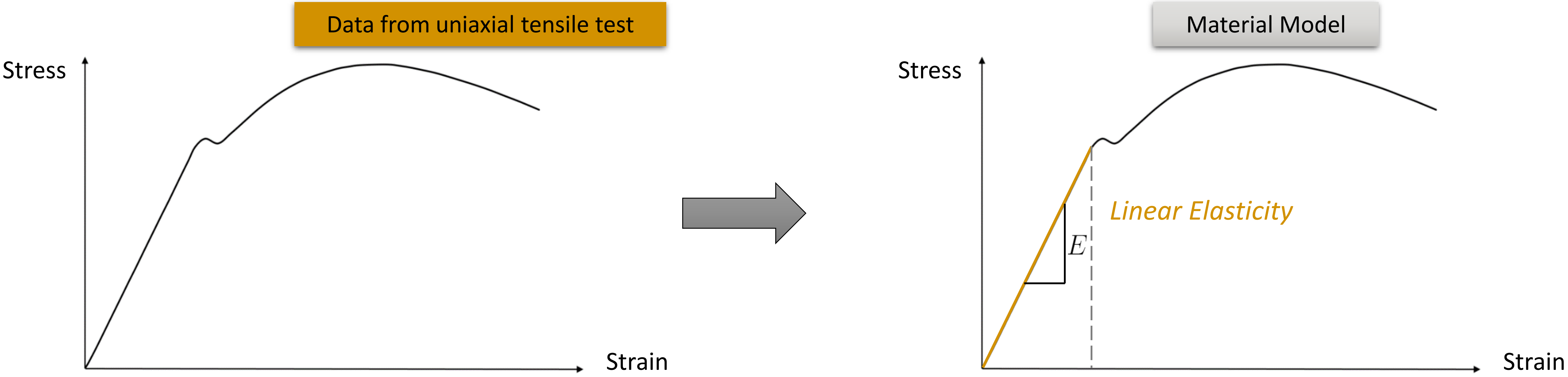
Yield Surface

Metal Plasticity – Lesson 2

Metal Plasticity

We learned the features of metal materials and plastic deformation behavior. Now the question is, how do we mathematically represent such materials' features in a solid mechanics analysis?

- In a testing lab, engineers can measure testing results of a material, e.g., data from a uniaxial tensile test.
- The general method to use such testing data in analysis is to fit them to a mathematical model.
- For example, if the relationship between stress and strain is measured to be almost a straight line for certain range, the material can be modeled as linear elasticity. The material's behavior can be captured by Young's modulus in the uniaxial direction.

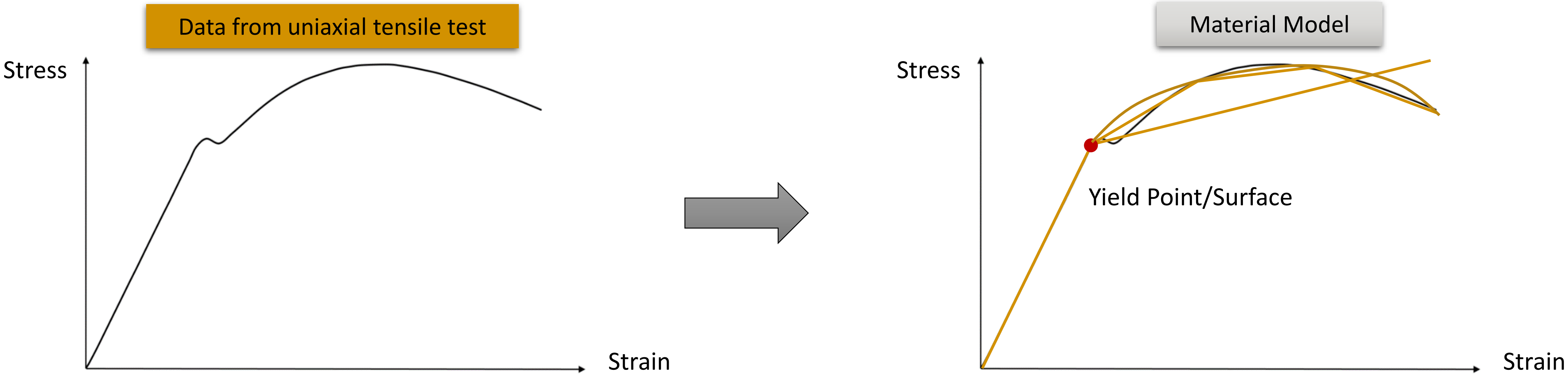


Metal Plasticity (cont.)

Beyond the linear elasticity range, we need plasticity models to capture the materials behavior.

To define a plasticity model:

1. Define the threshold for plasticity. For 1D, the threshold of plasticity is called the yield point. For a 3D problem, it becomes a surface, called the yield surface.
2. Define the material's behavior after the yield point. Depending on the accuracy requirement, it can be simplified to another line with a different tangent, or piece-wise straight lines or high-order nonlinear functions.

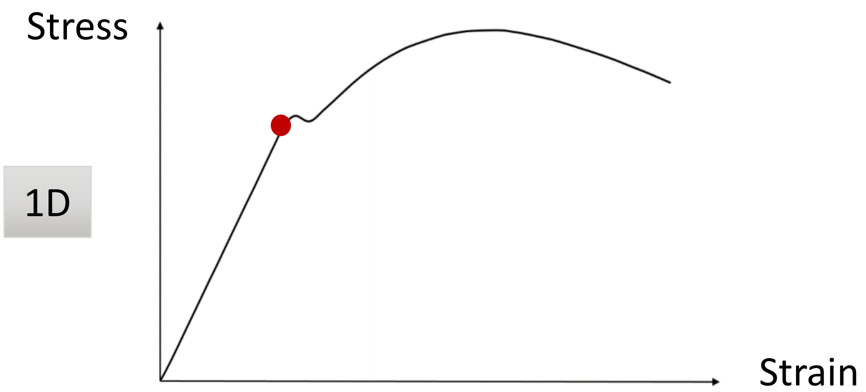


Metal Plasticity: Yield Surface

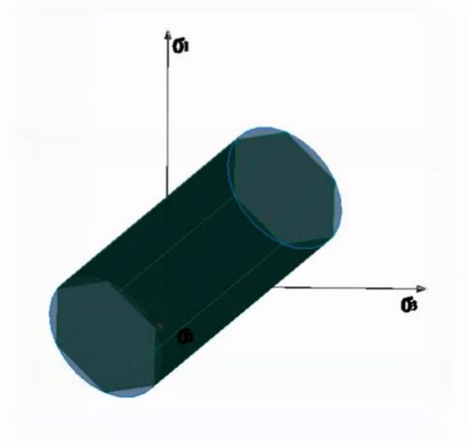
For 1D plasticity, the yield surface is a point, while in 3D the threshold for plasticity is a 3D surface. Once the material's status reaches this surface, the material is in the plasticity region. There are various definitions of yield surfaces, each with its own feature and application.

- Tresca yield surface, which uses maximum shear stress as the threshold of plasticity.
- Von Mises yield criteria, which calculates von Mises stress and uses it to decide if the material reaches plasticity.
- Plotting Tresca and von Mises yield surface on the same principal coordinate system, we will find that Tresca criteria form a hexagonal prism surface, and von Mises criteria form a cylindrical surface, which just wraps the hexagonal prism.
- There are also more complicated yield criteria, for example, the Drucker-Prager criteria, which is mostly used in brittle materials, such as concrete.

1D



3D



Tresca (Max. shear stress theory)

Von Mises (Max. distortion theory)

Metal Plasticity: Von Mises Yield Surface

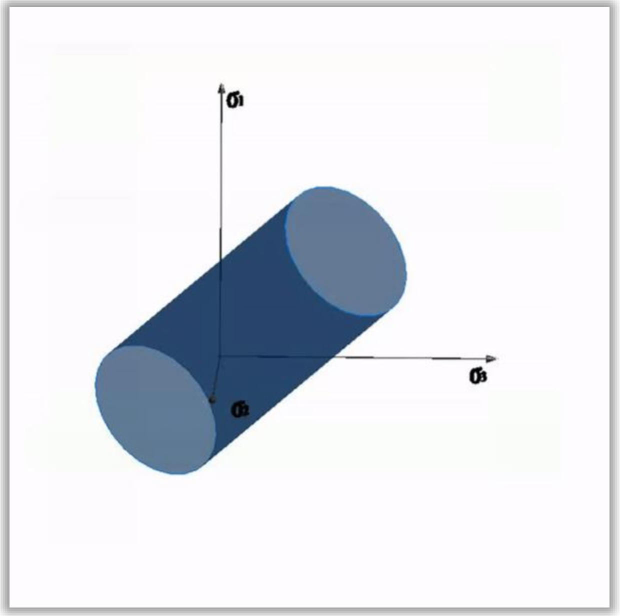
For metal material, one widely used yield criteria is von Mises yield criteria. By von Mises criteria, yielding begins when the distortion of material reaches a critical value. For this reason, the von Mises criterion is also known as the *maximum distortion strain energy criterion*.

- Expression of von Mises yield stress:

$$\sigma_v = \sqrt{\frac{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{xx})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2)}{2}}$$

- In a principal coordinate system

$$\sigma_v = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)}{2}}$$



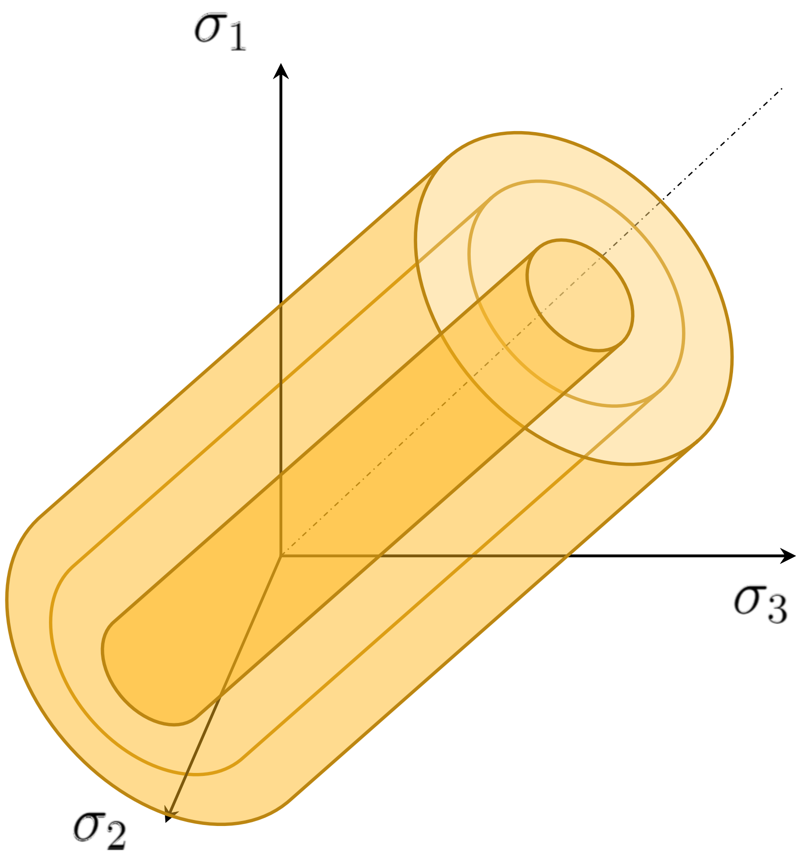
For von Mises yield surface in principal coordinate system, the terms in the three brackets are the differences between the three principal stresses. It is not the absolute value but that relative value of the three principal stresses determining the von Mises yield surface.

Check Equivalent Stress Module for more discussion of von Mises surface.

Metal Plasticity: Von Mises Yield Surface (cont.)

Let's see how to determine if a material is in the plasticity range.

1. For a material point, von Mises stress can be calculated from the 3x3 stress tensor.
2. Once the von Mises stress reaches the threshold value, the material is in the plasticity range.
3. If von Mises stress continues to grow, i.e., the distortion grows, the yield surface grows with the von Mises stress value. A stress status is *always on or inside the stress surface*. It can never be outside of the yield surface



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