Heat Exchanger Analysis: Design and Performance

How Heat Exchangers Work – Lesson 4

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Analysis of Heat Exchangers

- Heat exchanger analysis problems generally present one of two different challenges:
  1. Selecting a heat exchanger that will achieve a specified temperature change in a fluid stream of known mass flow rate
  2. Predicting the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger

- In this lesson we will discuss two methods used in the analysis of heat exchangers:
  - The log mean temperature difference (or LMTD) method, which is best suited for the first task
  - The effectiveness-NTU method, which is best suited for the second task

Analysis of Heat Exchangers (cont.)

- Heat exchangers operate with minimal change in their operating conditions for long periods of time and thus can be modeled as steady-flow devices
- Applying overall energy balances to the hot and cold fluids:

$$q = m_h c_{p,h}(T_{h,i} - T_{h,o})$$

$$q = m_c c_{p,c}(T_{c,o} - T_{c,i})$$

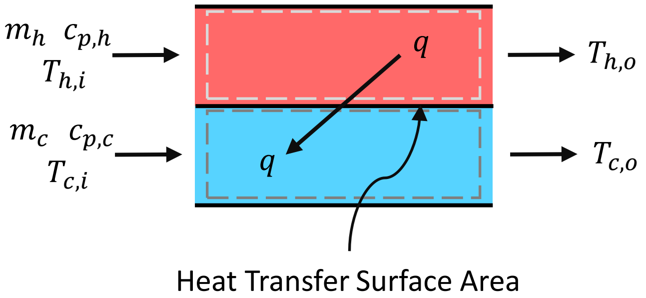
- We can obtain an expression relating the total heat transfer rate q to the temperature difference ΔT between the hot and cold fluids, where

$$\Delta T \equiv T_h - T_c$$

- Since ΔT varies with position in the heat exchanger, we need a rate equation of the form:

$$\mathrm{q} = \mathrm{UA}\Delta T_m$$

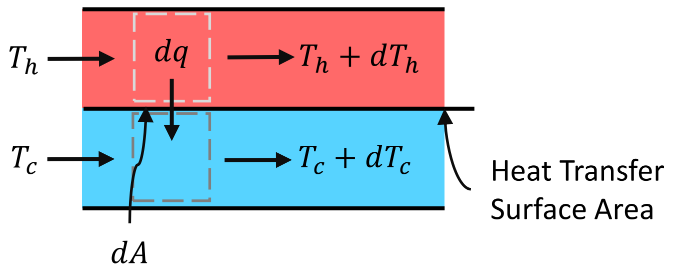
where ΔT_m is the appropriate mean temperature difference



Section of a Parallel Flow Heat Exchanger

Log Mean Temperature Difference

- The temperature difference between the hot and the cold fluid varies along the heat exchanger and thus, for heat transfer analysis, we need a mean temperature difference ΔT_m
- Let's consider a parallel-flow double-pipe heat exchanger. Applying an energy balance to a differential element of thickness dx in the hot and cold fluids as shown:



$$dq = -\dot{m}_h c_{p,h} dT_h \equiv -C_h dT_h$$

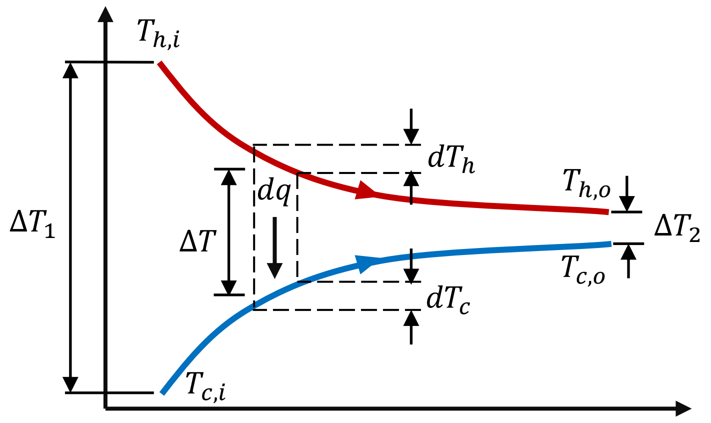
$$dq = -\dot{m}_c c_{p,c} dT_c \equiv C_c dT_c$$

$$dq = U \Delta T dA$$

$$d(\Delta T) = dT_h - dT_c$$

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_1^2 dA$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$



Temperature variation along the Section of a Parallel Flow Heat Exchanger

Log Mean Temperature Difference (cont.)

- Substituting for C_h and C_c, we get :

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{T_{h,i} - T_{h,o}}{q} + \frac{T_{c,o} - T_{c,i}}{q}\right)$$

- For a parallel-flow heat exchanger, $\Delta T_1 = (T_{h,i} - T_{c,i})$ and $\Delta T_2 = T_{h,o} - T_{c,o}$ thus we obtain:

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

$$q = UA\Delta T_{lm}$$

Log mean temperature difference

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

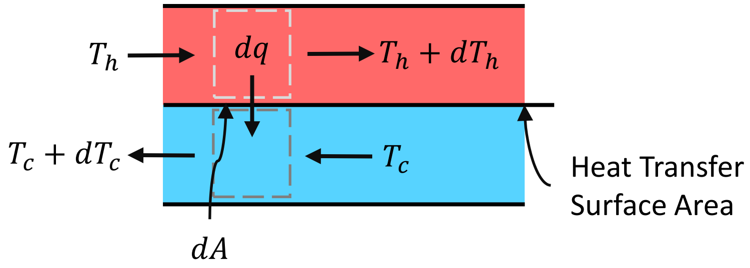
where $\begin{bmatrix}\Delta T_1 \equiv T_{h,1} - T_{c,1} \equiv T_{h,i} - T_{c,i} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} \equiv T_{h,o} - T_{c,o}\end{bmatrix}$

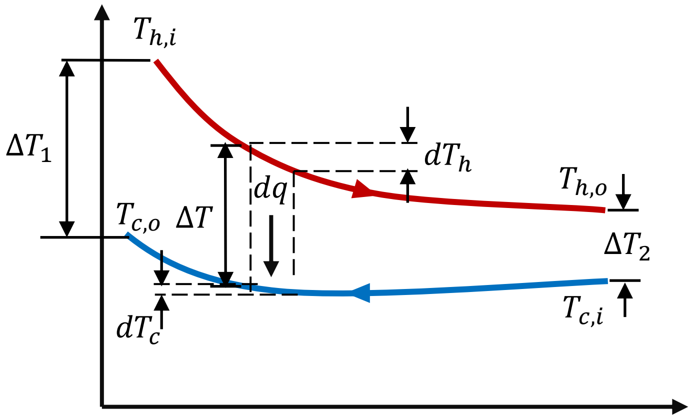
Log Mean Temperature Difference (cont.)

- The analysis performed for parallel flow is also applicable to a counter-flow arrangement. However, the endpoint temperatures in the case of a counter-flow exchanger must be defined as:

$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} \equiv T_{h,i} - T_{c,o} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} \equiv T_{h,o} - T_{c,i} \end{bmatrix}$$

- For the same inlet and outlet temperatures, the log mean temperature difference for a counter-flow configuration is higher than that for a parallel-flow configuration, i.e., $\Delta T_{lm,CF} > \Delta T_{lm,PF}$
- A counter-flow arrangement needs a lower surface area compared to a parallel-flow configuration for the same amount of heat transfer.





Temperature variation along the Section of a Counter Flow Heat Exchanger

LMTD – Use of a Correction Factor

- The LMTD relation developed in the previous slides is limited to parallel-flow and counter-flow heat exchangers.
- Similar expressions for cross-flow and multipass shell-and-tube heat exchangers can also be developed but are too complicated due to the complex flow conditions involved.
- In such cases, the equivalent temperature difference can be related to the log mean temperature difference relation for the counter flow case with the help of a correction factor, F:

$$\Delta T_{lm} = F \Delta T_{lm,CF}$$

- $\Delta T_{lm,CF}$ is the LMTD for the case of a counter-flow heat exchanger with the same inlet and outlet temperatures.
- The correction factor, F, depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.
- In general $F \leq 1$, and the limiting value of 1 corresponds to a counter-flow heat exchanger.
- F is a measure of the deviation of the ΔT_{lm} from the corresponding values for the counter-flow case.

LMTD – Correction Factor

Consider a shell and tube heat exchanger, where the heat transfer rate can be computed using:

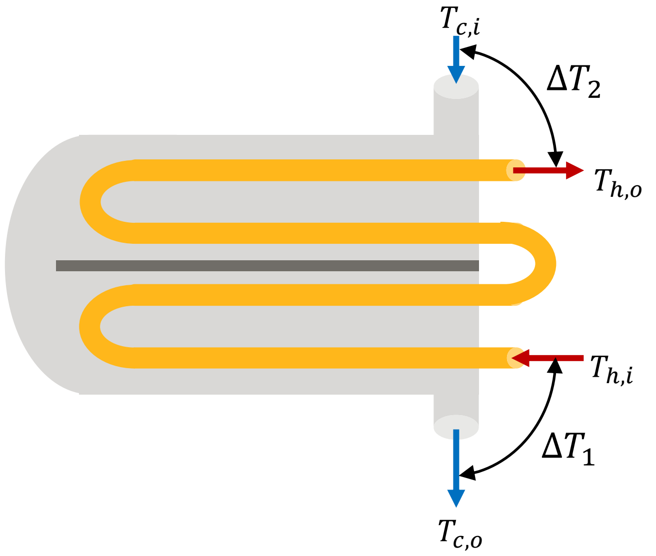
$$q = UAF\Delta T_{lm,CF}$$

where

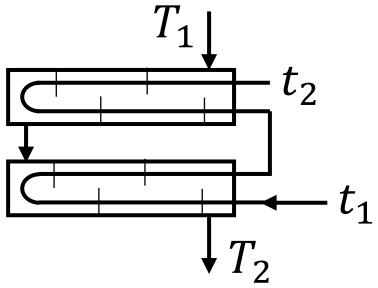
$$\Delta T_{lm,CF} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

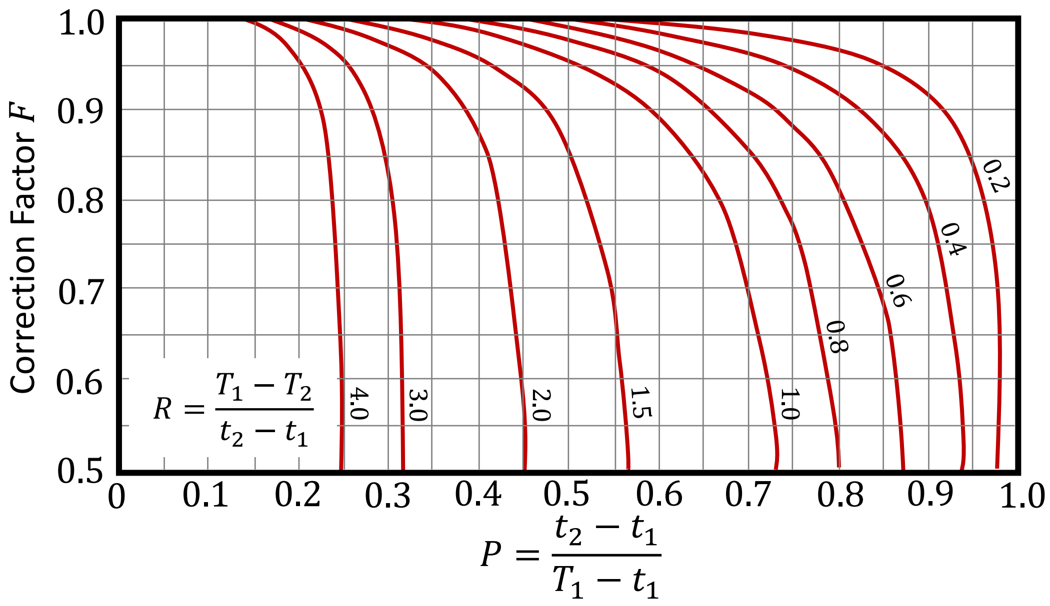
$$\begin{bmatrix} \Delta T_1 \equiv T_{h,1} - T_{c,1} \equiv T_{h,i} - T_{c,o} \\ \Delta T_2 \equiv T_{h,2} - T_{c,2} \equiv T_{h,o} - T_{c,i} \end{bmatrix}$$

And F is obtained from Correction Factor charts as shown.



Note that *T* and *t* represent the shell and tube side temperatures, respectively.





Condensers and Boilers

- Two special types of heat exchangers commonly used in practice are *condensers* and *boilers.*
- In these heat changers one of the fluids undergoes a phase-change process, and thus the rate of heat transfer can be expressed as

$$q = \dot{m} h_{fg}$$

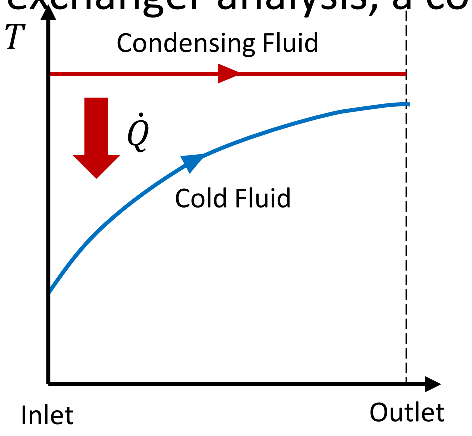
where $\dot{m}$ is the rate of evaporation and h_{fg} is the enthalpy of vaporization

- During a phase-change process the fluid absorbs or releases a large amount of heat at a constant temperature, and as a result the heat capacity rate of the fluid approaches infinity since the temperature change is practically zero.

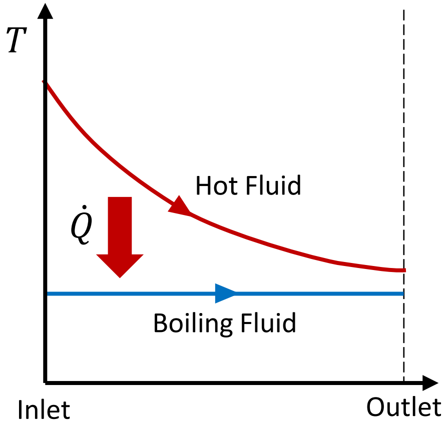
$$C = \dot{m} c_p \to \infty$$

when $\Delta T \to 0$ $\quad Q = \dot{m} c_p \Delta T$ is a finite quantity

- Therefore, in heat exchanger analysis, a condensing or a boiling fluid is modeled as a fluid whose heat capacity rate is infinity.



Condenser $C_h \to \infty$



Boiler $C_c \to \infty$

Effectiveness – NTU Method

- LMTD is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of hot and cold fluids are known or can be determined from the energy balance.
- If only the inlet temperatures are known, use of the LMTD method requires a cumbersome iterative procedure. It is therefore preferable to use an alternative approach called the effectiveness–NTU (or NTU) method.
- This method is based on a dimensionless parameter called the heat transfer effectiveness ε:

$$\varepsilon \equiv \frac{q}{q_{max}}$$

- The actual heat transfer rate is given by:

$$q = C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right)$$

- The maximum possible heat transfer rate for a heat exchanger can be estimated using the maximum temperature difference, i.e., $\Delta T_{max} = T_{h,i} - T_{c,i}$

$$C_c < C_h: \qquad q_{max} = C_c\left(T_{h,i} - T_{c,i}\right)$$

$$C_h < C_c: \qquad q_{max} = C_h\left(T_{h,i} - T_{c,i}\right)$$

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,i}\right)$$

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Effectiveness – NTU Method (cont.)

- Substituting these relations, we can get the following expressions for effectiveness:

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})}$$

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}$$

- Thus, if we know ε, $T_{h,i}$ and $T_{c,i}$, we can easily calculate the actual heat transfer rate using the following expression:

$$q = \varepsilon C_{min}(T_{h,i} - T_{c,i})$$

- For any heat exchanger:

$$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right)$$

- Here NTU (number of transfer units) is a dimensionless parameter given by:

$$NTU \equiv \frac{UA}{C_{min}}$$

Effectiveness – NTU Method (cont.)

- The effectiveness of a heat exchanger depends on the geometry of the heat exchanger as well as the flow arrangement.
- Let's determine the effectiveness-NTU relation for a simple parallel flow heat exchanger. Assuming $C_{min} = C_h$, we get

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$$

using

$$q = m_h c_{p,h}\left(T_{h,i} - T_{h,o}\right)$$

$$q = m_c c_{p,c}\left(T_{c,o} - T_{c,i}\right)$$

$$\frac{C_{min}}{C_{max}} = \frac{\dot{m}_h c_{p,h}}{\dot{m}_c c_{p,c}} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$$

$$ln\left(\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}\right) = -\frac{UA}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right) \rightarrow \left(\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}\right) = \exp\left[-NTU\left(1 + \frac{C_{min}}{C_{max}}\right)\right]$$

- Rearranging and simplifying we obtain the following expression for a parallel-flow heat exchanger,

$$\varepsilon = \frac{1 - \exp\left[-NTU\left\{1 + \left(\frac{C_{min}}{C_{max}}\right)\right\}\right]}{1 + \left(\frac{C_{min}}{C_{max}}\right)}$$

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Effectiveness – NTU Method (cont.)

- Similar expressions can be developed for different heat exchanger configurations as shown in the table below:

Flow Arrangement	Relation
Parallel flow	$\varepsilon = \dfrac{1-\exp[-NTU(1+C_r)]}{1+C_r}$
Counter-flow ($C_r < 1$)	$\varepsilon = \dfrac{1-\exp[-NTU(1-C_r)]}{1-\mathrm{C_r}\exp[-NTU(1-C_r)]}$
Shell and tube one shell pass	$\varepsilon_1 = 2\left\{1+C_r+(1+C_r^2)^{\frac{1}{2}}\times\dfrac{1+\exp\left[-(NTU)_1(1+C_r^2)^{\frac{1}{2}}\right]}{1-\exp\left[-(NTU)_1(1+C_r^2)^{\frac{1}{2}}\right]}\right\}^{-1}$
Shell and tube n shell pass	$\varepsilon_n = \left[\left(\dfrac{1-\varepsilon_1 C_r}{1-\varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1-\varepsilon_1 C_r}{1-\varepsilon_1}\right)^n - C_r\right]^{-1}$
Cross-flow (single pass)	$\varepsilon = 1-\exp\left[\left(\dfrac{1}{C_r}\right)(NTU)^{0.22}\{\exp[-C_r(NTU)^{0.78}]-1\}\right]$
All exchangers ($C_r = 0$)	$\varepsilon = 1-\exp(-NTU)$

$$C_r \equiv C_{min}/C_{max}$$

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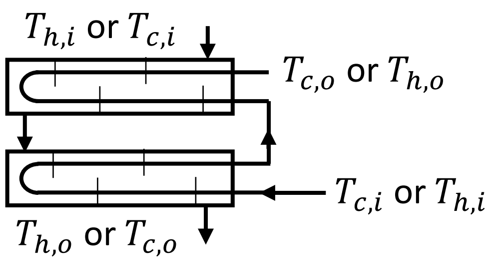
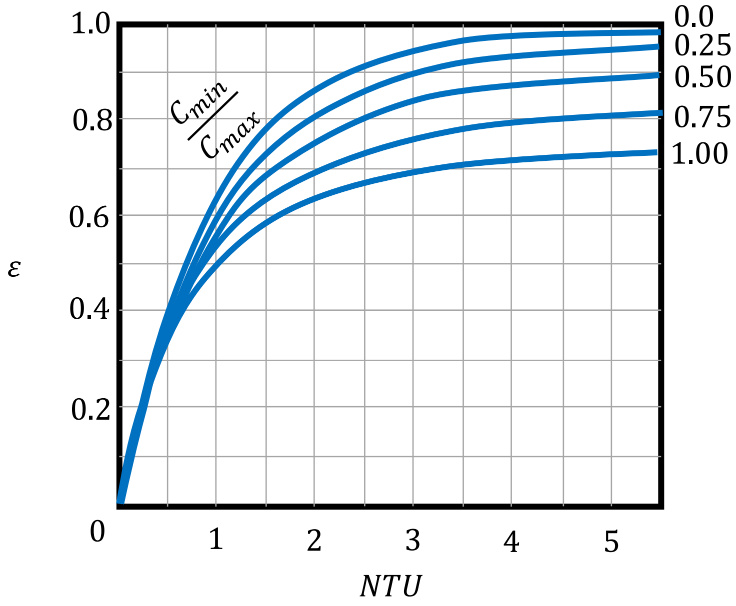
Effectiveness – NTU Method (cont.)

These expressions can also be represented graphically where the total number of transfer units are plotted along the *x*-axis and the effectiveness along the *y*-axis:





Effectiveness of a shell and tube heat exchanger with two shell passes and any multiple of four tube passes

Effectiveness – NTU Method (cont.)

- For most heat exchanger design calculations, it is more convenient to use $\varepsilon - NTU$ relations of the form:

$$NTU = f\left(\varepsilon, \frac{C_{min}}{C_{max}}\right)$$

Flow Arrangement	Relation
Parallel flow	$NTU = -\frac{ln[1-\varepsilon(1+C_r)]}{1+C_r}$
Counter-flow ($C_r < 1$)	$NTU = \frac{1}{C_r - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right)$
Shell and tube one shell pass	$(NTU)_1 = -(1+C_r^2)^{-\frac{1}{2}}\ln\left(\frac{E-1}{E+1}\right)$ $E = \frac{\frac{2}{\varepsilon_1}-(1+C_r)}{(1+C_r^2)^{\frac{1}{2}}}$
All exchangers ($C_r = 0$)	$NTU = -ln(1-\varepsilon)$

Heat Exchanger Design and Performance Calculations

- *Design Problem*: Specify a specific heat exchanger type and determine the size when the fluid inlet temperature and flow rates are known, and the desired hot and cold fluid outlet temperatures are prescribed.
- Typically used when we want to design a custom-built heat exchanger for a particular application.
- Solution:
  - Calculate ε and (C_{min}/C_{max})
  - Using the appropriate equation, obtain the NTU value
  - Determine A (Area) required for heat transfer

- *Performance Calculation*: Analyze an existing heat exchanger to determine the heat transfer rate and the fluid outlet temperature for specified flow rates and inlet temperatures.
- Typically used when we want to use off-the-shelf heat exchanger from a vendor.
- Solution:
  - Calculate NTU and (C_{min}/C_{max})
  - Using the appropriate equation, obtain the ε value
  - Determine q using $q = \varepsilon q_{max}$
  - Determine fluid outlet temperatures

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Summary

- In this lesson we talked about the analysis of heat exchangers and discussed how we can perform design and performance calculations.
- We investigated heat transfer in a heat exchanger using two methods:
  - Log mean temperature difference
  - Effectiveness – NTU method
- Lastly, we looked at the procedure used in heat transfer design and performance calculations.

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