Volumetric and Deviatoric Strain

Volumetric and Deviatoric Behavior

Ansys

Total Strain

- Consider a block of sides L_X, L_Y and L_Z that changes its dimensions by dL_x, dL_y and dL_z in three principal directions.
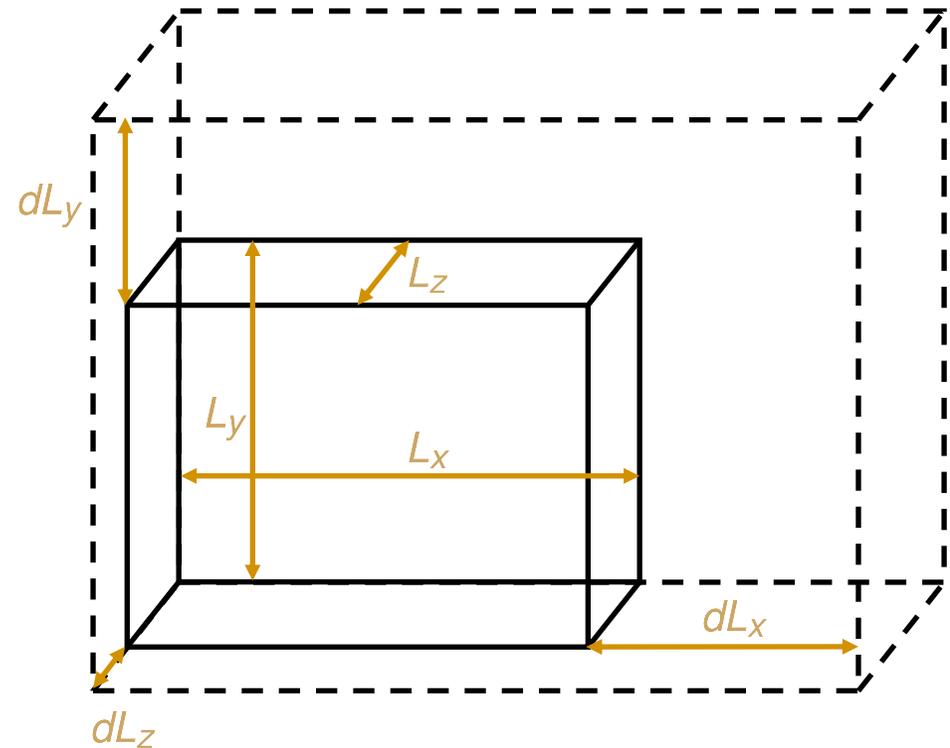
- The change in volume is given by

$$dV = \frac{\partial V}{\partial L_x}dL_x + \frac{\partial V}{\partial L_y}dL_y + \frac{\partial V}{\partial L_z}dL_z$$

- The initial volume $V=L_xL_yL_z$. So, dividing by V on both sides changes the relation to

$$\frac{dV}{V} = \frac{dL_x}{L_x} + \frac{dL_y}{L_y} + \frac{dL_z}{L_z}$$

- Replacing RHS terms with relevant strains

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Total Strain (cont.)

- ε_v is the volumetric strain and ε_x, ε_y and ε_z are the deviatoric strains.

- ε_v is also called dilation.

- Note that only the normal strains result in volume change; shear strains can distort but not change the volume.

Hydrostatic Pressure

- Imagine a component that is subjected to constrained compression.

- In this case, the body is being loaded in one direction and it is restricted from deforming in all other directions.

- So, there's no other way but for the body to either resist the load or change its volume.

- In this state, the body is experiencing tremendous stress from all principal directions.

- At this stage, we define hydrostatic pressure as the average of the three principal stresses acting on the body. It is simply the stress developed in the body while it is resisting change in volume.
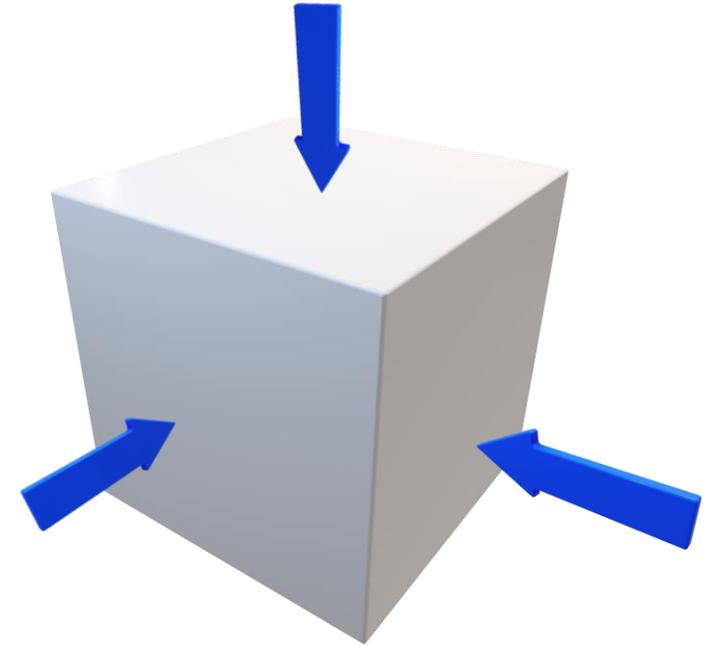
$$\sigma_h = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

Elastic and Bulk Modulus

- At any given point, the ratio of the deviatoric stress developed in the part to the deviatoric strain it is subjected to is just the elastic modulus.

- Therefore, it is the ability of the material to resist deformation.

$$E = \frac{\sigma_{dev}}{\varepsilon_{dev}}$$

- At any given point, the ratio of the hydrostatic stress developed in the part to the volumetric strain is just the bulk modulus.

- Therefore, the bulk modulus is the ability of the material to resist a change in volume.

$$K = \frac{\sigma_{vol}}{\varepsilon_{vol}}$$

Simple Example

- Let's study a simple steel cube of unit length subjected to two different loads to understand the physical meaning of elastic and bulk modulus.

- Case 1: The block is constrained only in one direction and it is free to deform in all other directions.

- Case 2: The block is constrained in all directions except for the direction of loading.
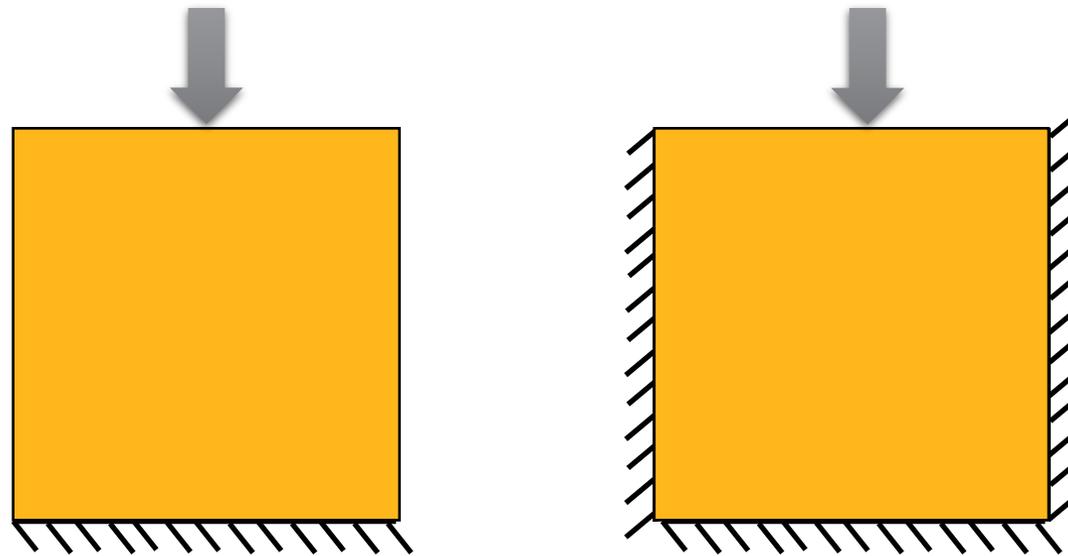
Simple Example (cont.)

- In case 1, the majority of the resistance offered by the body is deviatoric, as the body is free to deform in other directions.

- So, elastic modulus is the source for the majority of the resistance.

- In case 2, the majority of the resistance offered by the body is volumetric, as free deformation is not allowed.

- So, the bulk modulus is the source of most of the resistance.

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