Conservation of Mass Equation

Governing Equations of Fluid Dynamics – Lesson 3

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Introduction

- The first basic principle of fluid dynamics is the conservation of mass.
- This principle states that mass can neither be created nor destroyed, as was discussed in the previous lesson.
- More precisely, the net mass crossing a system boundary must be balanced by an accumulation or depletion of mass in the system.
- We know this law intuitively by observing the flow of water into a pipe – what goes in one end must come out the other!
- For compressible fluids, it is possible for the mass to increase or decrease within a fixed-volume system.

Deriving the Conservation of Mass Equation

- The Eulerian form of the conservation of mass equation is derived as follows:
  - Apply Reynolds Transport Theorem (with f = 1):

$$\frac{dM}{dt} = \frac{d}{dt}\iiint_{\Omega} \rho \, d\Omega = \iiint_{\Omega} \frac{\partial \rho}{\partial t} \, d\Omega + \oiint_{A} \rho \vec{V} \cdot \hat{n} \, dA = 0$$

  - Apply the Divergence Theorem:

$$\iiint_{\Omega} \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} \right) d\Omega = 0$$

  - The integral is satisfied for an arbitrary volume; hence its integrand must be zero. The result is the differential equation form of conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0$$

Conservation of Mass (Cartesian Form)

- It will be convenient to express our equations in the common Cartesian coordinate form. For conservation of mass, we can expand the vector form on the previous slide:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

- In two dimensions:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0$$

- Other forms (such as cylindrical and spherical coordinate forms) are possible and are available in the references.

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Incompressible Flow

- For incompressible flows, the density is assumed to be constant. Accordingly, the conservation of mass equation simplifies to

$$\nabla \cdot \vec{V} = 0$$

- This equation implies that, for an incompressible fluid, the net volume of fluid entering/leaving a system must be conserved.

Therefore, incompressible flows of liquids and gases at low speed are often stated in units of volume flowrate (e.g., gallons per minute for pumps and cubic feet per minute for fans and blowers).

Summary

- In this lesson we have derived the conservation of mass equation and its simplification to incompressible flow.
- Next, we will take up the more complicated conservation of momentum equation.

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