Strain Tensor Defined

Mechanical Strain in Deformation Analysis – Lesson 4

Ansys

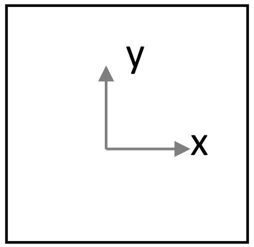
Three-Dimensional Strain

Let's first have a look at how deformation or deflection is represented 1D, 2D and 3D spaces. This involves recording how the coordinates change between the undeformed and deformed body.

Deformation

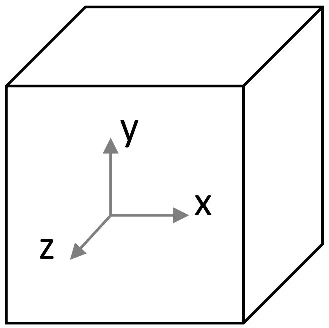
$$u = u_x$$

1D: Scalar



$$\boldsymbol{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

2D: Vector



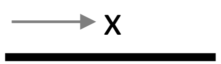
$$\boldsymbol{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

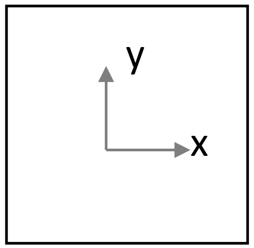
3D: Vector

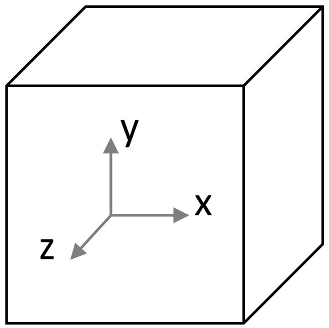
Three-Dimensional Strain (cont.)

We find that for a 1D problem, strain is scalar. For 2D and 3D problems, strain is in matrix format, which we call a tensor.

Strain







$$\varepsilon = \varepsilon_x$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

1D: Scalar

2D: Tensor

3D: Tensor

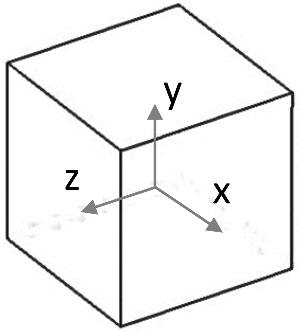
Three-Dimensional Strain: Normal Strain

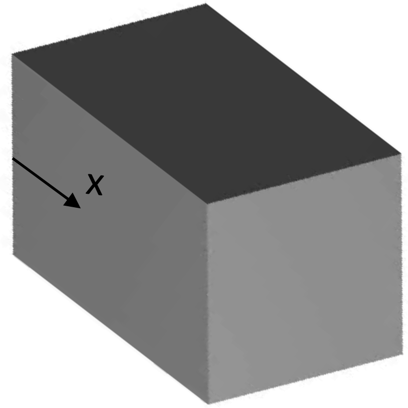
Since 3D space is more general, we'll skip the 2D case and directly discuss the 3D strain tensor. First, let's focus on the components along the diagonal in this 3 X 3 matrix. We call them normal strain.

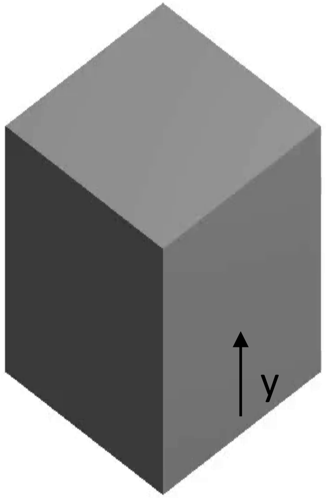
Normal Strain:

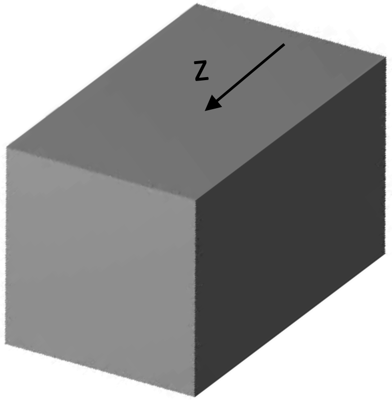
- Diagonal terms
- Representing the uniaxial deformation in x, y and z directions in the body.

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$









Three-Dimensional Strain: Shear Strain

Now, let's have a look at the off-diagonal terms in a strain tensor, which represent shear strain.

Shear Strain:

- Off-diagonal terms
- Symmetry $\varepsilon_{ij} = \varepsilon_{ji}$
- Represent shear deformation

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

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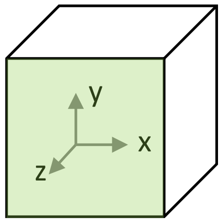
Three-Dimensional Strain: Shear Strain (cont.)

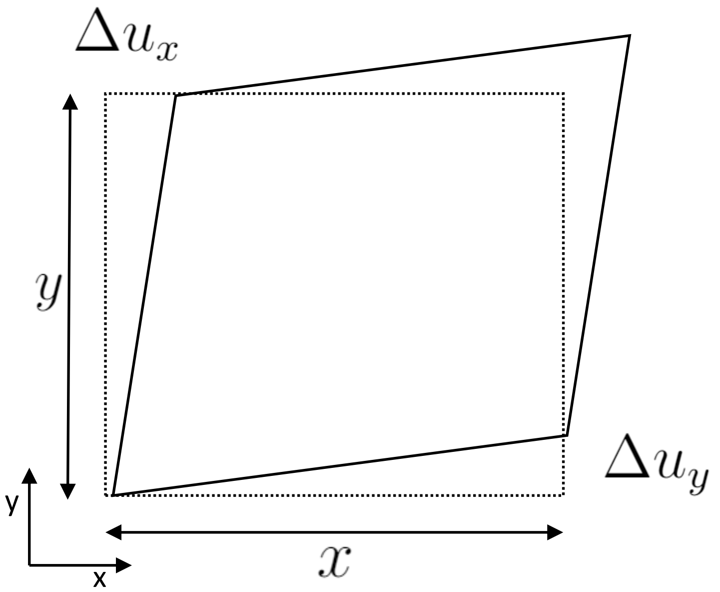
Definition of shear strain on xy surface:



$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

- Δu_x Deformation in the x **direction** with respect to the y **dimension**
- Δu_y Deformation in y **direction** with respect to the **x dimension**

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\frac{\Delta u_x}{y} + \frac{\Delta u_y}{x}\right)$$

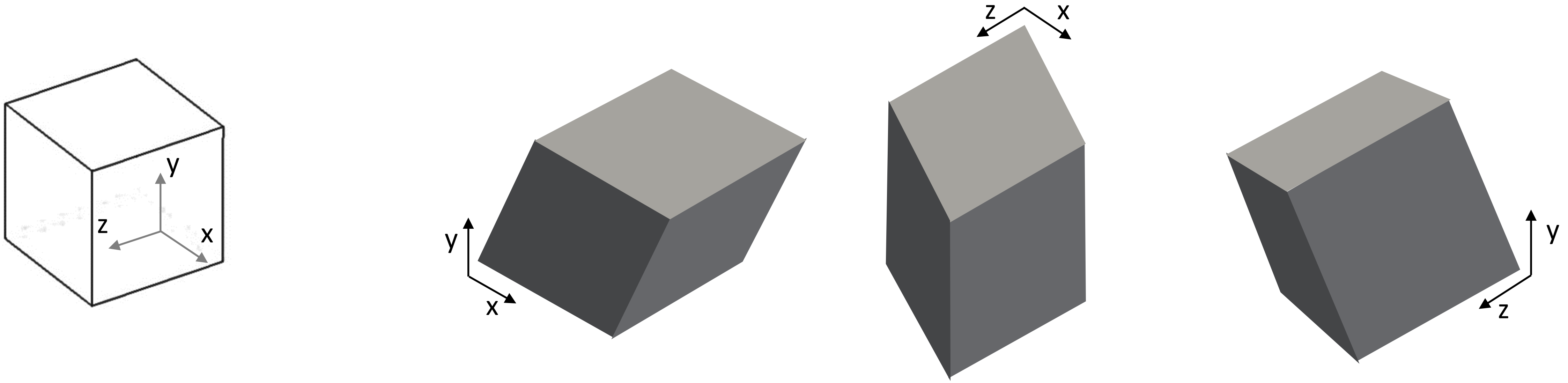


Note that x and y in this expression are interchangeable. This is one way to explain the symmetry of the strain tensor.

Three-Dimensional Strain: Shear Strain (cont.)

Illustration of pure shear deformation:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$



Note that only the angle on the denoted surface is changing; others remain perpendicular.

Three-Dimensional Strain: Voight Notation

A strain tensor as a 3 X 3 matrix can be inconvenient in deriving material relationships, so people have derived an easier notation of strain by transforming the 3 X 3 matrix to a vector, which is called Voight Notation.

Easier notation

$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \rightarrow \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{zx} \end{bmatrix}$$

Multiplier 2: for consistency in relating stress and strain.

- Three diagonal components
- Three off-diagonal components because of symmetry
- Multiplier 2 for the shear strain terms is for consistency in the stress–strain relationship

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