

# Intro to Transient Thermal Analysis

Thermal Capacitance in Heat Transfer Analysis –  
Lesson 1



# / Introduction

How long does it take for a cup of coffee to cool down to room temperature?

How much time do you have to shape melted glass?

Transient analysis can help determine the answers to such questions.

Transient thermal analyses determine temperatures and other thermal quantities that vary over time. Many heat transfer applications involve transient thermal analyses such as:

- Heat treatment problems
- Electronic package design
- Nozzles
- Engine blocks
- Pressure vessels



# Thermodynamics of Energy Conservation

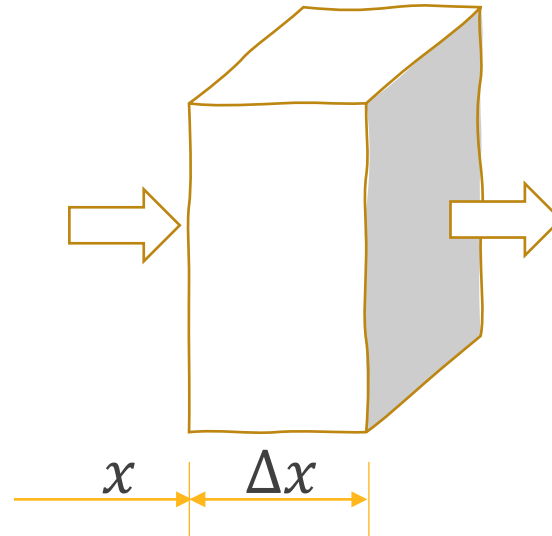
**Power input to the system**

**=**

**Power output from  
system**

**+**

**Rate of energy storage  
inside the volume**



# / Governing Equation

The governing equation for the heat conduction through a solid is given by:

$$k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q = \rho c \frac{\partial T}{\partial t}$$

Or

$$k \nabla^2 T + q = \rho c \frac{\partial T}{\partial t}$$

Rate of heat  
conduction

Rate of heat flux/convection/  
radiation/internal heat generation  
inside the volume

Rate of  
energy storage inside  
the volume

where,

$k$  = Thermal conductivity (in  $W/K \cdot m$ )

$t$  = Time

$T$  = Temperature ( in  $K$ )

$q$  = Rate of heat flux/convection/  
radiation/internal heat generation inside the  
volume (in  $W$ )

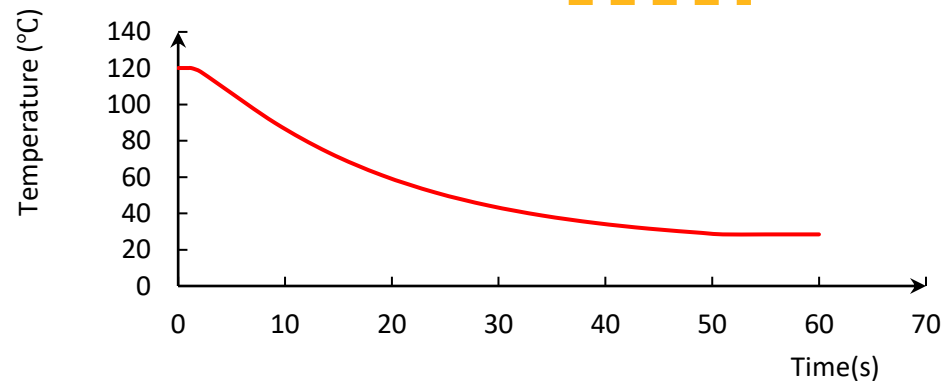
$\rho$  = Density of the material (in  $kg/m^3$ )

$c$  = Specific heat of the material (in  $J/kg \cdot K$ )

# / Governing Equation (cont.)

- In the equation for transient heat conduction, the term on the right is the rate of energy storage in the body. We will call this the transient term.
- This differentiates transient analysis from the steady-state, where temperature distribution does not change with time.
- For a long period of time, when the temperature distribution does not change any more, we say the problem has reached a steady-state condition that is governed by Fourier's law. For example, in the graph below the temperature profile stabilizes after 50 seconds.

$$k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q = \boxed{\rho c \frac{\partial T}{\partial t}} \rightarrow \text{Transient Term}$$



# / A Note About Initial Temperatures

- A transient thermal analysis involves loads that are functions of time. The first step in applying transient thermal loads is to establish the initial temperature distribution at time = 0.
- Initial temperatures do not matter in steady-state analyses.
- Initial temperatures are very important in transient analyses.
- For example, when leaving ice water and hot tea in the sun for 5 mins, the final temperatures will be different.

Ice water



Hot tea



# Thermal Capacitance and the Thermal Capacitance Term

- The product of density ( $\rho$ ), specific heat ( $c$ ) and volume ( $V$ ) for a body is the **thermal capacitance** ( $C$ ).
- We can call the product  $\rho c$  the **thermal capacitance term**, which indicates the ability of the body to store thermal energy.
- The larger the thermal capacitance term, the more time it will take to heat the body and vice-versa.

$$k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q = \underbrace{\rho c}_{\text{thermal capacitance term}} \frac{\partial T}{\partial t}$$

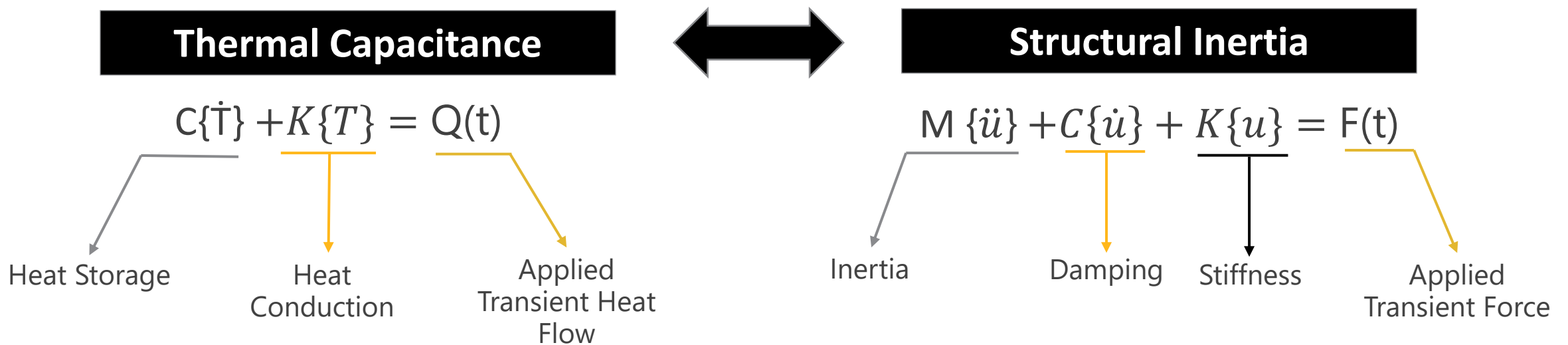
- In matrix form the transient thermal heat conduction can be written as:

$$C\{\dot{T}\} + K\{T\} = Q(t)$$

Thermal Capacitance Matrix      Thermal Conductivity Matrix      Heat Rate Vector

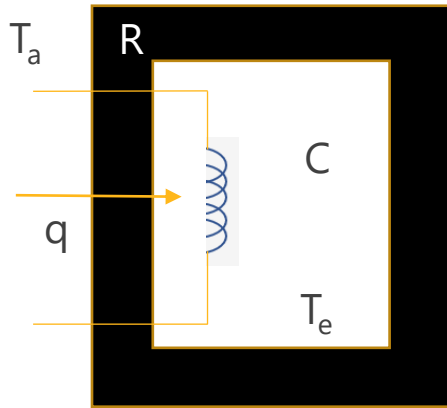
# Structural Analogy

- A body with larger thermal capacitance will take more time to heat compared to a body having smaller capacitance.
- This is analogous to the inertia of a structure seen in structural analysis. A body with large mass offers large resistance (inertia) to motion compared to a body of smaller mass.
- Because of this analogy thermal capacitance is also called **thermal mass**.



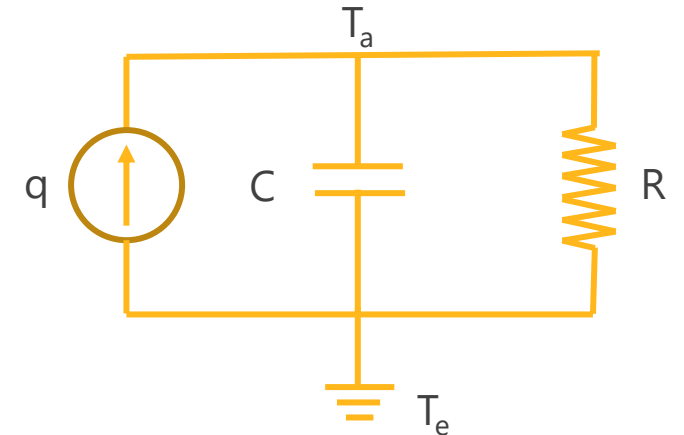
# Electrical Analogy

- The inverse of thermal conductivity is thermal resistance, which is analogous to electric resistance.
- Similarly, in a transient analysis thermal capacitance is analogous to electrical capacitance, which is related to storage of electric charge.



Heat in =  
Heat out + Heat stored

An insulated enclosure surrounded by wall where:  
R = Thermal resistance of the walls  
C = Thermal capacitance of the enclosure the heat  
q = Heat flow rate into the enclosure  
T<sub>e</sub> = Temperature of the enclosure  
T<sub>a</sub> = Constant ambient temperature



An electric circuit where:  
R = Electrical resistance, dissipates energy  
C = Capacitance of the capacitor, stores energy  
q = Current flow through the source  
T<sub>e</sub> = Temperature of one end of the capacitor  
T<sub>a</sub> = Temperature at the other end of the capacitor

 **Ansys**

