Engineering Strain and True Strain

Mechanical Strain in Deformation Analysis – Lesson 3

Ansys

Strain Measurement – Engineering Strain

Engineering strain is defined as the ratio between the change in length and the original length.

Assumption: for each step during deformation, the change of deformation is uniform, resulting in step-incremental deformation.

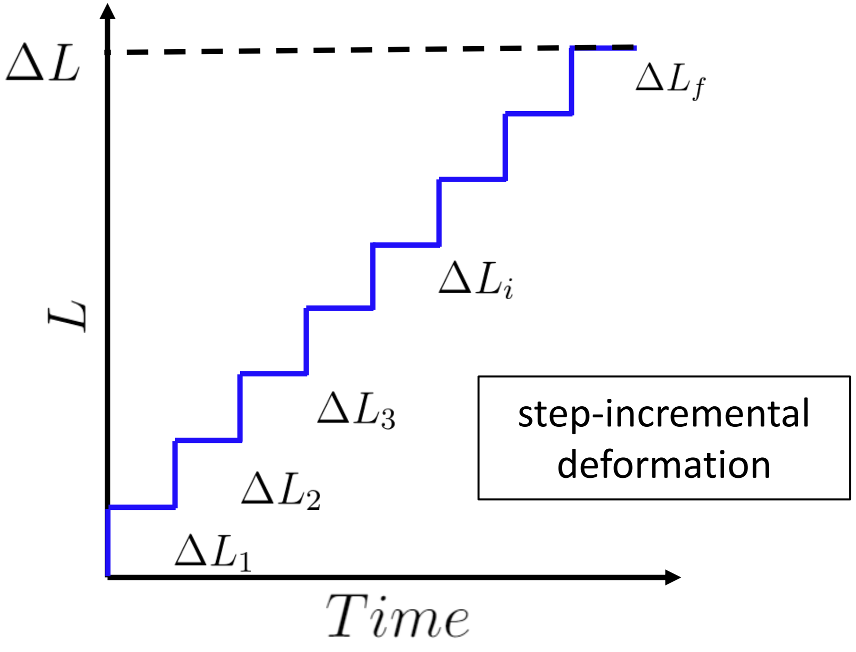
Engineering Strain

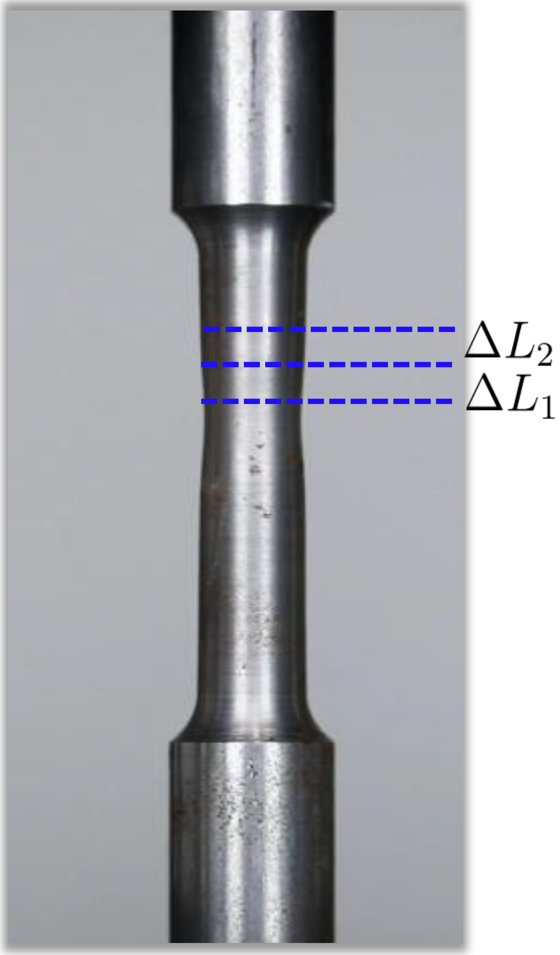
$$\epsilon_e = \frac{\Delta L}{L_0}$$

Strain Measurement – Engineering Strain (cont.)

Engineering Strain

$$\epsilon_e = \frac{\Delta L}{L_0}$$



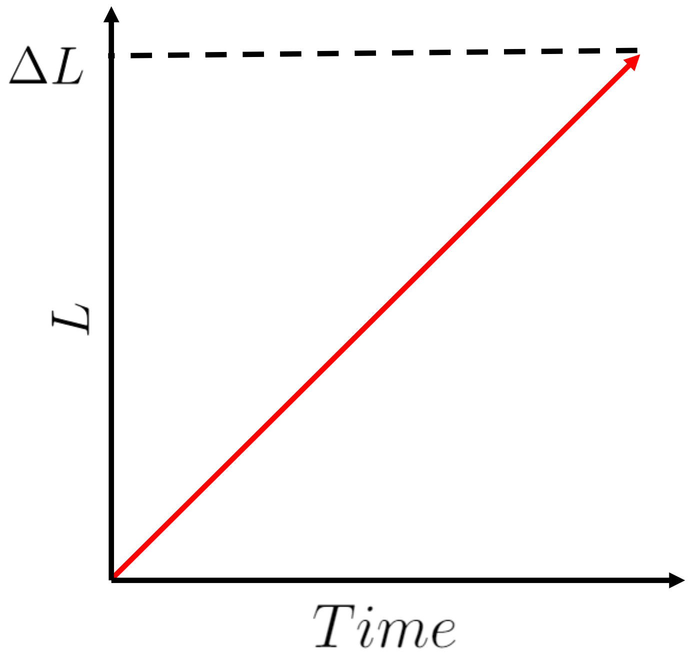


$$\epsilon_e = \frac{\Delta L_1}{L_0} + \frac{\Delta L_2}{L_0} \cdots + \frac{\Delta L_f}{L_0} = \frac{\Delta L}{L_0}$$

Strain Measurement – True Strain

True strain is the change in length with respect to the instant length.

True strain: deformation is continuous instead of stepwise.



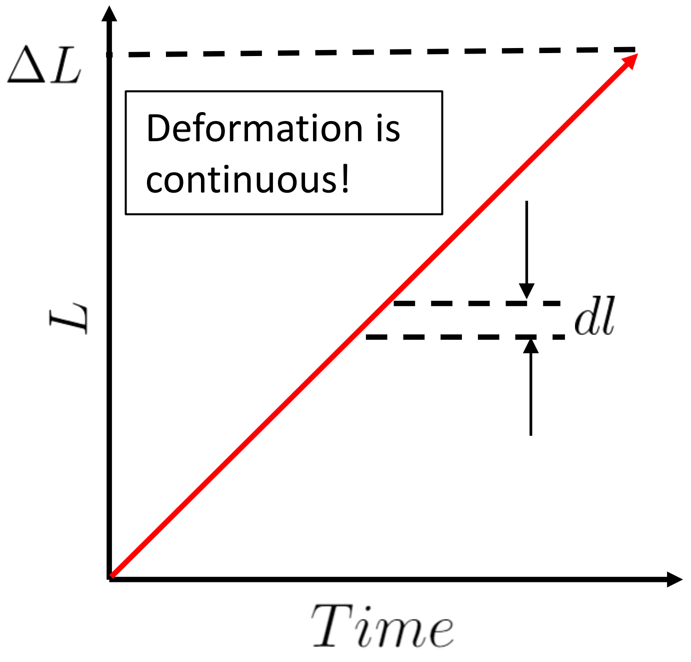
True Strain

$$d\epsilon = \frac{dl}{L_0}$$

$$\epsilon_t = \int_0^{\epsilon_t} d\epsilon$$

Strain Measurement – True Strain (cont.)

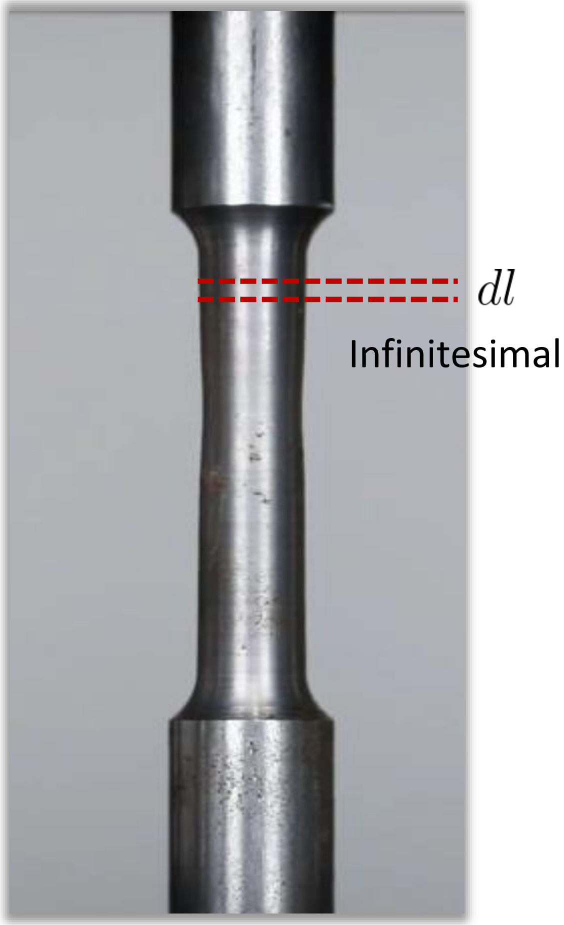
True Strain



$$d\epsilon = \frac{dl}{L_0}$$

$$\int_0^{\epsilon_t} d\epsilon = \int_{L_0}^{(L_0+\Delta L)} \frac{dl}{L_0} = \epsilon_t$$

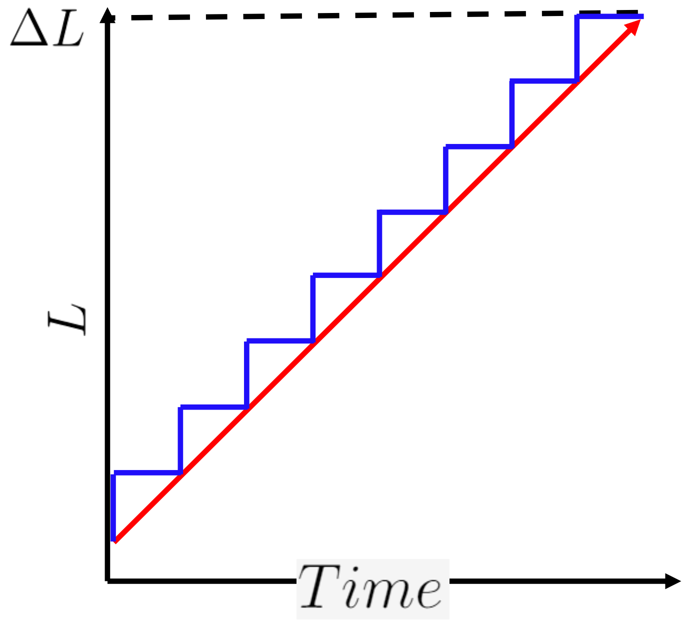
$$\epsilon_t = \ln\left(1 + \frac{\Delta L}{L_0}\right)$$



Strain Measurement

Let's put the two strain definitions on the same page and discuss the relationship between them.

Stepwise vs. Continuous



Engineering Strain

$$\epsilon_e = \frac{\Delta L}{L_0}$$

True Strain

$$\epsilon_t = \ln\left(1 + \frac{\Delta L}{L_0}\right)$$

Relationship between true and engineering strain:

$$\epsilon_t = \ln(1 + \epsilon_e)$$

Strain Measurement (cont.)

Let's check whether these two strain measurements meet the requirements of a strain definition:

No deformation	→	Zero Strain
Small deformation	→	Same strain value

No Deformation

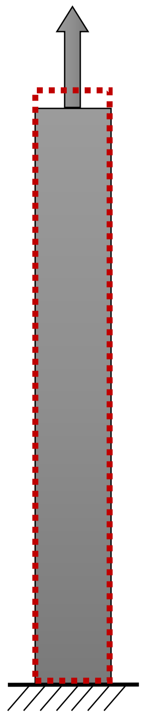
$$\Delta L = 0$$

Engineering Strain

$$\epsilon_e = \frac{\Delta L}{L_0} = 0$$ ✓

True Strain

$$\epsilon_t = \ln\left(1 + \frac{\Delta L}{L_0}\right) = 0$$ ✓



Strain Measurement at Small Deformation

No deformation	→	Zero Strain
Small deformation	→	Same strain value

Engineering Strain

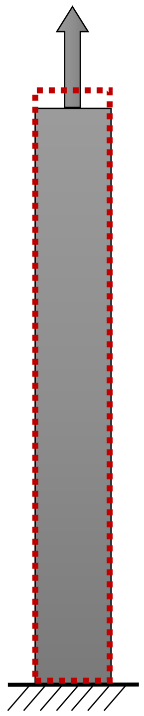
Small Deformation

$\Delta L = 0.05 L_0$

$$\epsilon_e = \frac{\Delta L}{L_0} = 0.05$$ ✓

True Strain

$$\epsilon_t = \ln\left(1 + \frac{\Delta L}{L_0}\right) = 0.049$$ ✓



Strain Measurement at Large Deformation

However, when the deformation is comparatively large, the difference between the two strain measurements will show up.

Large Deformation

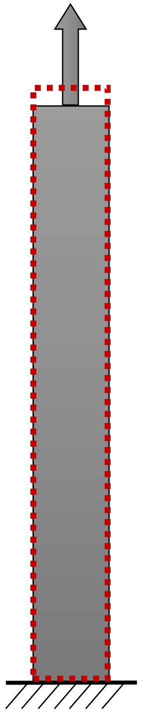
$$\Delta L = 0.3L$$

Engineering Strain

$$\epsilon_e = \frac{\Delta L}{L_0} = 0.3$$

True Strain

$$\epsilon_t = \ln\left(1 + \frac{\Delta L}{L_0}\right) = 0.26$$



When the change in length is 30% of the original length, the engineering strain is 0.3, but the true strain is calculated to be 0.26. That's more than a 10% difference.

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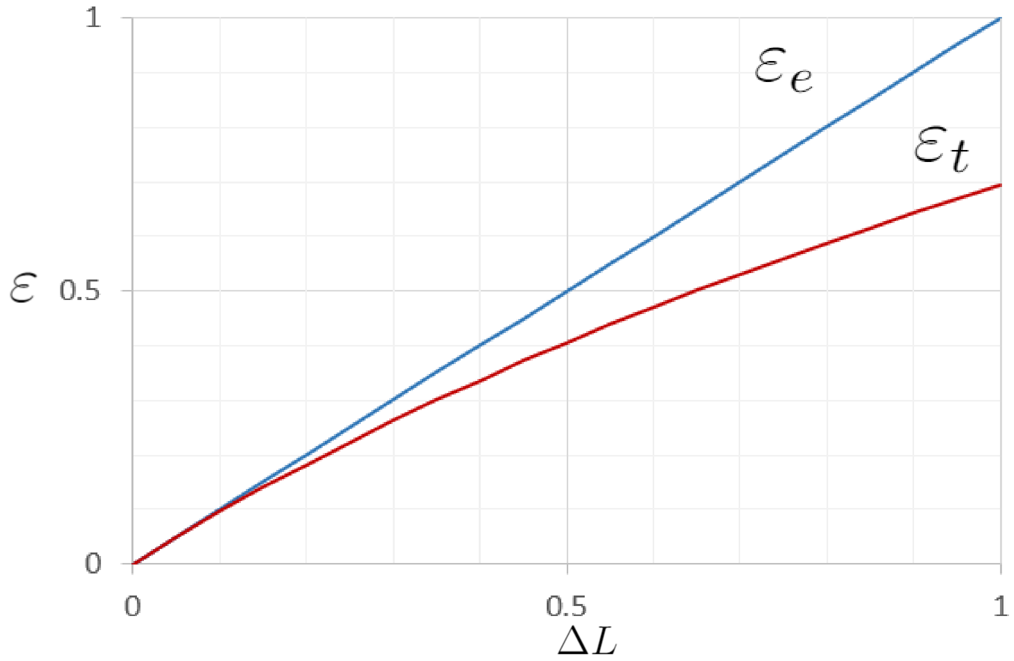
Strain Measurement

Let's plot the two strain measurements with length change as the horizontal axis and strain value as the vertical axis.

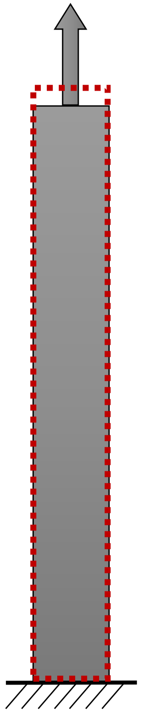


Engineering Strain

$$\epsilon_e = \frac{\Delta L}{L_0}$$

True Strain

$$\epsilon_t = \ln\left(1 + \frac{\Delta L}{L_0}\right)$$



Engineering strain is a straight line, while true strain is nonlinear. They deviate from each other with increasing deformation.

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