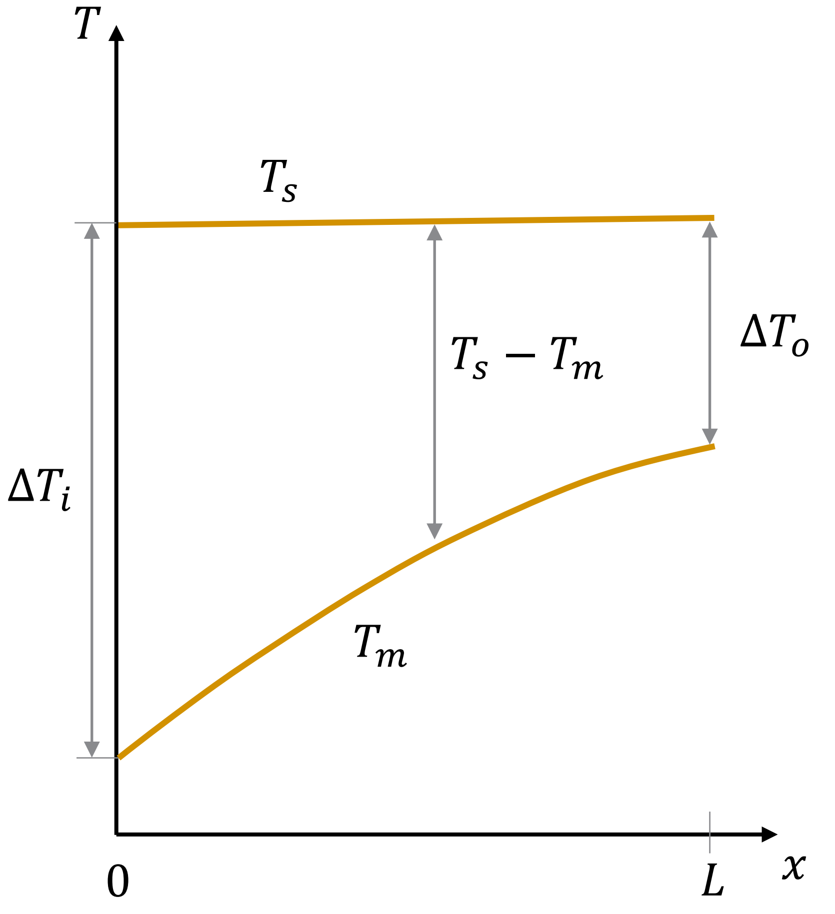
- Separating the variables and integrating along the tube (x) we obtain:

$$\frac{\Delta T_x}{\Delta T_i} = \frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right)$$

- Where $\bar{h}$ is the average convection heat transfer coefficient:

$$\bar{h} = \frac{1}{x}\int_0^x h dx$$

- This relation shows how the temperature difference decays exponentially as the flow moves downstream.



General Heat Transfer Analysis in Internal Flows (cont.)

- Applying the expression in the previous slide to the entire tube, from start to end, we obtain this similar relation:

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right)$$

- If we recast the heat transfer expression in terms of temperature variations, we can derive this expression:

PL

$$q_{conv} = \dot{m}c_p(\Delta T_i - \Delta T_o) = \bar{h}A_s\Delta T_{lm}$$

- Where ΔT_{lm} is the log mean temperature difference along the tube:

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln \Delta T_o / \Delta T_i}$$

Summary

- In this lesson we analyzed the heat transfer due to convection in internal flows.
- We defined the difference between hydrodynamic and thermally fully developed flows and found ways to estimate the respective entrance lengths.
- We learned how to perform a heat transfer analysis on a pipe and analyzed the special cases of constant heat flux and surface constant temperature wall conditions.

Ansys

Ansys