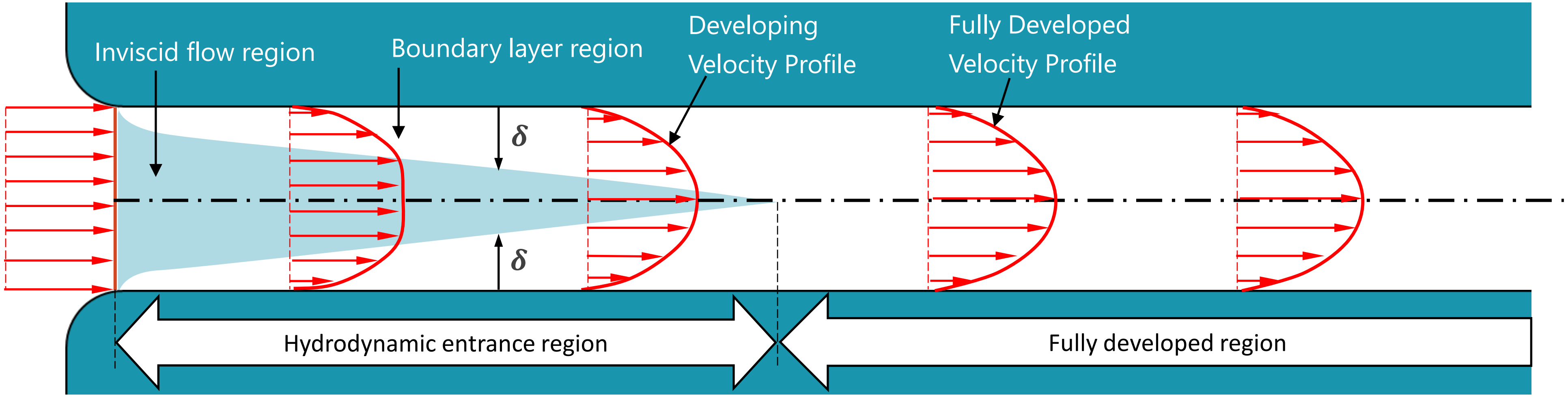
Fully Developed Internal Turbulent Flows in Ducts and Pipes

Real Internal Flows – Lesson 2

Entrance Region and Fully Developed Flow

- A fluid flowing through a pipe invariably enters the pipe at some location. A boundary layer along the wall starts developing at the entrance. The layer is thin near the entrance and viscous effects are restricted to the near-wall region.
- The boundary layer thickness grows as the fluid flows downstream, and eventually the layer edge reaches the pipe centerline, and the flow becomes fully developed.
- The length over which the flow evolves into the fully developed state is called the entrance length.
- The criteria for fully developed flow is for the rate of change of all mean quantities (except pressure) with respect to the coordinate in the flow direction to be zero.

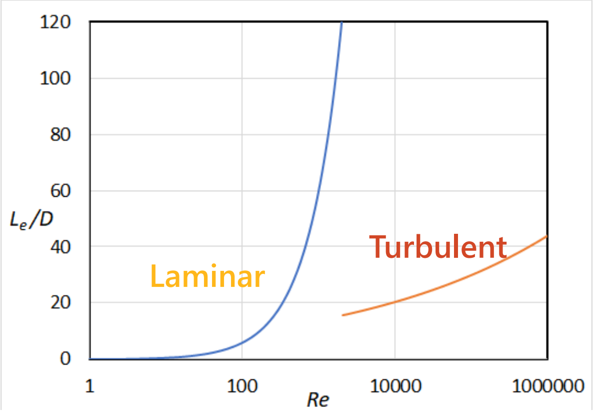


Entrance Region and Fully Developed Flow (cont.)

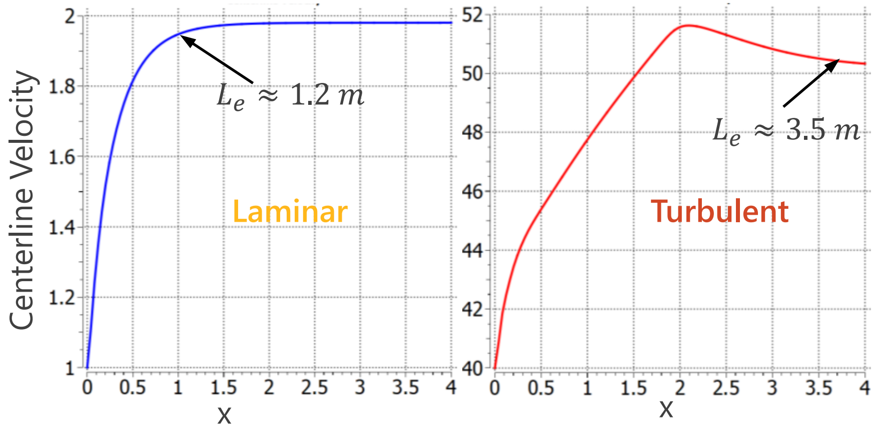
- The entrance length depends on whether the flow is laminar or turbulent. For a round pipe it can be estimated as:

Laminar: $\frac{L_e}{D} \approx 0.06\, Re$

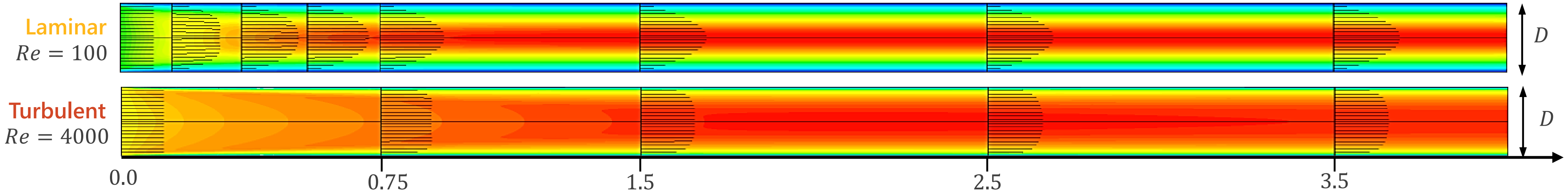
Turbulent: $\frac{L_e}{D} \approx 4.4\, Re^{1/6}$



- For large Reynolds numbers, the entrance length can be hundreds of diameters long!



- The pressure distribution and the velocity profile in the entrance region are complex, and correlations derived for fully developed flows will not hold. It is thus important to estimate the entrance length before applying fully developed flow analysis.



Fully Developed Turbulent Flow in a Circular Pipe

- We will consider fully developed turbulent flow in a circular pipe as a classical internal flow example.
- Laminar flow becomes unstable at around $Re = 2000$, and transitions to fully turbulent at $Re = 4000$.
- Recall that for a laminar flow, the exact Poiseuille solution was possible. For turbulent flows, however, no exact solutions exist, but we can employ physics-based dimensional analysis arguments like those used for flat-plate turbulent layer to approximate the velocity profile and other expressions.
- Under the fully developed condition, viscosity affects the flow across the entire pipe, and we can argue that the velocity should, in principal, have the universal law of the wall distribution:

$$\frac{\bar{u}}{v^*} = \frac{yv^*}{\nu}$$

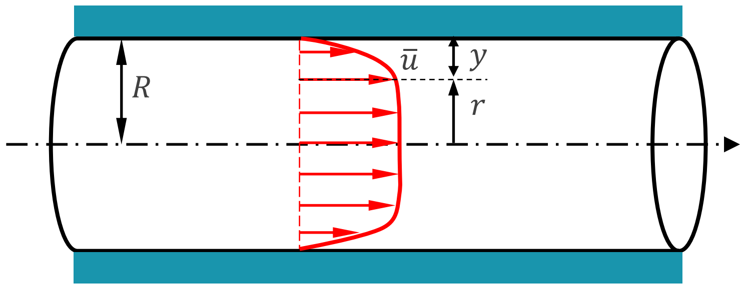
Laminar sublayer

$$\frac{\bar{u}}{v^*} = \frac{1}{\kappa}\ln\frac{yv^*}{\nu} + B$$

Log layer

$$\frac{V_c - \bar{u}}{v^*} = 2.5\ln(R/y)$$

Near centerline region



Here $y = R - r$ is the distance measured from the wall and V_c is the centerline velocity.

Fully Developed Turbulent Flow in a Circular Pipe (cont.)

- Assuming the log part of the layer is predominant across most of the pipe, we can evaluate the average pipe velocity:

$$V_{avg} = \frac{1}{\pi R^2}\int_o^R v^*\left[\frac{1}{\kappa}\ln\frac{(R-r)v^*}{\nu} + B\right]2\pi r dr = v^*\left(\frac{1}{\kappa}\ln\frac{Rv^*}{\nu} + B - \frac{3}{2\kappa}\right)$$

- The friction (or pipe-friction) factor is: $C_f = 2\tau_w/\rho V_{avg}^2$

- Thus: $\frac{V_{avg}}{v^*} = \left(\frac{2}{C_f}\right)^{1/2}$ $\frac{Rv^*}{\nu} = Re\left(\frac{C_f}{2}\right)^{1/2}$ $Re = \frac{2RV_{avg}}{\nu}$

- This shows the integrated expression for V_{avg} is a skin friction relation, which can be re-written in terms of Darcy's friction factor $f = 4C_f$ as:

$$f^{-1/2} = 1.99\log\left(f^{1/2}Re\right) - 1.02$$

Originally derived by Prandtl in 1935

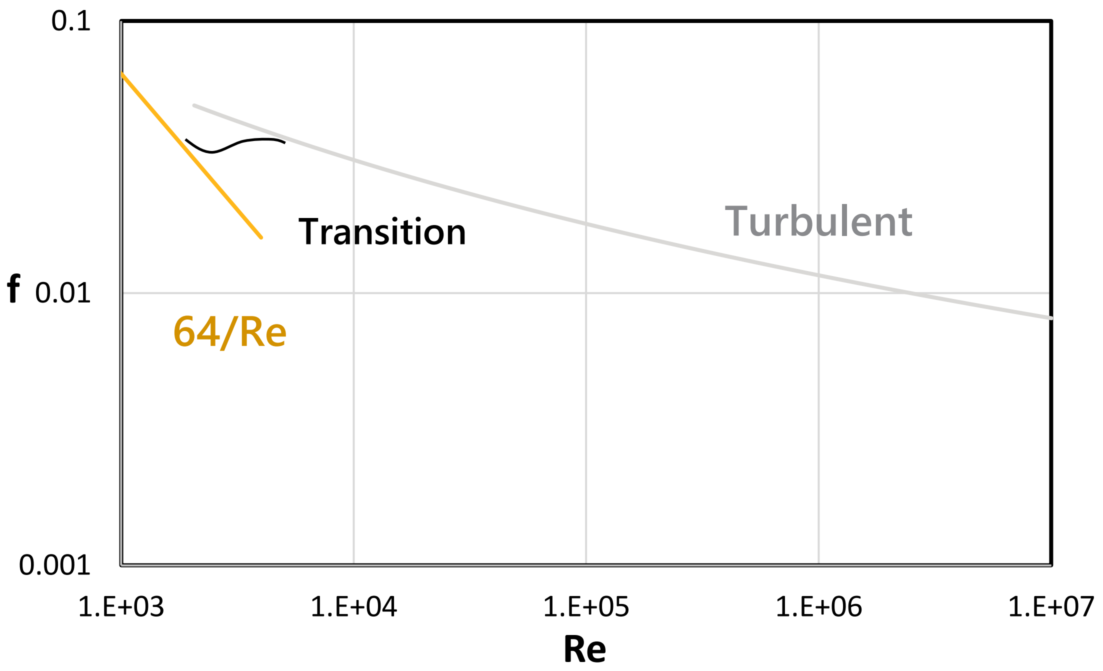
Here the constants κ and B are: $\kappa = 0.41$ and $B = 5.0$.

Fully Developed Turbulent Flow in a Circular Pipe (cont.)

- This formula for the Darcy friction factor was derived by neglecting the viscous sublayer and the centerline region. Prandtl adjusted the constants for a better fit with the experimental pipe-friction data:

$$f^{-1/2} = 2.0\log\left(f^{1/2}Re\right) - 0.8$$

- This expression has been proven to be valid for any $Re > 4000$.



Power Law Profile for Circular Pipe Turbulent Flow

- Another easy-to-use empirical correlation for the velocity profile is the power law:

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

  where n is a function of the Reynolds number.

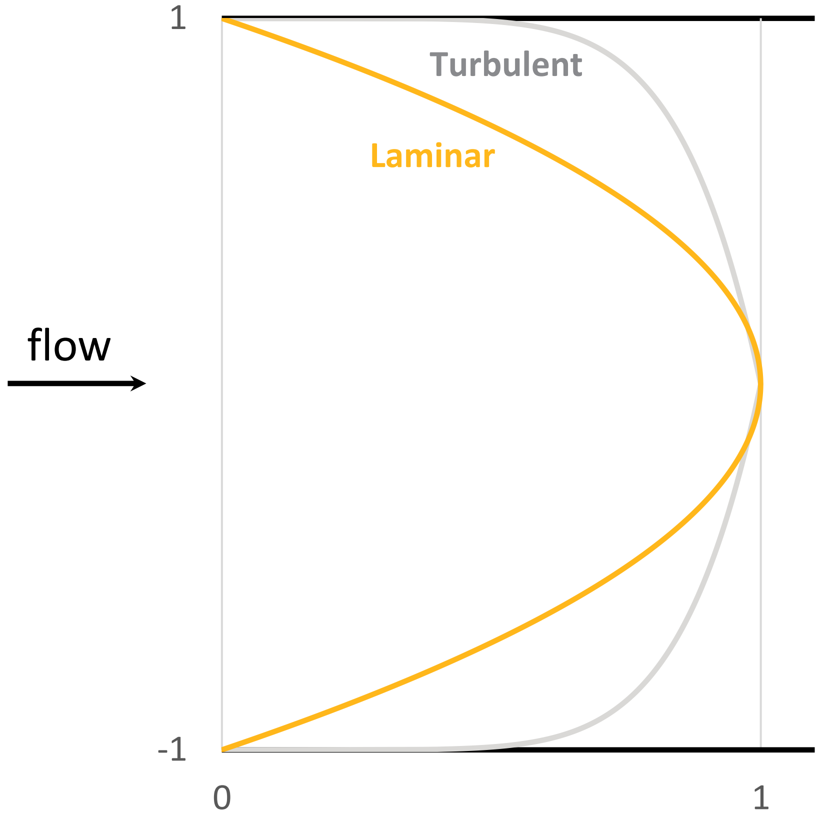
- This profile is a reasonable approximation for practical turbulent pipe flows.

- $n = 7$ reflects the range of many practical flows, thus this profile correlation is commonly used in its one-seventh power form.

- Note it is not valid near the wall as it yields infinite gradient at the wall. Likewise it is not valid at the pipe centerline since the gradient is not zero there. Still it provides favorable agreement with the experimentally measured profile across most of the pipe.

Comparison of Turbulent and Laminar Velocity Profiles

- Stark differences between laminar and turbulent velocity profiles in circular pipe flow are highlighted in the figure below.
- The turbulent profile is much flatter near the wall.



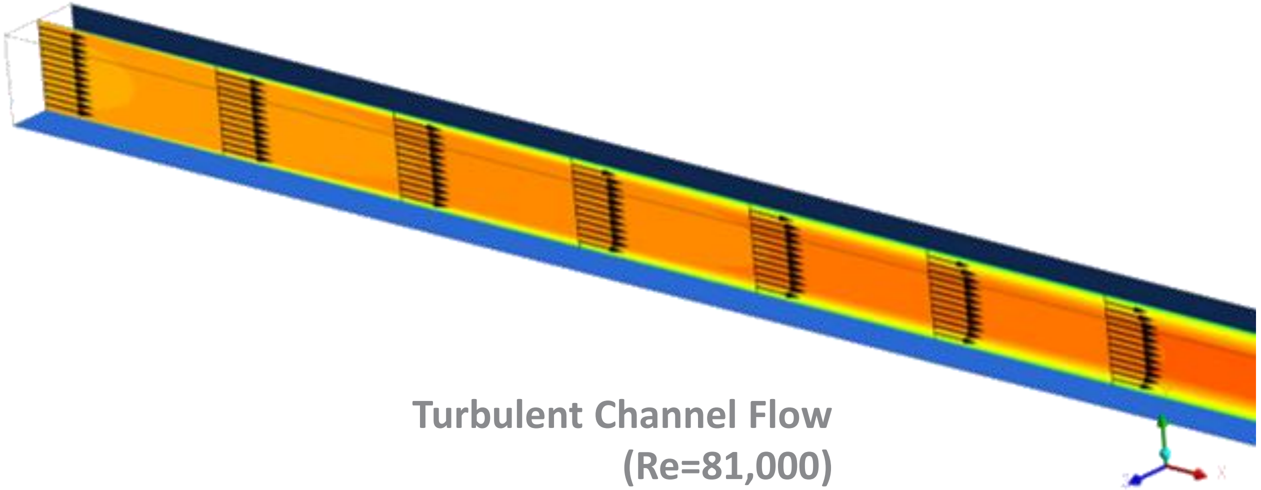
Fully Developed Turbulent Channel Flow

- Using an analysis similar to that of the circular pipe, an expression for the Darcy friction factor in a channel between parallel plates can be easily derived:

$$f^{-1/2} = 2.0\log\left(f^{1/2}Re_{D_h}\right) - 1.19$$

- Interestingly, this is close to the circular pipe expression, predicting 4% ($Re\sim10^8$) to 7% ($Re\sim10^5$) higher friction factor.

- This observation was the basis of a unified friction factor correlation for non-circular ducts.



Turbulent Channel Flow
(Re=81,000)

Turbulent Noncircular Duct Flows and Effective Diameter

- It turns out the Darcy friction correlation derived for the circular pipe can be utilized for non-circular ducts using the concept of effective diameter D_{eff} proposed in 1976:

$$f^{-1/2} = 2.0\log\left(f^{1/2} Re_{D_{eff}}\right) - 0.8$$

$$Re_{D_{eff}} = \frac{V_{avg} D_{eff}}{\nu}$$

Effective Reynolds number

$$D_{eff} = D_h \frac{16}{\left(C_f Re_{D_h}\right)_{laminar}}$$

Effective diameter

- In the effective diameter expression, the skin friction coefficient and Reynolds number are calculated from the laminar theory.

- This is a purely empirical correlation without a solid theoretical backing which was proven experimentally in tests on rectangular and concentric annular ducts.

- This correlation works reasonably well for ducts with blocky cross sections without unusually thin regions.

Summary

- In this lesson with discussed the basics of turbulent internal flows using flow in a circular pipe as a baseline.
- We covered the entrance flow effects and showed how to estimate the entrance length in both laminar and turbulent flow.
- Using the universal law of the wall, discussed earlier, to analyze the pipe flow yielded immediate results for the average velocity and friction factor without solving any differential equations.
- We noted a significant level of empiricism involved in deriving and adjusting correlation for turbulent flow.
- In the next lesson we will consider the evaluation of losses in internal flows.

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