Fluid Flow Rotation

Fluid Kinematics – Lesson 3

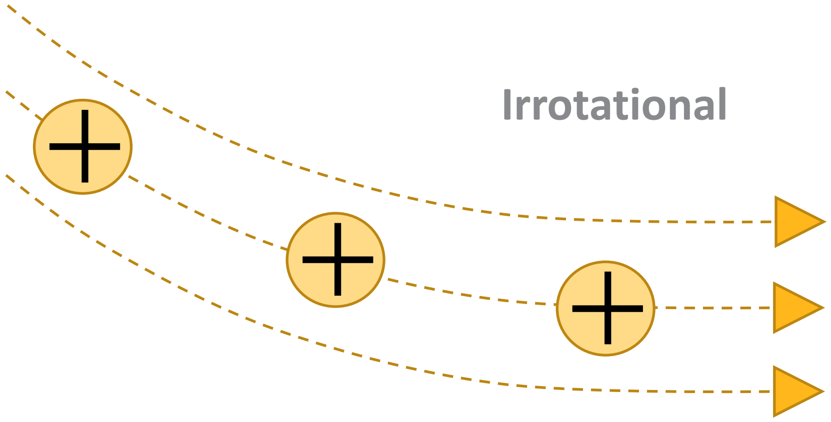
Ansys

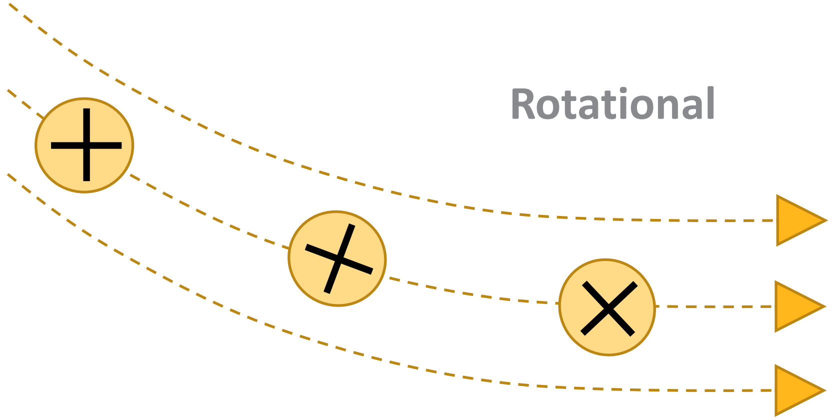
Irrotational Flow

- The definition of vorticity ($\vec{\omega}$) is:

$$\vec{\omega} = \nabla \times \vec{V}$$

- A fluid velocity field is said to be irrotational if the vorticity is zero everywhere.
- An example of irrotational versus rotational flow is shown on the right
  - As the irrotational flow moves downstream, a marker in the flow does not change orientation (no rotation)
  - If the flow is rotational, the marker would rotate as it moves downstream
- When is a flow irrotational?
  - If the flow entering a region is irrotational, we will see that (in the absence of viscous effects) the flow will remain irrotational. This is approximated for smooth flows outside boundary layers.





Irrotational Flow

- From vector calculus, the condition for the vorticity to be zero in an irrotational flow is given as:

$$\vec{\omega} = \nabla \times \vec{V} = 0$$

- This requires that the velocity field is a potential field, i. e., the velocity vector is expressed in terms of a potential function as:

$$\vec{V} = \nabla \phi$$

- This simplifies formulations of governing equations and makes certain classes of irrotational flows open to be solved analytically. We will discuss this in more detail in subsequent lessons.
- One property of an irrotational flow is that its circulation along any closed contour is zero:

$$\oint \vec{V} \cdot d\vec{r} = 0$$

- And, as a result, a streamline in an irrotational flow cannot be closed on itself.

Rotational Flow

- A flow is rotational if fluid elements undergo rotation about their axis while flowing along streamlines.
- The flow is rotational when its vorticity vector is non-zero in some of its regions.
- If the flow is rotational everywhere, then the divergence of the vorticity vector is zero:

$$\nabla \cdot \vec{\omega} = 0$$

- By applying the Gauss Divergence Theorem to the above equation, we can conclude that the vorticity flux across any closed surface is zero:

$$\oiint_A (\vec{\omega} \cdot \hat{n})\, dA = 0$$

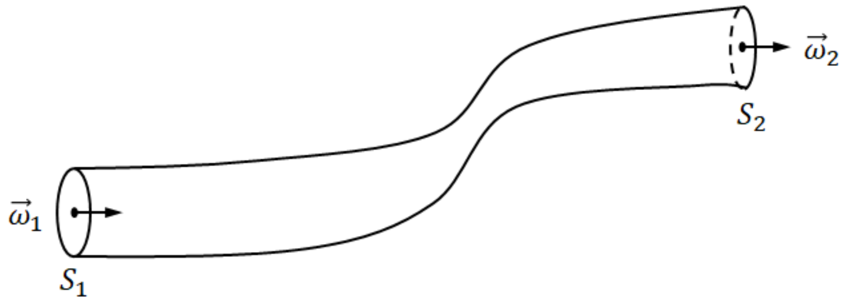
Vortex Lines and Vortex Tubes

- Similar to the streamline definition, a vortex line can be defined as a curve with the property that the vorticity vector is tangent at each point along the vortex line.
- Mathematically, the requirement of tangency results in the following mathematical definition of the vortex lines:

$$\frac{dx}{\omega_x} = \frac{dy}{\omega_y} = \frac{dz}{\omega_z}$$

- Vortex lines passing through each point of a closed contour form a so-called "vortex tube."
- The conditions of zero vorticity flux through a closed surface yields:

$$\oint_{S_1} \vec{\omega}_1 \cdot \hat{n}\, ds = \oint_{S_2} \vec{\omega}_2 \cdot \hat{n}\, ds$$



Vortex Lines and Vortex Tubes

- The integral quantity

$$\oint_S \vec{\omega} \cdot \hat{n}\, ds$$

is called the "strength" of the vortex tube, or simply vortex strength*, and it is the same everywhere along the tube.

- A tornado is a fascinating example of a devastating vortex tube observed in nature.

* Vortex strength is sometimes confusingly called vortex flux, which is not accurate, since it does not refer to the advection of vorticity.

Helmholt’z Vortex Theorems

- Helmholtz's theorems, named after Hermann von Helmholtz, state the following about vorticity fields:
    - Vortex lines move with the fluid, i. e., fluid elements lying on a vortex line at some instant continue to lie on that vortex line.
    - The strength of the vortex tube is constant along its length and in time.
    - Fluid elements initially free of vorticity, remain free of vorticity.
    - Vortex tubes can only extend to infinity, form a closed loop or end at a solid boundary.

- While rigorous mathematical proofs of these theorems is beyond the scope of this course, they can be formally derived in the framework of fluid kinematics by ignoring fluid compressibility and viscous (and other) forces.

- Helmholtz theorems played an important role in developing the classical lifting line theory for lining bodies (airfoils, wings), attributed to Prandtl (1921).

- While statements in Helmholtz's theorems may at first glance appear theoretical, they find confirmation in nature. A tornado always attempts to touch the ground as the most optimal way to satisfy the 4th statement above.

Summary

- Irrotational and rotational flow approximations provide simplifications to fluid flow descriptions, which make possible analytical solution approaches to certain classes of problems.

- Rotation flows are characterized by the vorticity vector, which can be represented and visualized by vortex lines and vortex tubes.

- Helmholtz's vortex theorems represent formal statements of vortex kinematics. They provided an important basis for the theory of lifting bodies developed in the first half of the 20th century.

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