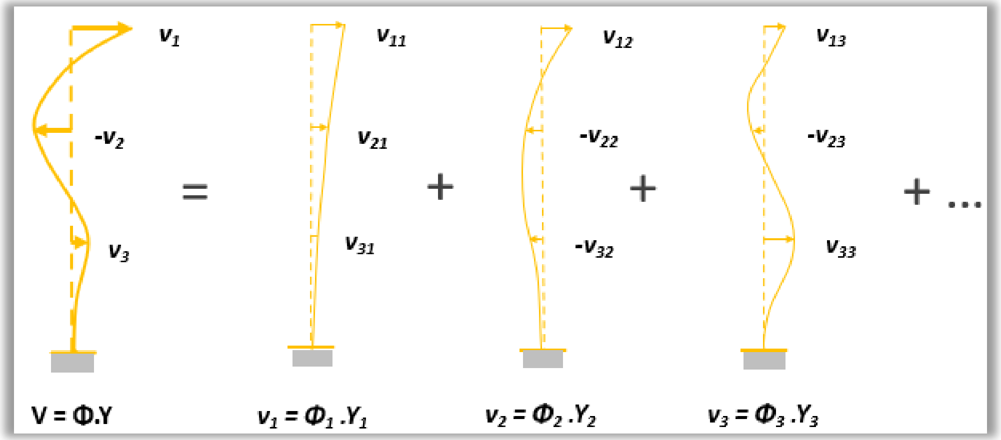
Mode Superposition Method

Concept, Discussion

Ansys

Concept of MSUP

- Full Analysis
  - Solve the governing equation directly using the static solver
- Mode Superposition Response Analysis
  - Express the displacement as a linear combination of mode shapes
  - Example: The shape of the curve on the left (which represents a simple beam) can be represented by adding together multiple curves(mode shapes) on the right in varying quantity and magnitude.



Here, the sum of ***mode shapes*** $\boldsymbol{f_1, f_2}$ ***and*** $\boldsymbol{f_3}$ approximates the final response.

Since mode shapes are relative, the coefficients $\boldsymbol{Y_1}$ $\boldsymbol{Y_2}$ are $\boldsymbol{Y_3}$ required.

- To achieve more complex shapes and accuracy, more modes are needed.

Concept of MSUP

- Understanding the General Concept
- This equation relates the physical DOFs (which are just the nodal displacements) to the modal coordinates.

$$\{v\} = \sum_{i=1}^{n} \emptyset_i * \{Y_i\}$$

where

$\{v\} = Displacement\ Vector\ (Physical\ DOFs)$

$\{\emptyset\} = Mode\ Shape\ Vector\ (Eigenvector)$

$\{Y\} = Modal\ Amplitude\ \ (Modal\ Coordinates)$

$n = Number\ of\ modes\ used$

$i = Mode\ number$

- First, we must solve a modal analysis to compute the eigenvectors $\{\phi\}$.
- It is after we have the eigenvectors that the solver can combine the mode shapes to represent the model deformations
- The Modal Coordinates $\{Y\}$ can be thought of as scale factors or coefficients for each mode and these are the DOFs we solve for. They are the unknows.
- Once we have the Modal Coordinates $\{Y\}$, and the eignenvectors $\{\phi\}$, the displacements $\{v\}$ can be computed using the above equation.

Concept of MSUP

Let's compare the MSUP Harmonic equations with the Full Harmonic equations

$$\left(-\boldsymbol{\Omega}^2[\boldsymbol{M}] + \boldsymbol{i\Omega}[\boldsymbol{C}] + [\boldsymbol{K}]\right)(\{\boldsymbol{u_1}\} + \boldsymbol{i}\{\boldsymbol{u_2}\}) = (\{\boldsymbol{F1}\} + \boldsymbol{i}\{\boldsymbol{F_2}\})$$

Harmonic equation of motion

- Typically, there are two options to solve this equation;
- **<u>The Full method</u>**
  - The unknows are the real and imaginary nodal displacement vectors $\{u_1\}$ and $\{u_2\}$, and we have multiple DOFs (3 for solids, 6 for shells) per node
  - These unknows are ***coupled***, meaning there are multiple unknows per equation
  - To solve the simultaneous set of equations we use a matrix solver which is computationally expensive

This is covered in more detail in the Forced Frequency Response/Harmonic Analysis Section

Concept of MSUP

- **The MSUP Method**
  - The equation of motion is converted into modal form (details omitted)

$$\left(-\boldsymbol{\Omega}^2 + \boldsymbol{i}2\boldsymbol{\omega}_j\boldsymbol{\Omega}\boldsymbol{\zeta}_j + \boldsymbol{\omega}_j^2\right)\boldsymbol{y}_{jc} = \boldsymbol{f}_{jc}$$

  - The unknows are the modal coordinates y_{jc} (which we introduced on the prior slide) and the number of DOFs is just equal to the number of modes.
  - We solve this ***uncoupled*** equation for each mode j and later perform a linear combination of orthogonal vectors (mode shapes).
  - The key takeaway is that MSUP method has ***fewer DOFs***, ***uncoupled equations***, resulting in a ***computationally less expensive solution***

This is covered in more detail in the Forced Frequency Response/Harmonic Analysis Section

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