Natural Convection Boundary Layers

Natural Convection – Lesson 2

Dimensionless Parameters for Natural Convection

- Let's start by analyzing the dimensionless parameters that govern natural convection. Consider the following nondimensional variables:

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L} \qquad v^* \equiv \frac{v}{u_0} \qquad u^* \equiv \frac{u}{u_0} \qquad T^* \equiv \frac{T - T_\infty}{T_s - T_\infty}$$

- Here L is a characteristic length and u_0 is the reference velocity.
- The x -momentum and energy equations reduce to the following:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{u_0^2} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

- The dimensionless parameter in the first term on the right-hand side of the x -momentum equation is the contribution of the buoyancy force.

Grashof Number

- If we choose the reference velocity u_0 such that

$$u_0^2 = g\beta(T_s - T_\infty)L$$

- The coefficient of the T^* term in the x-momentum equation becomes unity

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{u_0^2} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

1

and the corresponding Reynolds number can be written as

$$Re_L = \sqrt{\left[\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\right]}$$

- The square of this Reynolds number is defined as the Grashof number:

$$Gr_L \equiv \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

Grashof Number (cont.)

- The Grashof number is a measure of the ratio of the *buoyancy forces* to the *viscous forces* acting on the fluid.
- Thus, Gr plays the same role in natural convection that Re plays in forced convection.
- Heat transfer correlations in natural convection are of the form:

$$Nu_L = f(Gr_L, Pr)$$

- When forced and natural convection effects are comparable, it is more convenient to choose the reference velocity as u_∞.
- The coefficient of the T^* term is no longer unity but is of the form $\frac{Gr_L}{Re_L^2}$:

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{u_0^2} T^* + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

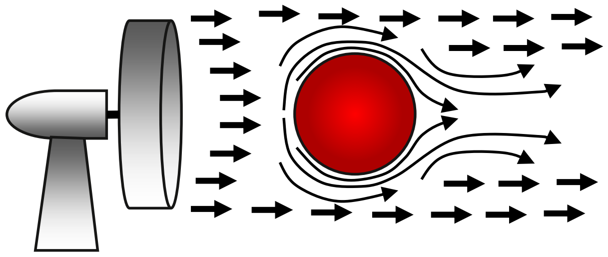
- The resulting Nusselt number correlations are of the form:

$$Nu_L = f(Re_L, Gr_L, Pr)$$

Grashof Number (cont.)

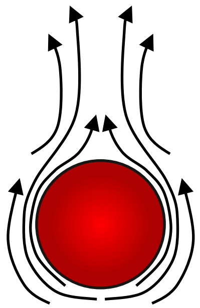
- In such situations, depending on the value of Gr_L/Re_L^2, we can have the following:

$\frac{Gr_L}{Re_L^2} \ll 1$ Natural convection effects may be neglected and $Nu_L = f(Re_L, Pr)$



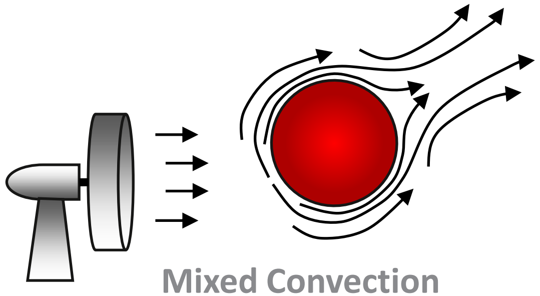
Forced Convection

$\frac{Gr_L}{Re_L^2} \gg 1$ Forced convection effects may be neglected and $Nu_L = f(Gr_L, Pr)$



Natural Convection

$\frac{Gr_L}{Re_L^2} \approx 1$ Combined effects of natural and forced convection needs to be considered and $Nu_L = f(Re_L, Gr_L, Pr)$

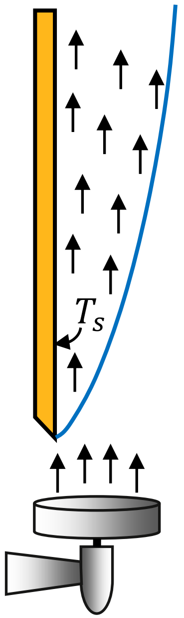


Mixed Convection

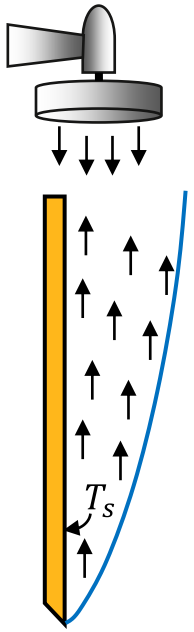
Mixed Convection

- In many situations, it may be inappropriate to neglect either natural or forced convection ($Gr_L/Re_L^2 \approx 1$). This is known as mixed (or combined) convection.
- Natural convection may help or hurt the forced convection heat transfer depending on the relative directions of buoyancy-induced and forced convection motions.

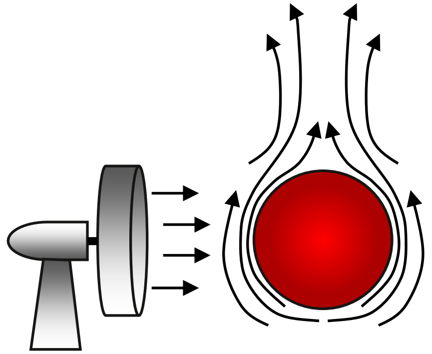
  - *Assisting flow:* The buoyant motion is in the same direction as the forced motion and therefore natural convection assists forced convection and enhances heat transfer. One example is an upward forced flow over a hot surface.
  - *Opposing flow:* The buoyancy-induced motion is in the opposite direction to the forced motion. Natural convection resists forced convection and decreases heat transfer. One example is an upward forced flow over a cold surface.
  - *Transverse flow:* The buoyancy-induced motion is perpendicular to the forced motion. The transverse flow enhances fluid mixing and, in the process, also enhances the heat transfer. One example is horizontal forced cross-flow over a hot cylinder.



Assisting Flow



Opposing Flow


Transverse Flow

Mixed Convection (cont.)

- The Nusselt number for mixed convection is often expressed using the following:

$$Nu_{mixed} = \left(Nu_{forced}^{n} \pm Nu_{natural}^{n}\right)^{\frac{1}{n}}$$

- Nu_{forced} and $Nu_{natural}$ are determined from existing correlations for pure forced and natural convection.
- The plus sign on the right-hand side of the equation is for assisting and transverse flows, whereas the minus sign applies to opposing flows.
- The value of the exponent n varies between 3 and 4, depending on the geometry involved.
- It is observed that $n = 3$ correlates experimental data for vertical surfaces well.
- Values of $n = 3.5$ and 4 are better suited for transverse flows involving horizontal plates and cylinders (or spheres), respectively.

Laminar Natural Convection on a Vertical Surface

- Consider natural convection on an isothermal vertical surface immersed in an extensive quiescent medium as shown.
- The governing equations of natural convection must be solved subject to the following boundary conditions:

$$y = 0: \quad u = v = 0 \quad T = T_s$$

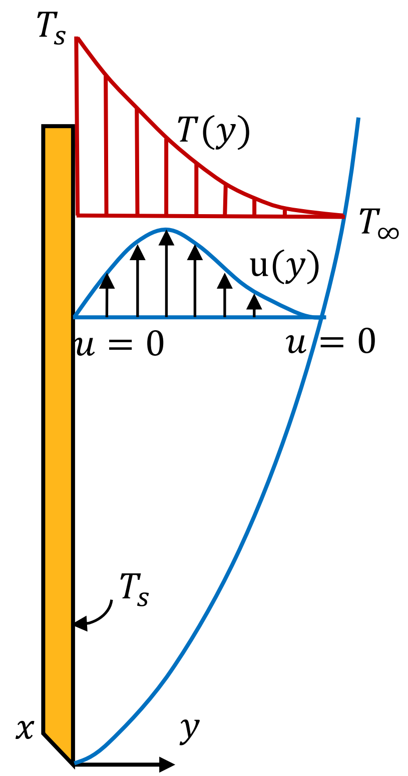
$$y \to \infty: \quad u \to 0 \quad T \to T_\infty$$

- Ostrach obtained a similarity solution by introducing a similarity parameter of the form:

$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$

- The velocity components can be defined in terms of the stream function shown below:

$$\psi(x,y) \equiv f(\eta)\left[4\nu\left(\frac{Gr_x}{4}\right)^{1/4}\right]$$



Boundary layer development on a hot plate immersed in a cooler fluid

Laminar Natural Convection on a Vertical Surface (cont.)

- The x-component of the velocity can be written using the stream function as:

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = 4\nu\left(\frac{Gr_x}{4}\right)^{1/4} f'(\eta)\frac{1}{x}\left(\frac{Gr_x}{4}\right)^{1/4} = \frac{2\nu}{x} Gr_x^{\frac{1}{2}} f'(\eta)$$

- Similarly, we can compute the y-component of the velocity as $v = -\partial\psi/\partial x$ and, using the dimensionless temperature used earlier, the three partial differential governing equations are reduced to two ordinary differential equations of the form:

$$f''' + 3ff'' - 2(f')^2 + T^* = 0$$

$$T^{*\prime\prime} + 3Pr f T^{*\prime} = 0$$

- Here f and T^* are functions of only η and the double and triple primes refer to second and third derivatives with respect to η respectively. The new transformed boundary conditions for this system of equations corresponds to:

$\eta = 0$	$f = f' = 0$	$T^* = 0$
$\eta \to \infty$	$f' \to 0$	$T^* \to 0$

Laminar Natural Convection on a Vertical Surface (cont.)

- From Newton's law of cooling for the local convection coefficient h, the local Nusselt number can be expressed as:

$$Nu_x = \frac{hx}{k} = \frac{\left[\frac{q_s''}{T_s - T_\infty}\right]x}{k}$$

- Using Fourier's law, we can obtain q_s'' as shown below:

$$q_s'' = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -\frac{k}{x}(T_s - T_\infty)\left(\frac{Gr_x}{4}\right)^{1/4}\frac{dT^*}{d\eta}\bigg|_{\eta=0}$$

- And therefore, we can write:

$$Nu_x = \frac{hx}{k} = -\left(\frac{Gr_x}{4}\right)^{1/4}\frac{dT^*}{d\eta}\bigg|_{\eta=0} = \left(\frac{Gr_x}{4}\right)^{1/4} g(Pr)$$

- The dimensionless temperature gradient at the surface is a function of the Prandtl number $g(Pr)$, which is also evident from the plots of the temperature profiles in the previous slide.

Laminar Natural Convection on a Vertical Surface (cont.)

- Using interpolation, we can express $g(Pr)$ as:

$$g(Pr) = \frac{0.75\, Pr^{1/2}}{(0.609 + 1.221Pr^{1/2} + 1.238Pr)^{1/4}}$$

For $0 \le Pr \le \infty$

- The local Grashof number can be expressed as:

$$Gr_x = \frac{g\beta(T_s - T_\infty)x^3}{\nu^2}$$

- Using this and the local Nusselt number relation, we can express the average convection coefficient for a surface of length L as:

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx = \frac{k}{L}\left[\frac{g\beta(T_s - T_\infty)}{4\nu^2}\right]^{1/4} g(Pr)\int_0^L \frac{dx}{x^{1/4}}$$

Laminar Natural Convection on a Vertical Surface (cont.)

- Integrating the equation, we get the following expression for $\overline{Nu_L}$:

$$\overline{Nu_L} = \frac{\bar{h}L}{k} = \frac{4}{3}\left(\frac{Gr_x}{4}\right)^{1/4} g(Pr)$$

- Substituting for $Nu_x|_{x=L}$, we get the following relationship for laminar natural convection on a vertical surface:

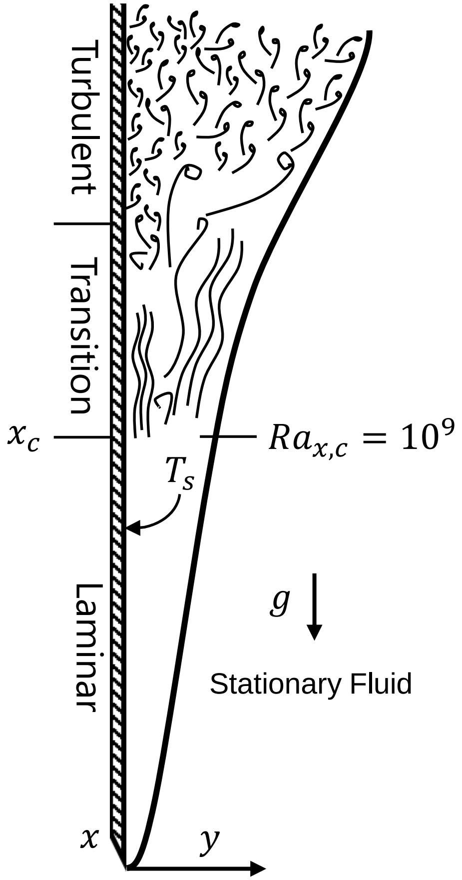
$$\overline{Nu_L} = \frac{4}{3} Nu_L$$

Transition to Turbulence

- Hydrodynamic instabilities may arise in natural convection boundary layers and, as a result, disturbances in the flow may amplify, leading to a transition from laminar to turbulent flow.
- Transition in a natural convection boundary layer depends on the relative magnitude of the buoyancy and viscous forces in the fluid.
- The Rayleigh number, $Ra_{x,c}$, can be used to determine the occurrence of transition. It is simply the product of Grashof and Prandtl numbers:

$$Ra_{x,c} = Gr_{x,c}Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha}$$

- The Rayleigh number can be viewed as the ratio of buoyancy forces and the product of thermal and momentum diffusivities.
- For vertical plates, the critical Rayleigh number is $Ra_{x,c} \sim 10^9$



Summary

- In this lesson, we started with a discussion on nondimensionalization of the natural convection governing equation, which led to the definition of the Grashof number, an important nondimensional number that is used for characterizing natural convection.
- We learned that based on the value of Gr_L/Re_L^2 we may be able to simplify the convective heat transfer problem by either neglecting the forced convection ($Gr_L/Re_L^2 \gg 1$) or natural convection ($Gr_L/Re_L^2 \ll 1$).
- In the case of $Gr_L/Re_L^2 \approx 1$, both natural and forced convection need to be included in the analysis.
- We analyzed the laminar natural convection on a vertical surface and obtained a similarity solution.
- Lastly, we looked at the effect of turbulence and the critical Rayleigh number at which the transition can occur for a vertical flat plate.

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