Rayleigh & Fanno Flows

Compressible Internal Flows – Lesson 2

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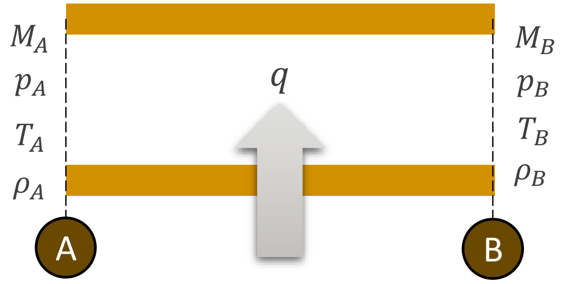
Intro

- We have considered 1D compressible flows to be isentropic, which implies that friction effects are small and can be neglected.
- The flow of gases in pipelines, however, requires consideration of both compressibility and friction effects, which produce pressure loss in the pipe. For long pipe lengths, this loss can be significant.
- Another important consideration in 1D flows is heat addition, which is important for applications such as turbojet and ramjet engine burners, where heat is added in the form of a fuel-air combustion.
- In this lesson, we will introduce heat addition and friction effects into our 1D compressible flow model and analyze its effects.

1D Flow with Heat Addition: Rayleigh Flow



- Consider a 1D flow with heat addition (or removal) between regions A and B

- Governing equations:

$$\rho_A V_A = \rho_B V_B$$

$$p_A + \rho_A V_A^2 = p_B + \rho_B V_B^2$$

$$h_1 + \frac{V_1^2}{2} + q = h_2 + \frac{V_2^2}{2}$$

- Assuming a calorically perfect gas, $h = c_p T$, and using the definition of total temperature

$$q = c_p\left(T_{0_B} - T_{0_A}\right)$$

- This equation clearly shows that the heat addition directly changes the total temperature of the flow.
  - Heat addition → T_0 increases
  - Heat removal → T_0 decreases

- Using the isentropic relations, we can obtain the ratio of the properties between regions A and B in terms of Mach numbers M_A and M_B.

Rayleigh Flow

- Setting sonic flow as a reference condition and $M_A = 1$, the corresponding flow properties can be denoted by $p_A = p^*$, $T_A = T^*$, $\rho_A = \rho^*$ and $T_{0_A} = T_0^*$.

- We can then obtain the flow properties at any other value of M using the following relations:

$$\frac{p}{p^*} = \frac{1+\gamma}{1+\gamma M^2}$$

$$\frac{T}{T^*} = M^2\left(\frac{1+\gamma}{1+\gamma M^2}\right)^2$$

$$\frac{\rho}{\rho^*} = \frac{1}{M^2}\left(\frac{1+\gamma M^2}{1+\gamma}\right)$$

$$\frac{p_0}{p_0^*} = \frac{1+\gamma}{1+\gamma M^2}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_0}{T_0^*} = \frac{(1+\gamma)M^2}{(1+\gamma M^2)^2}[2+(\gamma-1)M^2]$$

- The current flow setup represents a *nonadiabatic* process. In this case the flow properties T^*, p^*, and ρ^* are conditions in a 1D flow that would exist if enough heat is added to achieve Mach 1.

Rayleigh Flow (cont.)

- We can get some insights into the physical trends of such flows based on the ratio equations.

- Supersonic flows in region A, i.e., $M_A > 1$, for heat addition
  - Mach number decreases, $M_B < M_A$
  - Pressure increases, $p_B > p_A$
  - Temperature increases, $T_B > T_A$
  - Total temperature increases, $T_{0_B} > T_{0_A}$
  - Total pressure decreases, $p_{0_B} < p_{0_A}$
  - Velocity decreases, $V_B < V_A$

- Subsonic flows in region A, i.e., $M_A < 1$, for heat addition
  - Mach number increases, $M_B > M_A$
  - Pressure decreases, $p_B < p_A$
  - Temperature increases for $M_A < \gamma^{-\frac{1}{2}}$ and decreases for $M_A > \gamma^{-\frac{1}{2}}$
  - Total temperature increases, $T_{0_B} > T_{0_A}$
  - Total pressure decreases, $p_{0_B} < p_{0_A}$
  - Velocity increases, $V_B > V_A$

All the above trends are opposite in the case of heat removal

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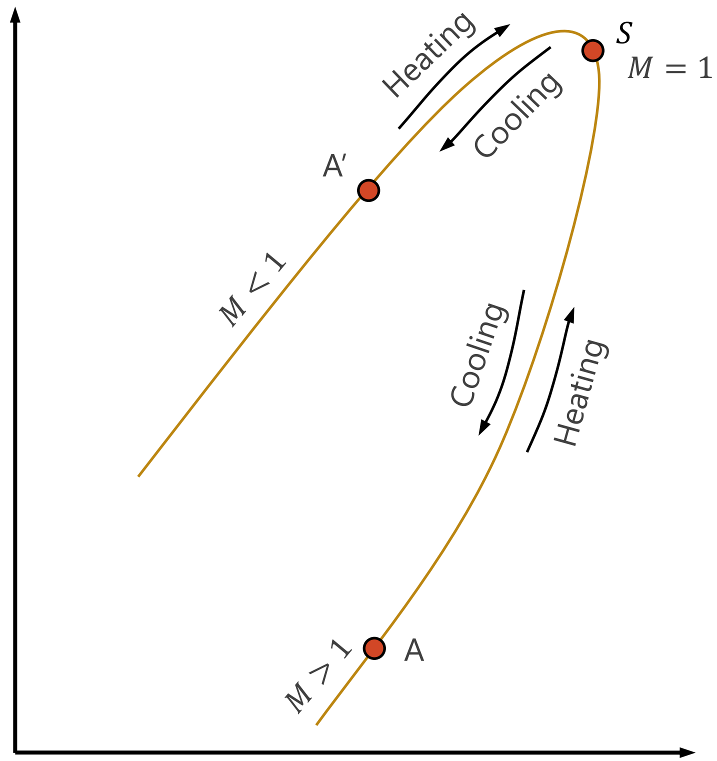
The Rayleigh Curve

- The figure on the right represents a 1D heat addition process on a Mollier diagram ($h - s$) for a given set of initial conditions. This curve is known as the Rayleigh Curve.
- Heat addition always drives the Mach numbers toward 1. It decelerates a supersonic flow and accelerates a subsonic flow.
- If point A represents the conditions in region A, then the Rayleigh curve is the locus of all possible states in region B, where each point corresponds to a different value of q added or removed.
- At point S, the flow is sonic and represents the maximum entropy state for a particular flow. The upper branch represents subsonic flow and the lower branch represents supersonic flow.



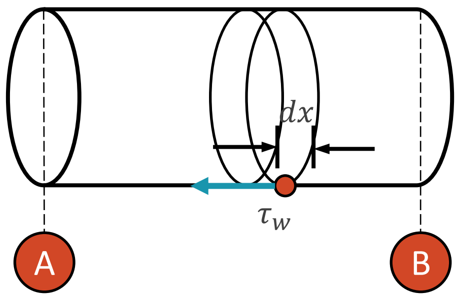
- If the flow in region A is supersonic and is represented by point A on the curve, then any heat addition will make the conditions in region B move closer to the point S . For a certain value of q the flow becomes sonic (Point S) and becomes choked as no further increase in q is possible without a drastic change in the upstream conditions.

The Rayleigh Curve (cont.)

- If q is further increased beyond this point, then a normal shock is formed and the conditions in region A will suddenly become subsonic.
- On the other hand, if the initial flow conditions in region A were subsonic (represented by A') on the curve, then upon heat addition the conditions downstream in region B will be driven toward the point S, and for a certain value of q, the flow will reach sonic conditions.
- The flow in this situation is again choked and any further increase in heat addition is not possible without an adjustment in the initial conditions.
- Any further increase in q would cause a series of pressure waves to propagate upstream and help adjust the conditions in region A to a lower subsonic Mach number, i.e., move the point A'toward the left.
- Another interesting observation from the Rayleigh curve is that a supersonic flow can be made subsonic by first heating it until the flow becomes sonic and then subsequently cooling it thereafter. Similarly a subsonic flow can be made supersonic by first heating it to sonic conditions and then cooling it thereafter.
- Irrespective of the flow Mach number, heat addition always decreases the total pressure. This is significant in the design of jet engines.

1D Flow with Friction

- Consider a compressible inviscid 1D flow with frictional effects modeled as shear stresses at the wall acting on the fluid, with uniform properties over any cross-section.



- We can analyze this flow following an approach like the one used for Rayleigh flow. However, this time we need to include the frictional effects in the momentum equation:

$$p_A A_A + \rho_A V_A^2 A_A = p_B A_B + \rho_B V_B^2 A_B + \int_0^L \pi D \tau_w dx$$

- Since $A_A = A_B = \pi D^2/4$, this can be simplified to:

$$(p_B - p_A) + (\rho_B V_B^2 - \rho_A V_A^2) = -\frac{4}{D}\int_0^L \tau_w dx$$

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Fanno Flow

- As the shear stress varies with the distance along the pipe, we can simplify the integral by considering a small section dx:

$$dp + d(\rho V^2) = -\frac{4}{D}\tau_w dx$$

- Note that $\rho u = const$, and the shear stress can be expressed in terms of a friction coefficient f using $\tau_w = \frac{1}{2}\rho u^2 f$:

$$dp + \rho V dV = -\frac{1}{2}\rho V^2 4\frac{f}{D}dx$$

- For a calorically perfect gas we can express this relation in terms of Mach number as:

$$\frac{2}{\gamma M^2}(1 - M^2)\left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{-1}\frac{dM}{M} = 4\frac{f}{D}dx$$

- Integrating this between the two points A and B gives the following equation relating the Mach numbers at two different sections to the effect of friction between the two points:

$$\left[-\frac{1}{\gamma M^2} - \frac{\gamma + 1}{2\gamma}\ln\left\{\frac{M^2}{1 + \frac{\gamma - 1}{2}M^2}\right\}\right]_{M_A}^{M_B} = \int_{x_A}^{x_B} 4\frac{f}{D}dx$$

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Fanno Flow (cont.)

- We can use sonic flow reference conditions denoted by $p_A = p^*$, $T_A = T^*$, $\rho_A = \rho^*$ and $T_{0_A} = T_0^*$ to obtain the following relations:

$$\frac{p}{p^*} = \frac{1}{M}\left[\frac{1+\gamma}{2+(\gamma-1)M^2}\right]^{1/2}$$

$$\frac{T}{T^*} = \left(\frac{1+\gamma}{2+(\gamma-1)M^2}\right)^{1/2}$$

$$\frac{p_0}{p_0^*} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{\rho}{\rho^*} = \frac{1}{M}\left(\frac{2+(\gamma-1)M^2}{1+\gamma}\right)^{1/2}$$

- Defining $x = L^*$ as the location where the flow becomes sonic, i.e., $M = 1$, and integrating the last equation from the previous slide, we arrive at the following expression for choking length:

$$\frac{4\bar{f}L^*}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right]$$

where $\bar{f}$ is defined as the average friction coefficient given by: $$\bar{f} = \frac{1}{L^*}\int_0^{L^*} f\,dx$$

Fanno Flow (cont.)

- We can get some insights into the physical trends of such flows based on the ratio equations.

- Supersonic flows in region A, i.e., $M_A > 1$, the effect of friction on the downstream flow is such that:
  - Mach number decreases, $M_B < M_A$
  - Pressure increases, $p_B > p_A$
  - Temperature increases, $T_B > T_A$
  - Total pressure decreases, $p_{0_B} < p_{0_A}$
  - Velocity decreases, $V_B < V_A$

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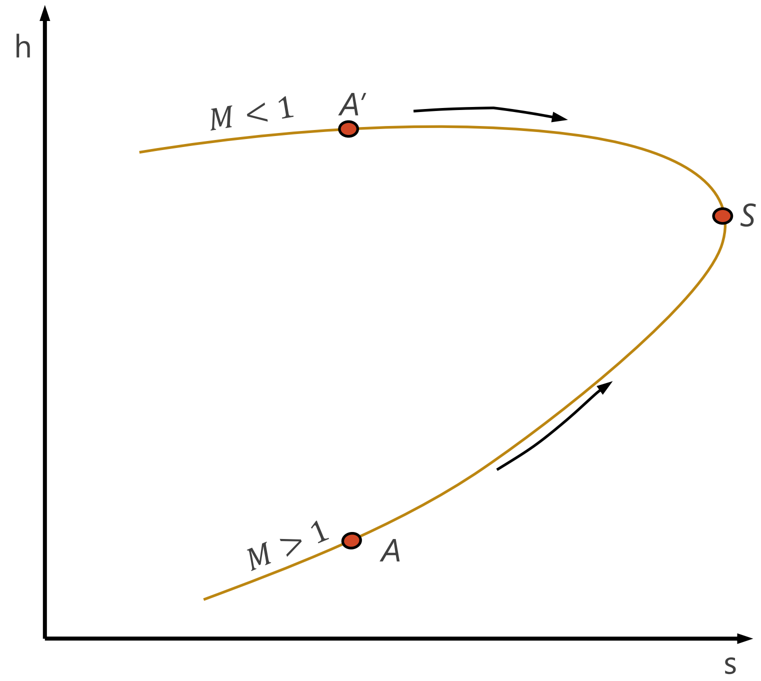
Note that the friction always drives the Mach Number toward 1, accelerating a subsonic flow and decelerating a supersonic flow.

The Fanno Curve

- The figure on the right represents a 1D flow with frictional effects on a Mollier diagram ($h - s$) for a given set of initial conditions. This curve is known as the *Fanno Curve*.
- At the point *S*, the flow is sonic and represents the maximum entropy state for a particular flow. The upper portion represents subsonic flow and the lower portion represents supersonic flow.
- If point A represents the conditions in region A, then friction causes the flow downstream to move closer to point S by decreasing the flow Mach number.



- Each point between A and S corresponds to a certain length of the pipe having frictional effects. For a certain value of length, the flow reaches the point S and becomes choked, because any further increase in length is not possible without a drastic change of the inlet conditions. An analogous observation can be made for subsonic inlet conditions corresponding to point A’.
- Friction always causes the total pressure to decrease irrespective of the inlet conditions. Moreover, unlike the Rayleigh curve, the upper and lower branches of the Fanno curve cannot be traversed by the same flow.

Summary

- We have covered the analysis of 1D flows with heat addition and frictional effects.
- Heat addition always drives the flow toward sonic conditions and decreases the total pressure, which is an important consideration in designing jet engines.
- Similarly, frictional effects drive the flow downstream toward sonic conditions and decreases the total pressure. This is important for operating gas pipelines, etc.
- We also looked at the Rayleigh and Fanno curves and discussed the corresponding choked conditions.

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