# Free-Free Modal and Pre-stressed Modal Analysis

Modal Analysis – Lesson 5

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# Modal Analysis: Free-Free Modal Analysis

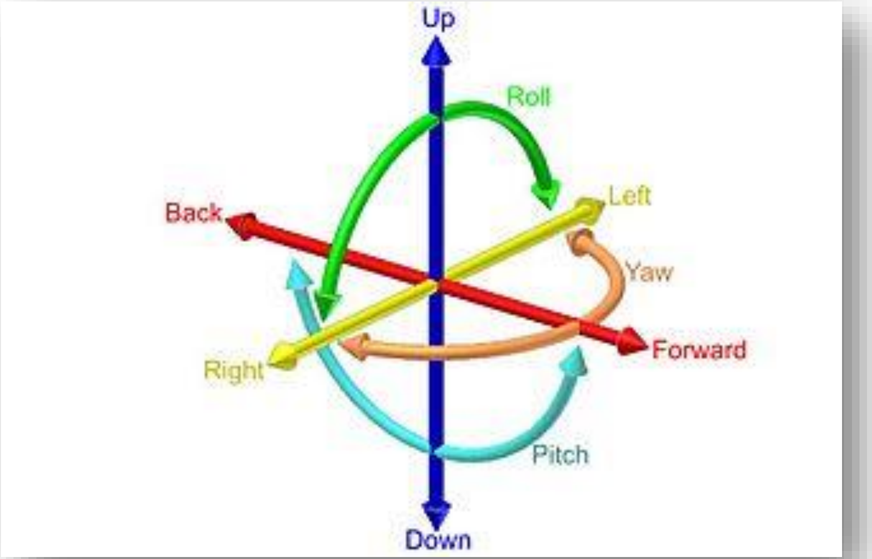
Let’s consider the two systems below: an airplane flying in the sky and a skyscraper. If an engineer plans to perform modal analysis on these two structures, what will be the biggest difference despite the geometry and material differences?

- The airplane is flying in the sky. It’s free of boundary conditions.
- Th skyscraper is “fixed” on the ground.

# Modal Analysis: Free-Free Modal Analysis (cont.)

If the modal analysis is for a structure without boundary conditions, it is called a "free-free" modal analysis.

- “Free-free” indicates that the structure is free of load and free of boundary conditions.
- The structure is free to have rigid movement in 6 degrees of freedom in three-dimensional space.



# Modal Analysis: Free-Free Modal Analysis (cont.)

For a free-free modal analysis, theoretically, the first 6 natural frequencies must be 0.

- Zero natural frequency means that the structure can have rigid movement without any excitation. This is expected as there is no boundary condition.
- Solving the free-free modal analysis numerically, e.g., by finite element software, you might find the first 8  natural frequencies are not absolutely 0, but they should be numbers very close to 0.0.

| Mode | Frequency (Hz) |
|---|---|
| 1 | 0. |
| 2 | 0 |
| 3 | 1.9361e-005 |
| 4 | 3.2459e-005 |
| 5 | 3.584e-005 |
| 6 | 5.3911e-005 |
| 7 | 7.8648 |
| 8 | 10.665 |
| 9 | 14.728 |
| 10 | 15.921 |
| 11 | 23.857 |
| 12 | 23.879 |
| 13 | 24.706 |
| 14 | 24.989 |
| 15 | 25.077 |

# Pre-stressed Modal Analysis

In industry and life, many structures are designed to work under stress. When performing modal analysis on such structures, the pre-stress effect must be considered, because the stress state changes the natural frequency of a structure.

- Guitar strings are pressed at different locations to sound a different note when plucked.
- The concrete plate and beam of a large-span bridge is reinforced by pre-stressed steel bars.

# Pre-stressed Modal Analysis: Formulation

How do we consider pre-stress in modal analysis?

- First, a static analysis is performed to find the stress status of the structure.
- The pre-stress state changes the structural stiffness by adding a stress stiffness matrix to the original structural stiffness.
- The eigenvalue solution is based on the new structural stiffness.

$$[\boldsymbol{K}]\{\boldsymbol{u}\} = \{\boldsymbol{F}\} \rightarrow \boldsymbol{\sigma}_0 \rightarrow [\boldsymbol{S}]$$

**A linear static analysis is performed**

**A stress stiffness matrix is calculated from the structural analysis**

$$([\boldsymbol{K}] - \omega_i^2[\boldsymbol{M}])\{\phi\}_i = \{0\}$$

$$\downarrow$$

$$([\boldsymbol{K} + \boldsymbol{S}] - \omega_i^2[\boldsymbol{M}])\{\phi\}_i = \{0\}$$

**The original free vibration equation is augmented to include the [S] term**

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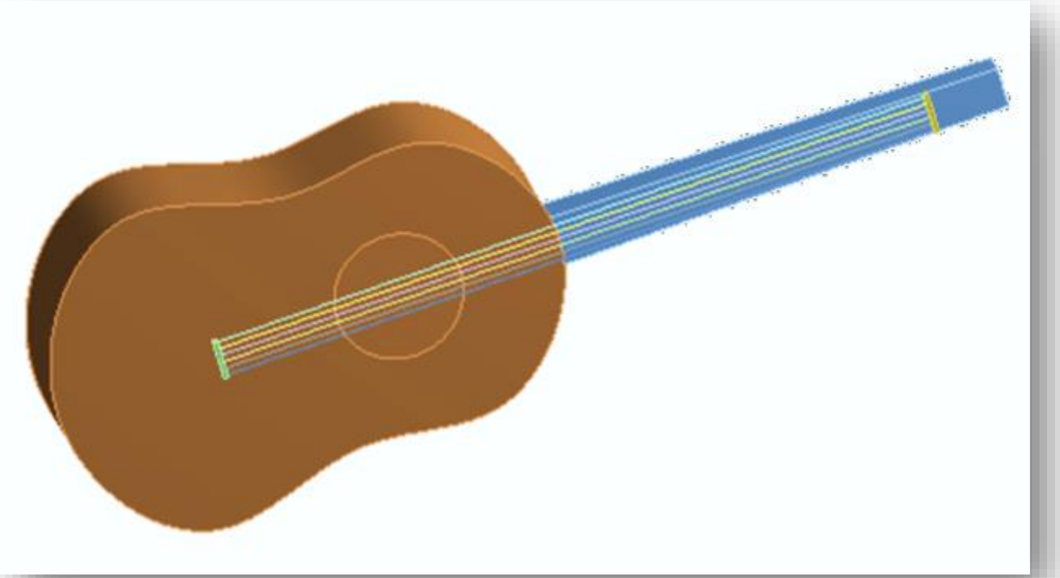
# Pre-stressed Modal Analysis

Let’s calculate the natural frequency of guitar strings to see how pre-stress affects the system.

- A displacement control is applied to the strings to put them in tension.
- The natural frequencies of the strings are drastically increased because of the pre-stressed effect.
- Usually, pre-stressing in tension causes natural frequencies to increase, while pre-stressing in compression causes natural frequencies to decrease.



**Before Pre-stress**

| Mode | Frequency (Hz) |
|---|---|
| 1 | 1.7677 |
| 2 | 1.7677 |
| 3 | 3.0051 |
| 4 | 3.01 |
| 5 | 4.87 |
| 6 | 4.87 |
| 7 | 5.1264 |
| 8 | 5.1264 |
| 9 | 8.2838 |
| 10 | 8.2838 |

**After Pre-stress**

| Mode | Frequency (Hz) |
|---|---|
| 1 | 187.54 |
| 2 | 187.54 |
| 3 | 187.89 |
| 4 | 187.89 |
| 5 | 188.5 |
| 6 | 188.5 |
| 7 | 190.56 |
| 8 | 190.56 |
| 9 | 193.95 |
| 10 | 193.95 |

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