Thermal Conductivity of Materials

Thermal Conductivity in Heat Transfer – Lesson 2

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What Is Thermal Conductivity?

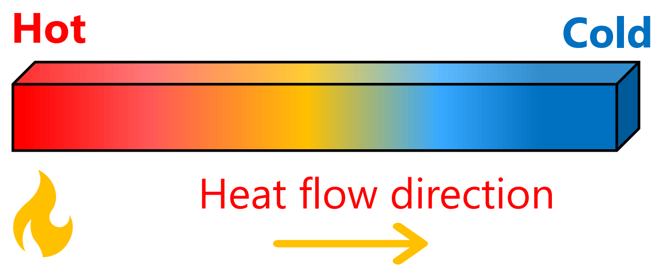
Based on our life experiences, we know that some materials (like metals) conduct heat at a much faster rate than other materials (like glass). Why is this? As engineers, how do we quantify this?

- **Thermal conductivity** of a material is a measure of its intrinsic ability to conduct heat.

Metals conduct heat much faster to our hands, which is why we use oven mitts when taking a pie out of the oven.

Plastic is a bad conductor of heat, so we can touch an iron without burning our hands.

What Is Thermal Conductivity?



- Let's recall Fourier's law from the previous lesson:

$$q = -k\nabla T$$

where q is the heat flux, ∇T is the temperature gradient and k is the thermal conductivity.

- If we have a unit temperature gradient across the material, i.e.,$\nabla T = 1$, then $k = -q$. The negative sign indicates that heat flows in the direction of the negative gradient of temperature, i.e., from higher temperature to lower temperature.

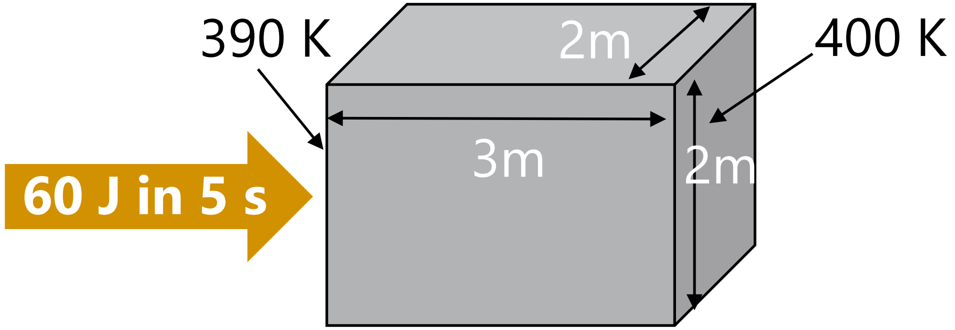
Thus, thermal conductivity of a material can be defined as the heat flux transmitted through a material due to a unit temperature gradient under steady-state conditions. It is a material property (independent of the geometry of the object in which conduction is occurring).

Measuring Thermal Conductivity

- In the International System of Units (SI system), thermal conductivity is measured in watts per meter-Kelvin (W $m^{-1}K^{-1}$).

$$q = -k\nabla T \longrightarrow k = -\frac{q}{\nabla T} \xrightarrow{\text{Unit}} \frac{\mathrm{W \cdot m^{-1}}}{\mathrm{K}} \quad (\mathrm{W \cdot m^{-1} \cdot K^{-1}})$$

- In imperial units, thermal conductivity is measured in British thermal unit per hour-feet-degree-Fahrenheit (BTU $h^{-1}ft^{-1}\,{}^{o}F^{-1}$).



In the above example, since 60 J of heat flows through the block in 5 s, the rate of heat flow = 60 J/5 s = 12 W. As the heat flow is normal to the x-y face with an area of 2 m × 2 m = 4 m^2, the heat flux = 12 W/4 m^2 = 3 W m^{-2}. Since the heat flows in x-direction, the temperature gradient in that direction = (390K – 400K)/3 m = -10/3 K m^{-1}. Therefore, the thermal conductivity of this material = -3 / (-10/3) = 0.9 W $m^{-1}K^{-1}$.

Thermal Conductivity of Different Materials

- Metals are typically good conductors of heat, and hence have high thermal conductivity. Gases generally have low thermal conductivity and are bad conductors of heat (or, in other words, are good insulators).
- The table below shows typical values of thermal conductivity for various materials at 0°C:

Material	Silver	Copper	Pure iron	Steel	Plastic	Glass	Wood	Water	Air
Thermal conductivity ($W\ m^{-1}K^{-1}$)	429	403	86.5	51.0	0.17-0.5	0.4 – 1.0	0.16 – 0.25	0.599	0.0226

Good conductors (Silver, Copper, Pure iron, Steel)

Good insulators (Plastic, Glass, Wood, Water, Air)

Reducing thermal conductivity →

- Thermal conductivity of different materials may differ vastly by orders of magnitude, e.g., thermal conductivity of metals is 1000- to 10,000-times higher than that of air.

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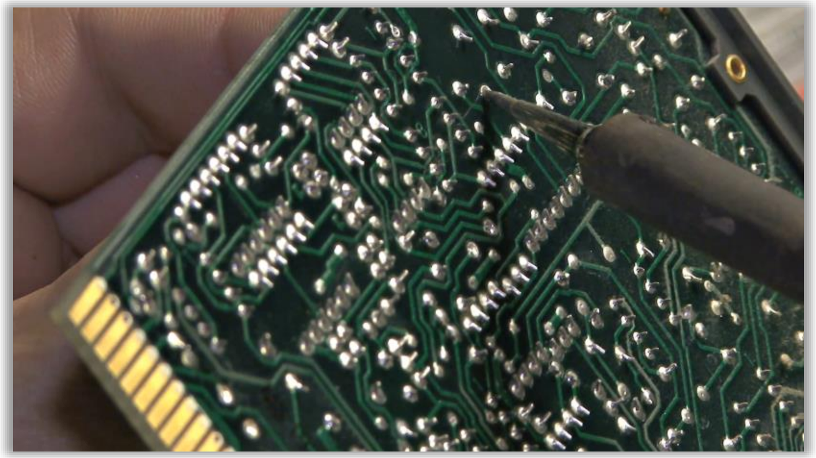
Thermal Conductivity of Different Materials

In many cases, thermal conductivity is an important factor that engineers need to consider for product design.



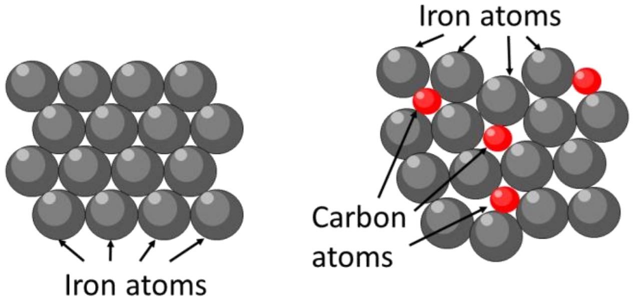
The tip of a **soldering iron** is made of copper. Copper, a good conductor of heat, reaches high temperatures quickly and allows it to melt the soldering wire.

Thermos flasks have double walls with a vacuum in between the walls. The vacuum is an effective insulator, ensuring that hot liquids stay hot for a long times.

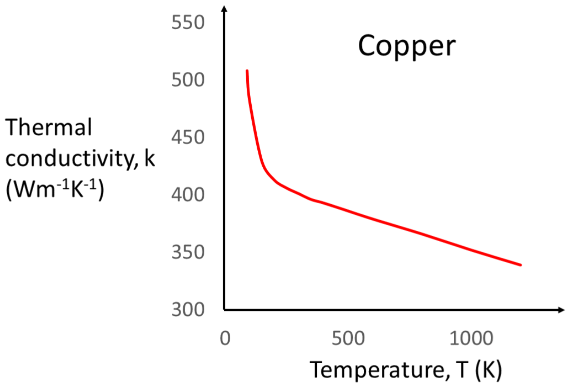
Although the kettle might be made of metal, it has a **plastic handle** since plastic is a bad conductor of heat, making it safe for bare hands to handle the hot kettle.

Factors Affecting Thermal Conductivity

- **Impurities:** Increasing the amount of impurities reduces the thermal conductivity, e.g., the thermal conductivity of steel with 1% carbon is 40% lower than that of pure iron.
- **Porosity:** Because the pores are filled with air (which has low thermal conductivity), increasing the porosity of a material reduces its overall thermal conductivity (e.g., fiberglass and Styrofoam have low thermal conductivity which makes them good insulators.
- **Temperature:** Thermal conductivity of most materials changes with the operating temperature.



Porous styrofoam

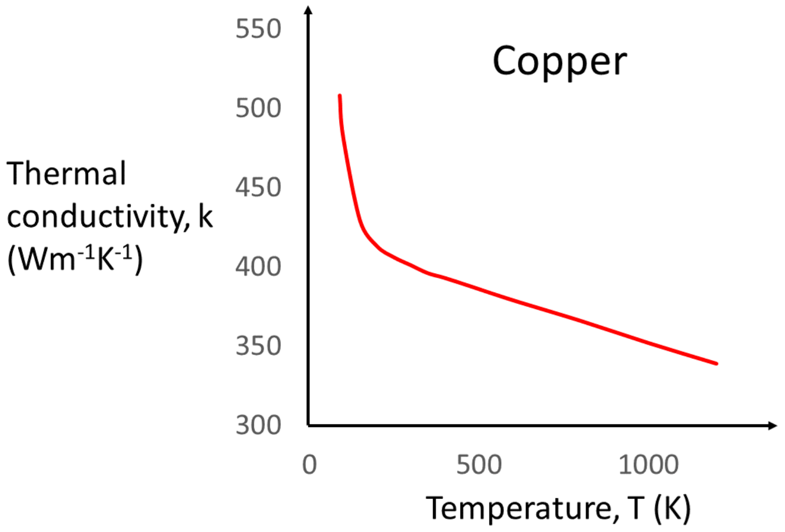


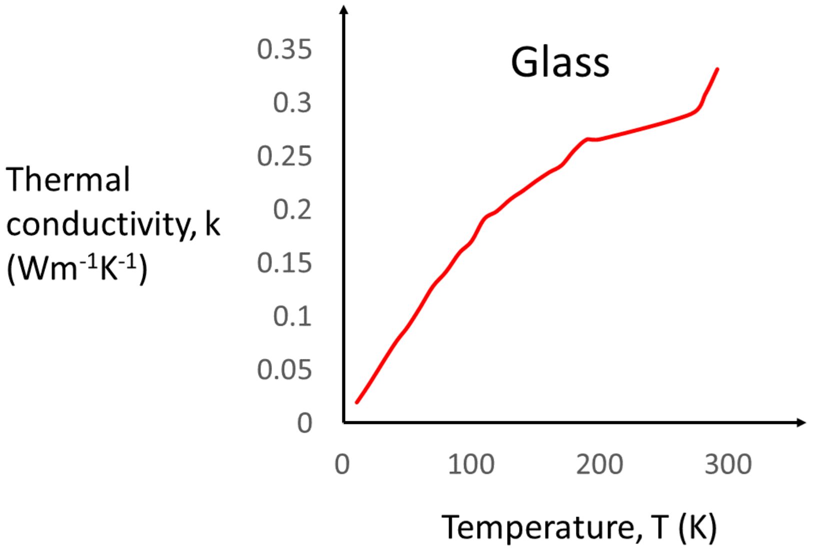
The dependence of thermal conductivity on temperature causes Fourier's law to become a nonlinear equation and we need to use the Newton-Raphson method to solve such problems.

Temperature-Dependent Thermal Conductivity

The thermal conductivity of most materials changes with temperature.

- For instance, the thermal conductivity of most metals (copper, etc.) decreases with an increase in temperature, while the thermal conductivity of nonmetallic materials (such as glass) increases with temperature.





Temperature-Dependent Thermal Conductivity

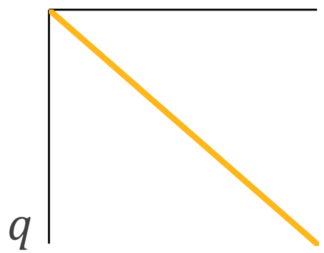
- If the thermal conductivity changes with temperature, Fourier's law is modified as:

$$q = -k(T)\nabla T$$

where thermal conductivity k is a function of temperature T, then

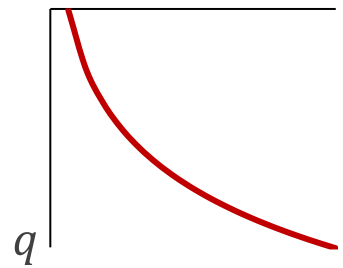
- If we consider heat conduction in 1D, when thermal conductivity is constant, we have

$$q = -k\,\frac{dT}{dx} \qquad q\int_0^l dx = -k\int_{T_1}^{T_2} dT \qquad ql = -k(T_2 - T_1) \qquad q = -k\,\frac{(T_2 - T_1)}{l}$$



- Now let's assume $k = \frac{1}{T}$. Then the heat flux is calculated as:

$$q = -k\,\frac{dT}{dx} \qquad q\int_0^l dx = \int_{T_1}^{T_2} -\frac{1}{T}\,dT \qquad ql = -k(\log T_2 - \log T_1) \qquad q = -\frac{k}{l}\,log\,\frac{T_2}{T_1}$$



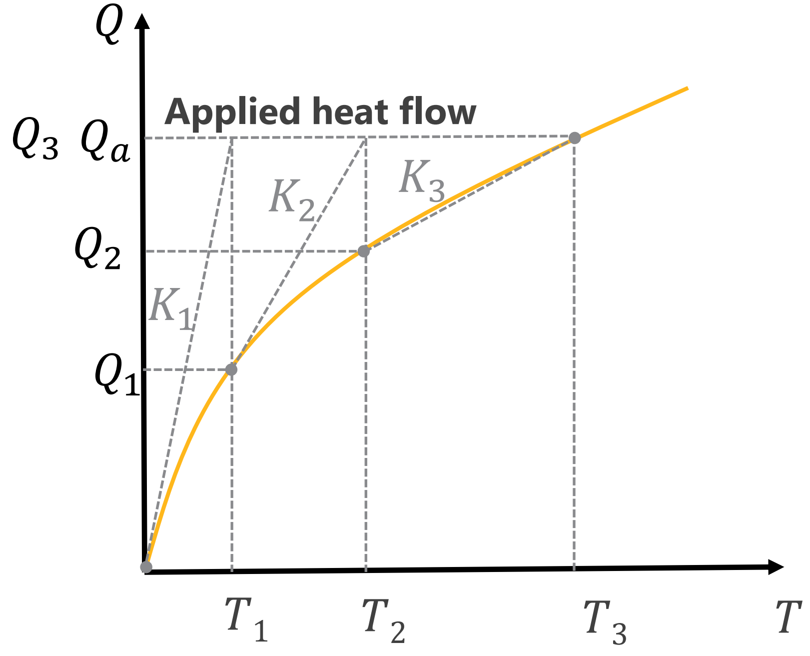
- The dependence of k on temperature may not always be so simple always and we may be dealing with a complicated geometry (not 1D). In these cases, we use the Newton-Raphson method to solve the problem.

Temperature-Dependent Thermal Conductivity

- When conductivity is temperature-dependent, in the FEA governing equation, the conductivity stiffness $[K]$ varies with temperature.

$$[K]\{T\} = \{Q\}$$

- To solve this type of nonlinear system, **Newton-Raphson method** is commonly used with FEA to find numerical solution through iterations.



Anisotropic Thermal Conductivity

- Until now, we have been writing Fourier's equation in a scalar form:

$$\boxed{q} = -\boxed{k}\,\nabla T$$

Scalars

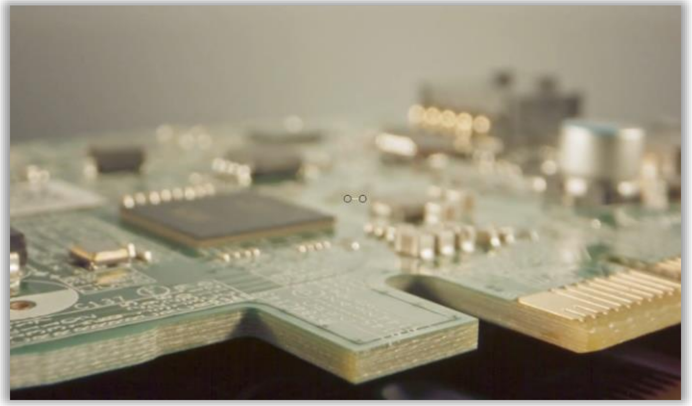
- This is based on the assumption that we are dealing with isotropic materials, which have identical values of a property (say, thermal conductivity) in all directions.
- However, in the most general form, Fourier's equation is written as:

$$\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \begin{bmatrix} dT/dx \\ dT/dy \\ dT/dz \end{bmatrix} \qquad \boldsymbol{q} = -\,\boldsymbol{k}\,\nabla T$$

- Here, heat flux $\boldsymbol{q}$ is a vector (and can have different values in different directions), thermal conductivity $\boldsymbol{k}$ is a tensor and $\boldsymbol{\nabla T}$ is a vector.
- These materials can have different thermal conductivities in different directions and are known as anisotropic materials.

Anisotropic Thermal Conductivity

- Typically, homogeneous materials are isotropic in behavior, but some materials (rocks, composite materials, etc.) may exhibit different properties in different directions.
- For example, printed circuit boards (PCBs) are made with alternating layers of copper and polymer. The effective conductivity of PCBs is about 100-times smaller in the through-thickness direction, as compared to in-plane directions (0.264 $Wm^{-1}K^{-1}$ through-thickness, 21.56 $Wm^{-1}K^{-1}$ in-plane directions).
- Careful consideration must be given while performing thermal analysis on these types of composite materials.



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