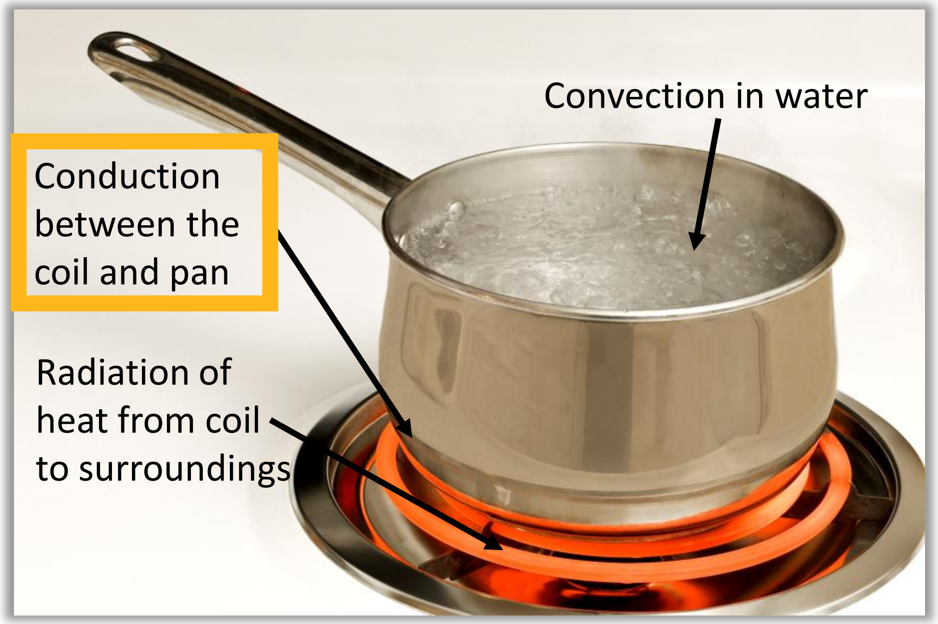
Introduction to Fourier's Law

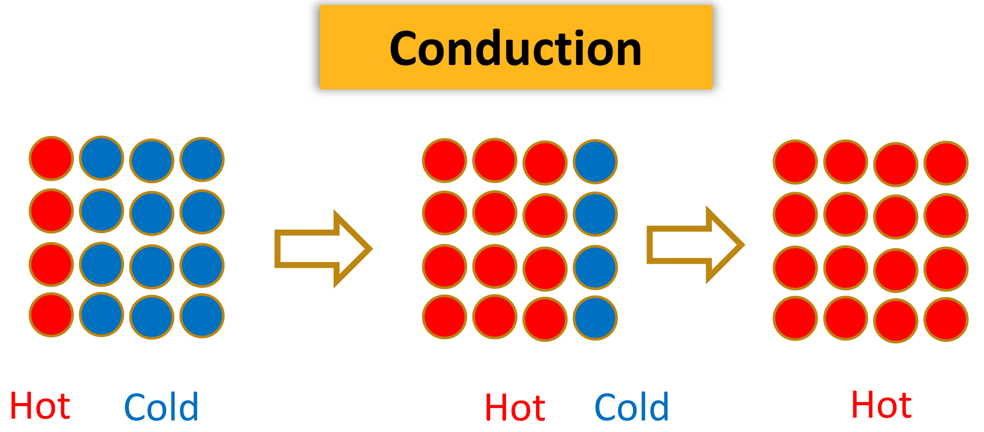
Thermal Conductivity in Heat Transfer – Lesson 1

Ansys

Introduction to Fourier's Law

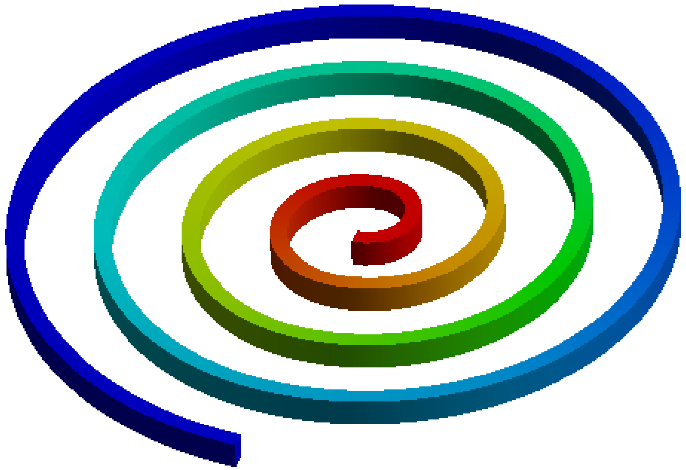
We know that heat conduction occurs when molecule vibration increases. In the collision between neighboring molecules, heat energy is transferred from the hotter to cooler area. In science and engineering, which law does the process adhere to and how is the heat energy quantified?





We need a way to mathematically describe conductivity and quantify the conduction process.

Introduction to Fourier's Law



- For this electric heater, the amount of heat transferred per unit time into the coil is Q, also known is **heat flow**.
- Dividing Q by the cross-section area A, we obtain the heat flow per unit area, which is **heat flux** q.
- The relationship between heat flux and temperature is governed by **Fourier's law**.

Fourier's Law

$$q = -k\,\frac{dT}{dx}$$

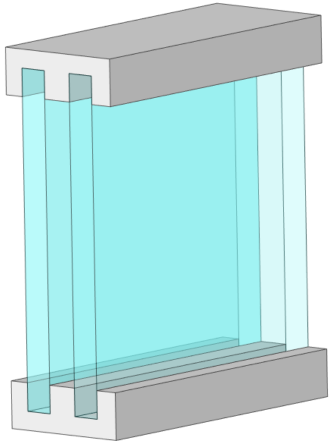
Temperature gradient

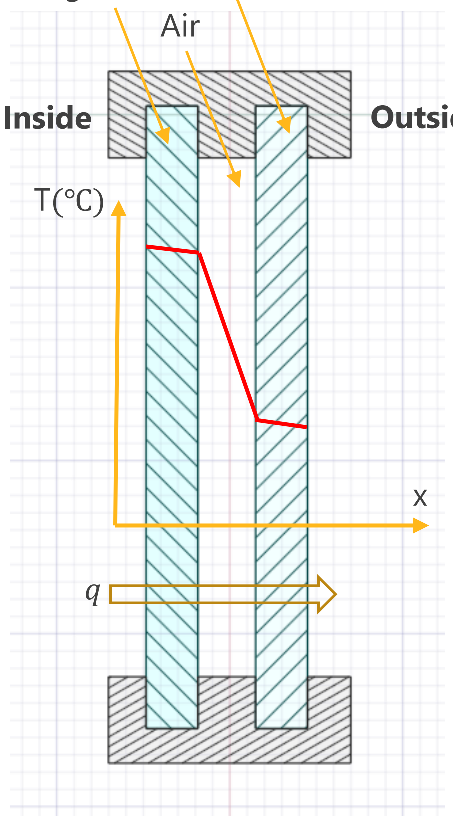
Heat flux

Thermal conductivity

Thermal conductivity k is a material parameter obtained from experiment.

Introduction to Fourier's Law





Fourier's Law

$$q = -k \frac{dT}{dx}$$

Heat flux

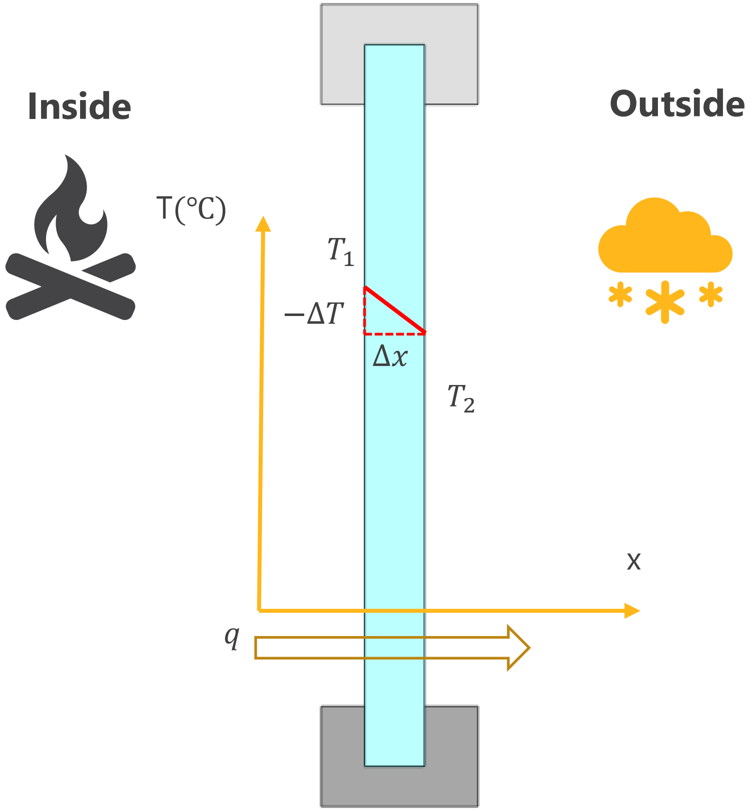
Temperature gradient

Thermal conductivity

- Thermal conductivity represents the ability to transfer heat through conduction. For a give amount of heat flux, the larger k is, the smaller the temperature gradient is.

- The image on the left illustrates temperature distribution through a composite window. The conductivity of glass is much larger than the air, thus the temperature in the glass is almost uniform.

Introduction to Fourier's Law



Fourier's Law

Temperature gradient

$$q = -k\frac{dT}{dx}$$

- Why a negative sign before k?

  Using the glass window as an example and setting the heat flux direction (inside to outside) as the reference, the temperature gradient is $-\Delta T/\Delta x$. Since k and q are both positive, we need a negative sign in Fourier's law.

- The heat flow always goes to the lower temperature side according to the second law of thermodynamics.

Introduction to Fourier's Law

Fourier's Law:

$$q = -k\frac{dT}{dx}$$

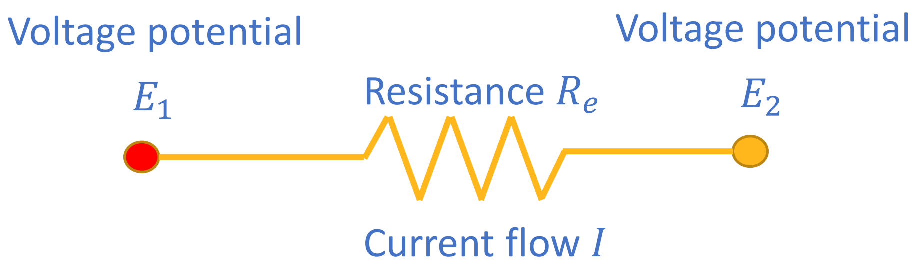
Assuming temperature is distributed linearly along the bar, we can derive the following equations:

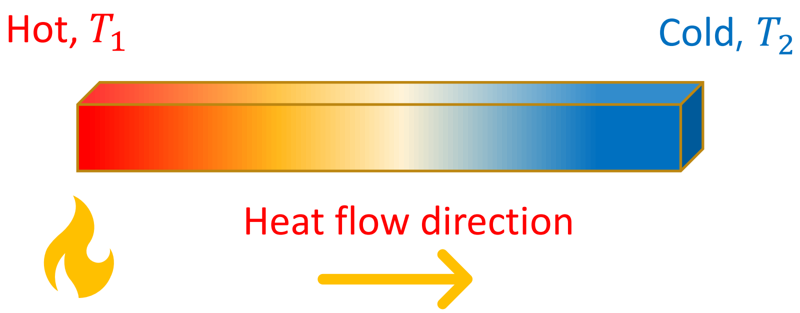
$$q = -k\frac{(T_2 - T_1)}{L} \Rightarrow Q = \frac{Ak}{L}(T_1 - T_2) \Rightarrow Q = \frac{(T_1 - T_2)}{R_k} \qquad R_k = \frac{L}{Ak}$$

Heat flow

Thermal resistance

Analogy to Electric Circuits





Ohm's Law:

$$I = \frac{(E_1 - E_2)}{R_e}$$

Fourier's Law:

$$Q = \frac{(T_1 - T_2)}{R_k}$$

Steady-State Analysis

Fourier's law governs the thermal equilibrium for steady-state thermal analysis.

1D Fourier's Law

$$q = -k\frac{dT}{dx}$$

Although thermal conductivity tells how rapidly a material transfers heat energy, there is no time involved in Fourier's law.

When time is not involved in a heat transfer problem, we call it a **steady-state thermal analysis.**

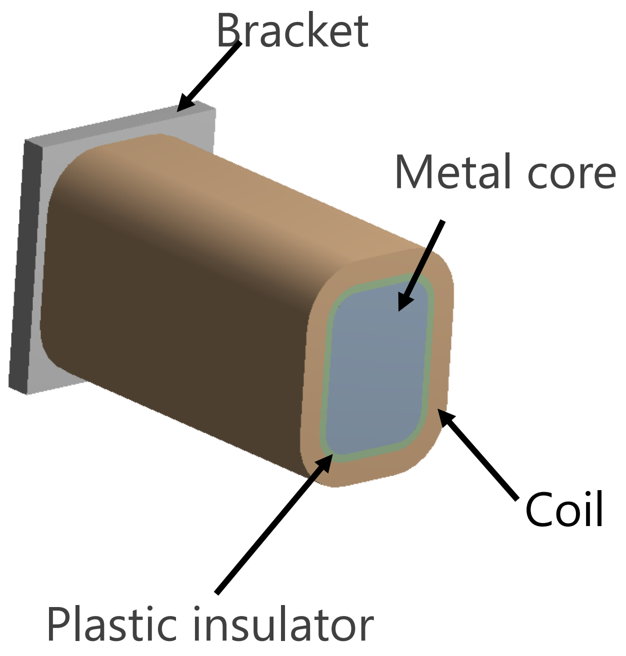
Using the stainless steel pot as an example, with a heat source temperature of 225 °F, can you predict the temperature on the pot handle after 1 hour, without calculating it?

- The answer is yes, the pot handle will be nearly 225°F, since both the handle and the pot are made of highly conductive stainless steel.

Steady-State Analysis

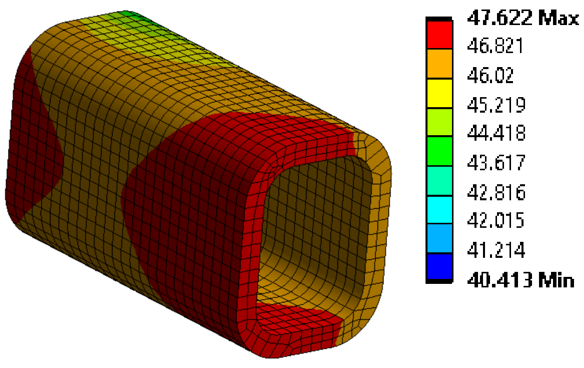
Here we use simulation to further illustrate a steady-state problem. An electrical coil is heated inside the iron coil and covered by a plastic insulator. There is a constant heat loss at the free surfaces.

- Let's conduct two types of simulations, one considers time (**transient analysis)** and the other does not (**steady state analysis).**
- Let's run many transient analyses and heat the coil for a different time duration in each analysis.
- Finally let's compare the transient analysis over a long duration with the steady-state analysis.

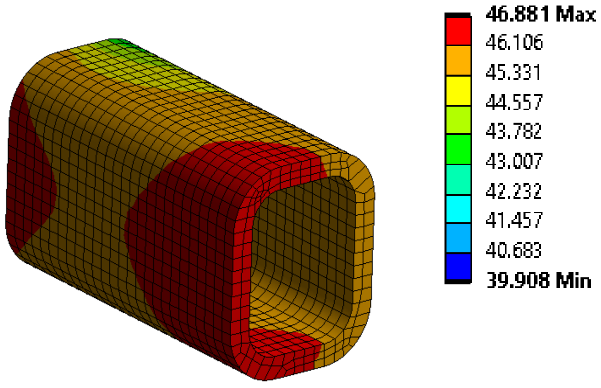


Steady-State Analysis

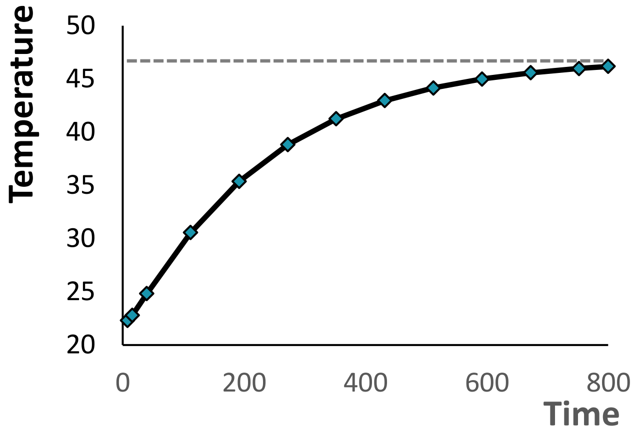
Steady-State



Transient t=600 s



Max T vs. Time duration



- The above images plot the temperature distribution in the plastic cover for two types of analyses.
- We can see that, as the duration increases, the maximum temperature in the transient analysis approaches a constant value.
- When the time duration is sufficiently long and there is little temperature change in the model, we can say the problem has reached a steady state.

Analogy to Structural Analysis in Finite Element Method

Steady-State Thermal Analysis

$$[K]\{T\} = \{Q\}$$

- $[K]$ → Conductivity stiffness
- $\{T\}$ → Temperature
- $\{Q\}$ → Heat "load": Heat flux, heat flow, etc.

Static Structural Analysis

$$[K]\{u\} = \{f\}$$

- $[K]$ → Structural stiffness
- $\{u\}$ → Displacement
- $\{f\}$ → Structural load: Force, traction, etc.

- The thermal analysis is **steady-state** and does not consider the effect of time.
- Like structural analysis, a thermal boundary condition is needed in a thermal analysis.
  - In the analysis of this boiling pot, if we only provide a heat source without a boundary condition (for example, the temperature in the coil), the resulting temperature will rise infinitely high because of "rigid body motion."

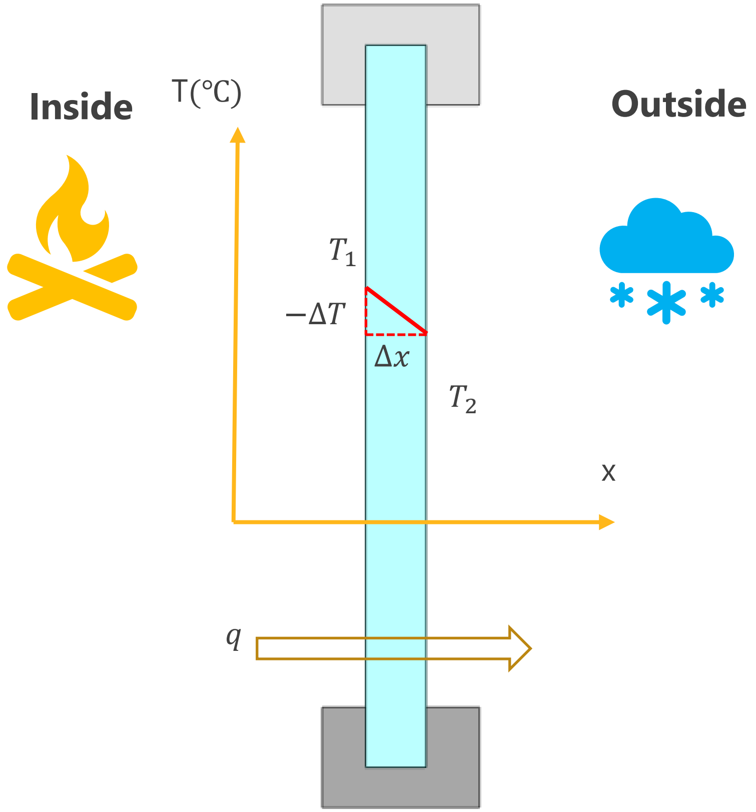
Steady-State Analysis

In Fourier's law, heat flux has a unit of $J \cdot s^{-1} \cdot m^{-2}$

Fourier's Law $$q = -k\frac{dT}{dx} \qquad [K]\{T\} = \{Q\}$$

- Although the heat flux represents the heat transfer per unit time per unit area, it does not mean the system is in a transient state.
- A steady state does not necessarily mean temperature becomes uniform.

Let's again look at the window example. There is an indoor heater to keep the room at a constant temperature, and a snowy, cold day outside, so there is a continuous heat flow from inside to outside through the window, yet the problem is still steady-state because the temperature does not change with time.



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