

Fundamentals of Damping

Damping Effects – Lesson 2



/ What is Damping?

- There are several mechanisms for dissipation of energy, often these mechanism work in tandem to dampen the system.
- For instance, in the automobiles, there are several dissipating mechanisms such as;
 - Shock absorbers
 - Brakes
 - And even tires dissipate some energy via friction and repeated loading and unloading
- The rate of dissipation depends on several factors such as operating velocity, materials, and even the frequencies of vibrations can affect it.
- Damping as a physical phenomenon is quite complex and may not always be well understood, but numerically its implementation is quite simple.
- Now let's take a look at some of the common types of damping, keeping in mind that overall damping of a system (automobile, building, airplane, etc.) will often be comprised of a combination of one or more actual physical damping phenomena.



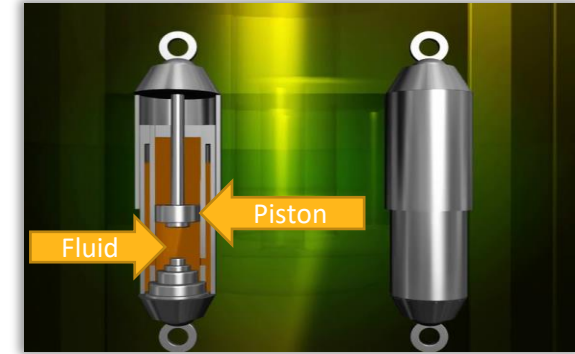
/ Types of Physical Damping

Viscous Damping

- Viscous damping is a very common type of damping and it relies on the viscous heat dissipation by fluids, and an automotive shock absorber is a common viscous damper.
- The fluid passes through orifices in the piston, between chambers resulting in frictional energy losses in the fluid.
- The damping force is proportional to the velocity $\{\dot{\mathbf{u}}\}$ of the damper, so fast motions resist greater than slower motions. We specify $[\mathbf{C}]$ to account for the damping.

$$\frac{F_{\text{inertia}}}{[M]\{\ddot{\mathbf{u}}\}} + \frac{F_{\text{damping}}}{[C]\{\dot{\mathbf{u}}\}} + \frac{F_{\text{stiffness}}}{[K]\{\mathbf{u}\}} = \frac{F_{\text{applied}}}{\{F(t)\}}$$

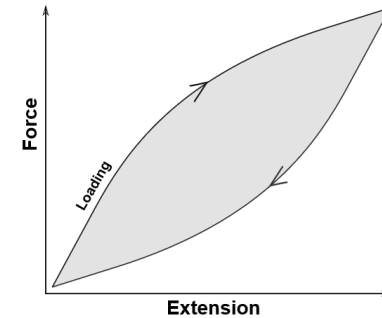
- Another example of viscous damping would be a loudspeaker where external fluids such as air provide viscous damping to the cone of a loudspeaker. The large face of the cone is interacting directly with the air affecting the vibration characteristics.



/ Types of Physical Damping

Material Damping (also known as solid, structural or hysteretic damping)

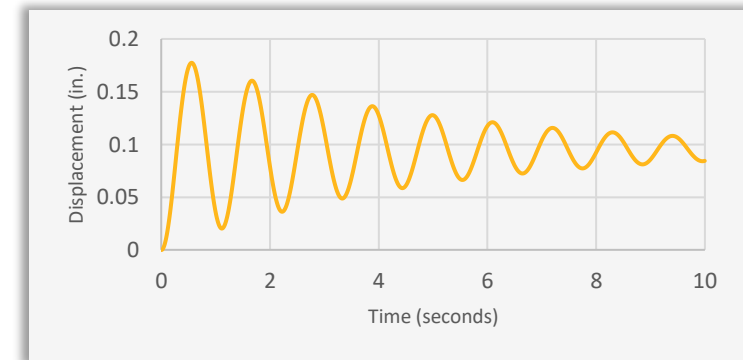
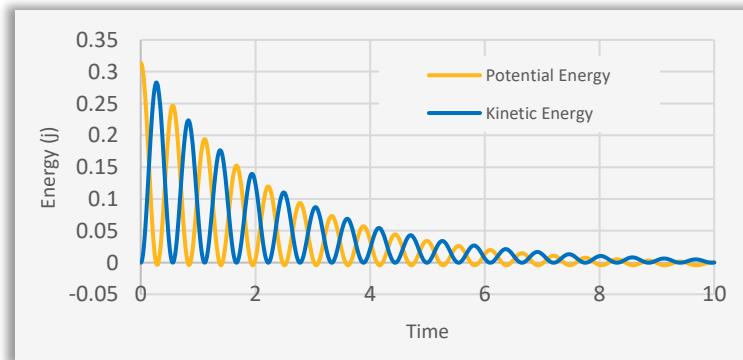
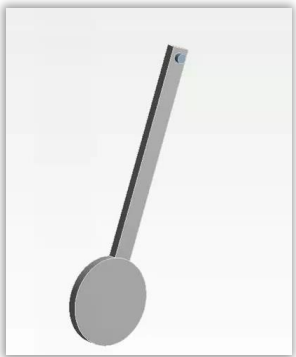
- Material damping arises from complex molecular interactions within a material.
- What happens at the material level is not our focus, but we can think of the mechanism as a type of internal frictional loss, which will often be dissipated as heat.
- As the material deforms, energy is lost as shown in the graph as the area enclosed by the stress-strain hysteresis plot which is for a steel material.
- This damping is independent of the frequency of the excitation, but the damping is proportional to the displacement.
- In some materials, such as viscoelastic materials, the damping is frequency dependent (this is discussed shortly).
- Rubber vibration isolator pads are a good example of this type of material damping, as the rubber absorbs the kinetic energy from the vibrations and stores as internal energy and dissipates as heat energy upon unloading.
- This washing machine isolator pad prevents the strong vibration from transmitting into the floor, reducing the overall noise and vibration.



Types of Physical Damping

Frictional Damping

- Frictional or Coulomb Damping arises from the energy lost in the sliding of parts in contact.
- The total energy decreases, dissipating into heat and noise.
- A simple example of this is shown with a pendulum swinging on a pin with friction.
- The energy alternates between potential and kinetic, as the total energy is lost via friction.
- Notice the progressive decay in the amplitude of the pendulum displacement.



/ Characterization of Damping

- In the next slides we will see several different ways to characterize damping, but they are often interrelated.
- Why are there are so many and which to choose?
- Depending on the industry, the type of vibration (free or forced), the type of material or even the type of testing performed, there are different common ways to characterize the damping.

Characterization of Damping

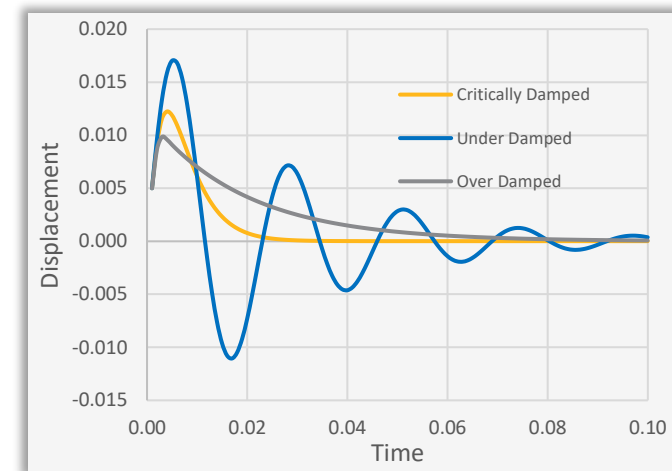
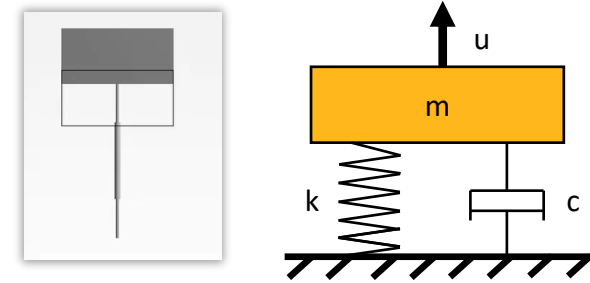
- An important concept in dynamics and vibrations is the **Damping Ratio**
- It is a ratio of the actual damping to the critical damping.

$$\zeta = \frac{c}{C_c} = \frac{\text{Actual Damping}}{\text{Critical Damping}}$$

- The critical damping is the damping at the point the system response changes from oscillatory to non-oscillatory.
- Have a look at the graph where the time history response of a vibration is plotted, and the decay of amplitude is evident. The mass is given an initial displacement u and then let go to freely vibrate.

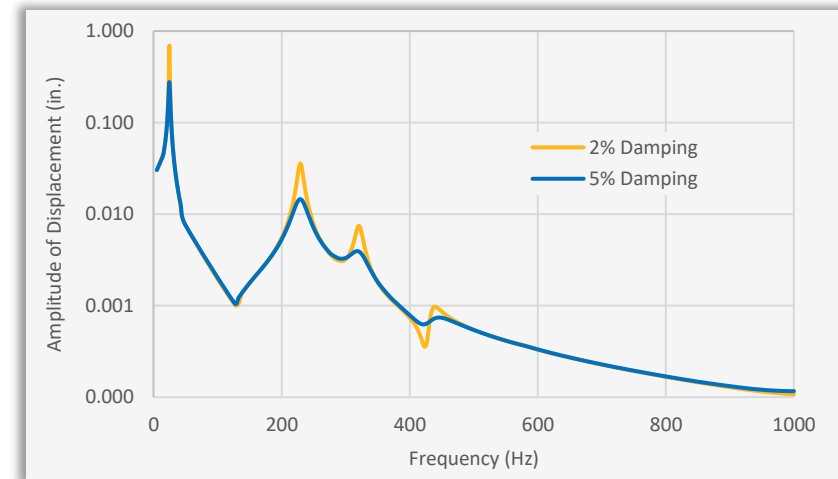
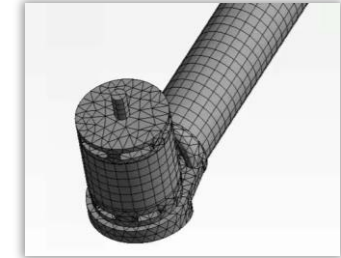
$\zeta = 0$	No Damping
$0 > \zeta < 1$	Oscillatory behavior with varying degree of damping
$\zeta = 1$	Critical Damping
$\zeta > 1$	Over Damped

- In most engineering metals the range for zeta (ζ) is below .05 (5%), with the majority less than .02 (2%), but for some elastomeric materials, such as rubbers, it will generally be higher.



Characterization of Damping

- We saw how different levels of damping can affect the free response of a structure.
- But what if the structure is experiencing a forced steady sinusoidal (Harmonic) excitation. In that case what does different levels of damping do the response?
- The damping decreases the maximum amplitude of the sinusoidal response.
- In this example of a drone arm, we can see the effect the damping has on the amplitude of the vibration response. The peak curves soften with higher amounts of damping and we predict smaller amplitudes of steady state vibration.



/ Characterization of Damping

- We may not always know the details of the physical phenomenon that is causing the damping in a structure, but typically we can specify equivalent values to numerically account for it.
- For example, the natural decay in shaking of a building after a seismic event may come from the viscous motion of the building against the air, and the hysteretic damping of the steel material, energy dissipated in the vibration isolator, and even the slight slippage at bolted connections, but knowing how much from each of these physical phenomenon may not be easily determined.
- So then how to determine the damping of an actual structure, which we can specify to account for this effective damping?
- In practice, what to specify for the damping ratio(s) is one of the most likely unknowns. References can be found, and they can often be a good starting point.
- There are ways to calculate the damping ratio(s). We will cover two of the common methods;
 - 1 Logarithmic Decrement Method (free vibration)
 - 2 Half-Power Bandwidth Method (forced harmonic vibration)

Characterization of Damping

1 Logarithmic Decrement Method

- Recall we plucked the guitar string and observed the decay in the peak amplitudes.
- The logarithmic decrement is defined as;

$$\delta = \frac{1}{n} \ln \frac{x(t)}{x(t+nT)}$$

where

$x(t)$ = Amplitude of a peak

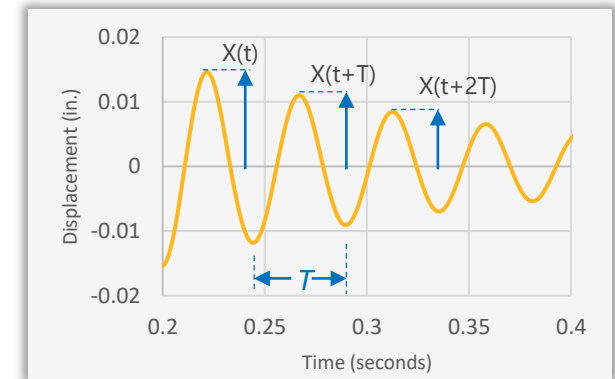
$x(t+nT)$ = Amplitude of peak n periods away

T = Period

- Therefore, the damping ratio can then be calculated as;

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}}$$

- So by exciting the structure and observing the response over time, the effective viscous damping ratio can be computed.
- The response is typically measured using an accelerometer or strain gauge.
- This method is most commonly used when there is a single dominate frequency.



Characterization of Damping

2 Half-Power Bandwidth Method

- As an example, a Drone arm experiences resonances at its natural frequencies as shown by the peaks in the frequency response graph.
- The excitation may be from the increasing of the motor speed or testing the drone's dynamic behavior on a shaker table.
- We can compute the damping ratio at a peak by first computing the Amplification Factor Q ;

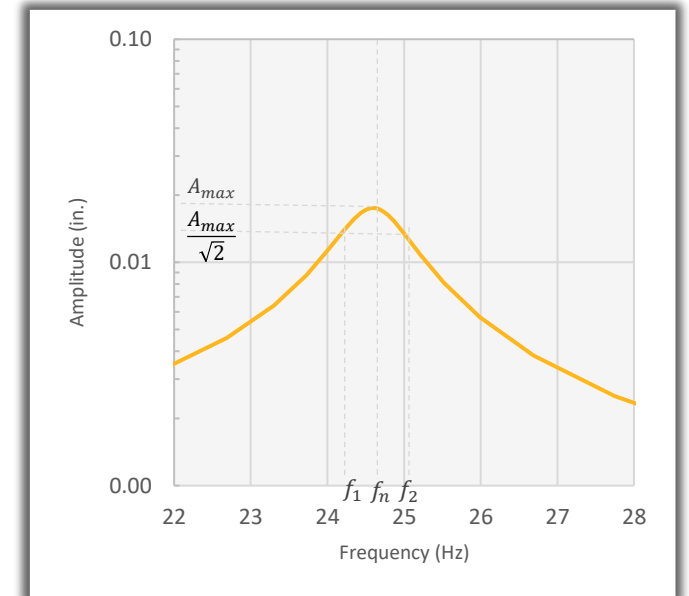
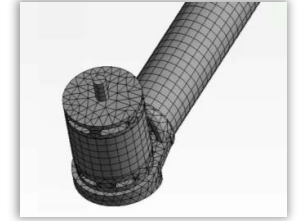
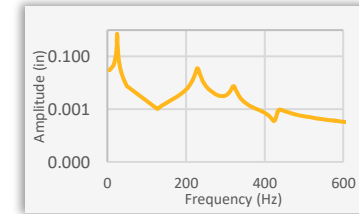
$$Q = \frac{f_n}{\Delta f}$$

where $f_n = \text{natural frequency}$
 $\Delta f = f_2 - f_1 = \text{frequency width between half power points}$

- For clarity, f_1 and f_2 are determined from drawing a line at an amplitude of $\frac{A_{max}}{\sqrt{2}}$ and extracting the corresponding frequencies.
- Finally, the damping ratio is computed from Q ;

$$\zeta = \frac{1}{2Q}$$

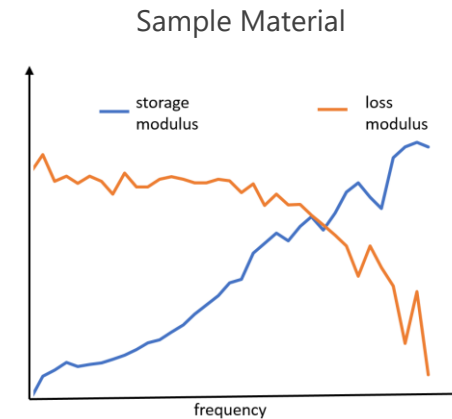
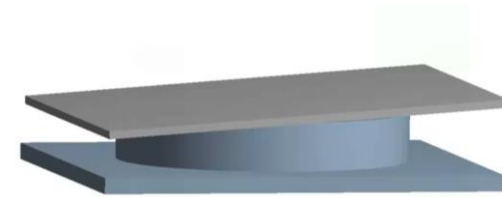
- This method is most commonly used with frequency-based testing such as impact hammer or shaker table which produce these frequency response plots (Amplitude vs frequency).



Characterization of Damping

- Elastomeric materials, such as rubber or rubber like materials may have damping that is highly frequency dependent.
- This type of damping is called viscoelastic damping.
- The damping of a viscoelastic material is typically characterized using dynamic mechanical analysis (DMA).
- The output from the tests provide the complex modulus (storage and loss) as shown in a sample graph to the right where E' is the storage modulus and E'' is the loss modulus, both as a function of frequency.
- The loss factor $\eta(f)$ is frequency dependent and is a common measure to characterize damping.
- The loss factor $\eta(f)$ can be related to the moduli, and is a common measure of the damping capability of a rubber like material;

$$\eta(f) = \frac{E''(f)}{E'(f)}$$



Characterization of Damping

- As we have discussed the damping ratio is a common measure, but damping is also be characterized by other parameters that are interrelated.
- Let's have a look at the common relationships as one might be provided with a different measure, and conversion between measures may be necessary.
- The following measures that one may come across in dynamics and vibrations. For low levels of damping, the following relationship are commonly used;

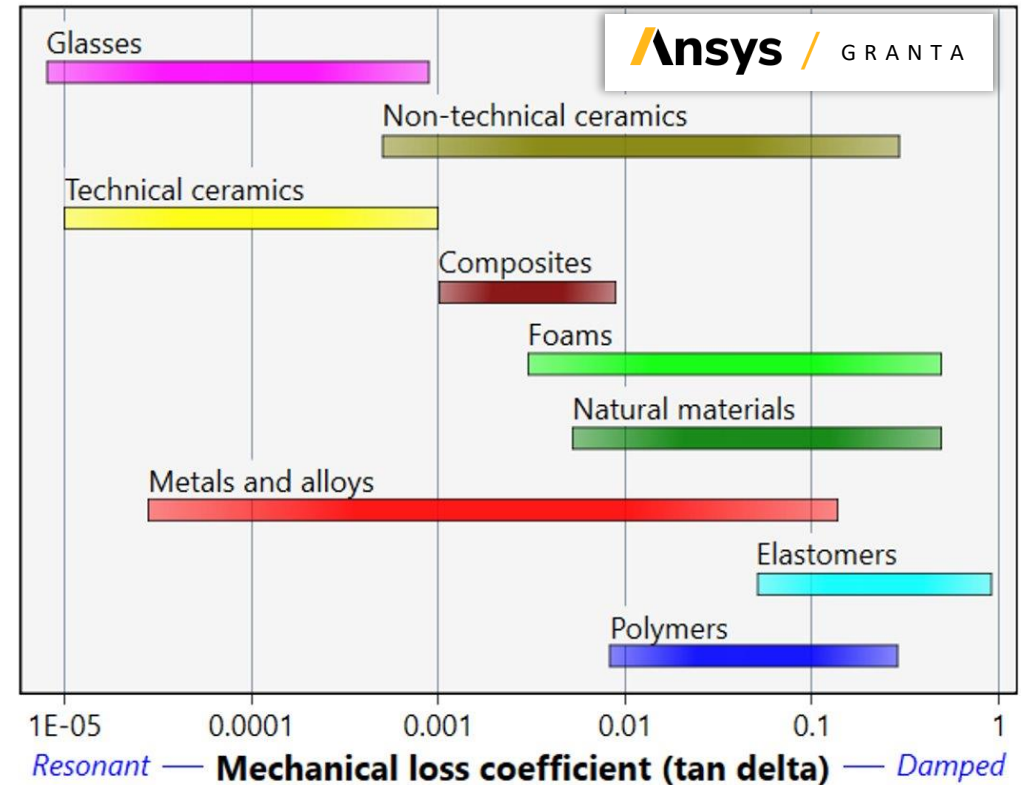
$$\zeta = \frac{c}{c_c} = \frac{1}{2Q} = \frac{\eta}{2}$$

where $\zeta = \text{Damping Ratio}$
 $c = \text{Damping Coefficient}$
 $c_c = \text{Critical Damping Coefficient}$
 $\eta = \text{loss factor}$
 $Q = \text{Amplification or Quality Factor}$

Characterization of Damping

- So what are some typical values for loss factor?
- We can there is no single value for a material.
- The range can be due to the dependence on exact composition, temperature and frequency of excitation.
- One clear trend is that relatively, rubber like materials have high loss factors compared to metallics.

Range of Typical Loss Factors near Room Temperature



Characterization of Damping

- There are yet other ways to characterize damping such as proportional or Rayleigh damping.
- Rayleigh damping is convenient mathematically as it is still linear and proportional to the mass and the stiffness as we will see with just two terms.
- Let's again recall the general matrix form of the equation of motion;

$$\overbrace{[M]\{\ddot{u}\}}^{F_{\text{inertia}}} + \overbrace{[C]\{\dot{u}\}}^{F_{\text{damping}}} + \overbrace{[K]\{u\}}^{F_{\text{stiffness}}} = \overbrace{\{F(t)\}}^{F_{\text{applied}}}$$

- We can then express the damping matrix to be linearly proportional to the Mass and Stiffness matrices.
- The two terms alpha (α) and beta (β) are what are to be specified.

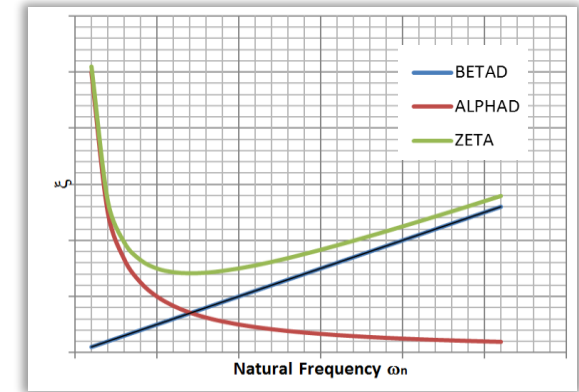
$$[C] = \overbrace{\alpha[M]}^{\text{Mass damping}} + \overbrace{\beta[K]}^{\text{Stiffness damping}}$$

Characterization of Damping

- The Mass (α) and stiffness (β) multiplier terms can be specified which results in viscous damping that is proportional to the linear combination of the mass and stiffness matrices.
- What to specify for (α) and (β)? They are not typically known directly but can be calculated from the modal damping ratio ζ_i at a particular natural circular frequency ω_i using;

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$$

- Recall that $\omega_i = 2\pi f_i$ where f_i is the natural frequency in Hz (cycles/second).
- Keep in mind that these terms (α) and (β) have no physical meaning. They are not a material property and typically need to be derived from testing, research, or established best practices.
- Keep in mind that this method may have difficulty matching a desired damping ratio over a larger frequency range.



Characterization of Damping

- While we have already seen our equation of motion during our discussion of damping, one important clarification is how the damping ratio can be accounted for directly.

$$\overbrace{[M]\{\ddot{u}\}}^{F_{\text{inertia}}} + \overbrace{[C]\{\dot{u}\}}^{F_{\text{damping}}} + \overbrace{[K]\{u\}}^{F_{\text{stiffness}}} = \overbrace{\{F(t)\}}^{F_{\text{applied}}}$$

- Dynamic simulations can use a modal based method called the mode superposition method (MSUP), which is discussed in the session titled Mode Superposition.
- Using the MSUP method, one can specify the damping ratios (ζ_j) directly and even on a per mode basis, as shown here with the harmonic modal coordinates equation, which is covered in more detail in the section titled Forced Frequency Response/Harmonic Analysis.

$$y_{jc} = \frac{f_{jc}}{(w_j^2 - \Omega^2) + i(2w_j\Omega\zeta_j)}$$

 **Ansys**

