

Stress Tensor

Stress tensor and vector, Normal and shear stress

Solid Mechanics I – Understanding the Physics

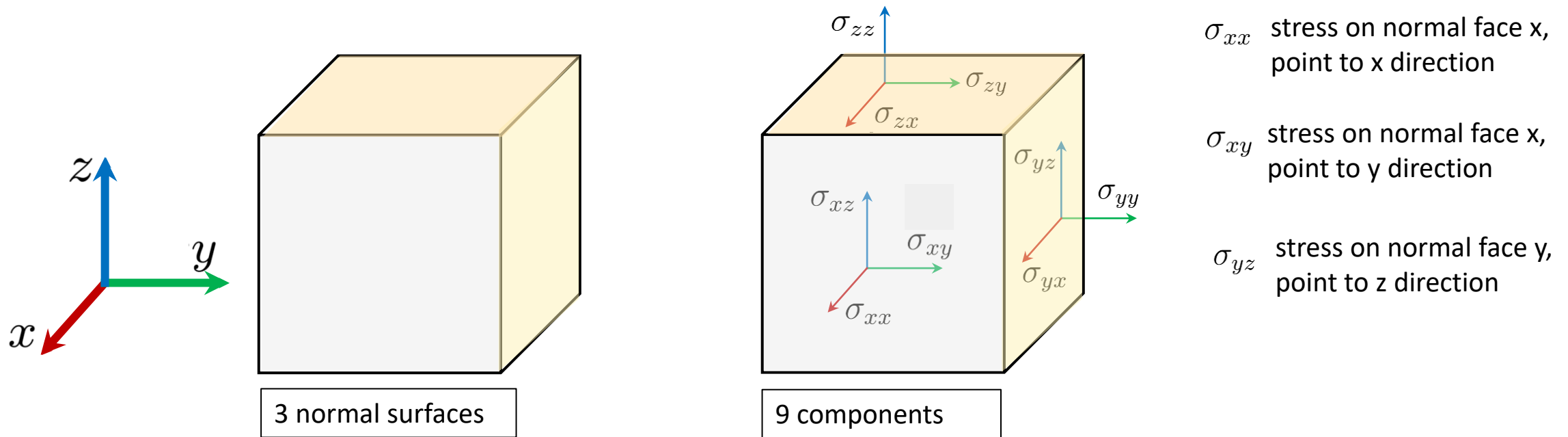
Stress and local equilibrium



Stress Tensor

With the concept of the infinitesimal cube in mind, let's have a look of the nine components of a stress.

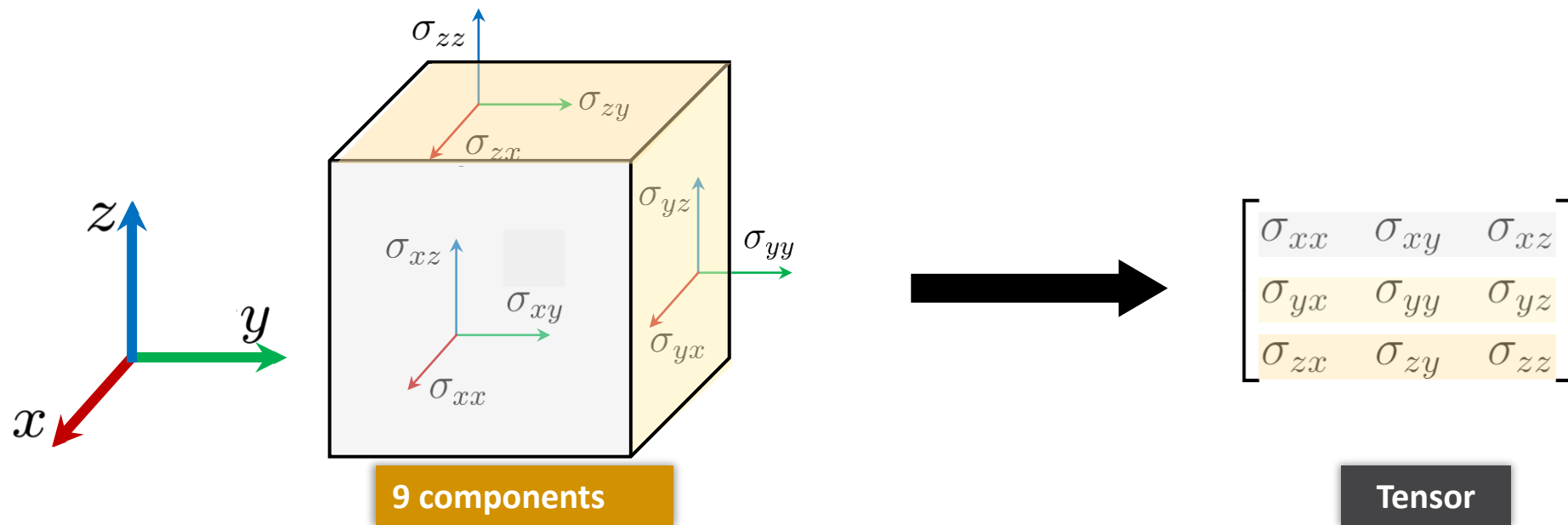
- The three normal faces to the coordinate system is marked by different colors.
- Note that the two indices of these components, the first index represents the normal face, the second index represent the direction of the component.



Stress Tensor

Thee nine stress components on an infinitesimal cube naturally form a mathematic object, Tensor, which is written as a 3 by 3 matrix.

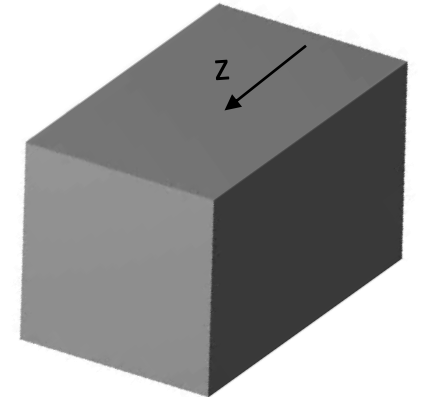
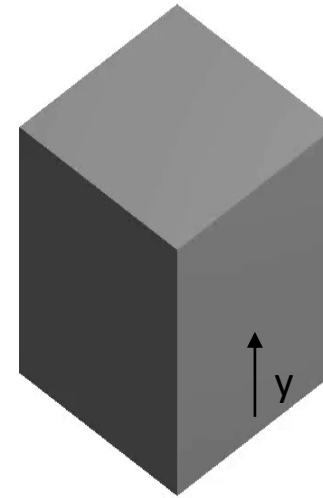
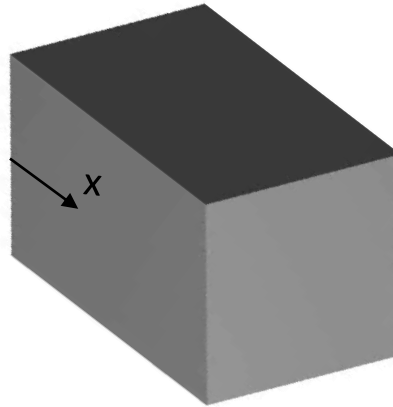
- The first row of the tensors are components on normal face x
- The first row of the tensors are components on normal face y
- The first row of the tensors are components on normal face z



Stress Tensor: Normal Stress

Each component of the stress tensor corresponds to a load condition. The diagonal components are normal stress, representing uniaxial load condition in normal direction.

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$



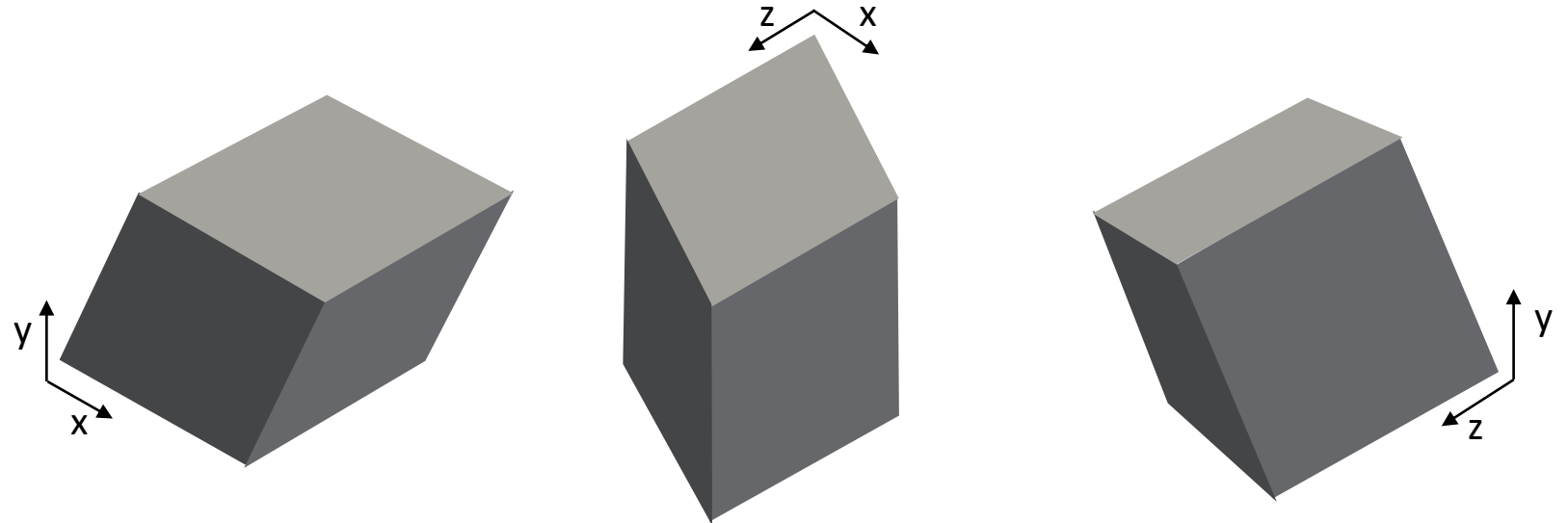
Normal Stress

Stress Tensor: Normal Stress

The off-diagonal components are all shear stress, representing shear load condition.

- Stress tensor is symmetric. For off-diagonal components, $\sigma_{ij} = \sigma_{ji}$
- Symmetry of stress tensor is from moment balance of internal force, which will be discussed later

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

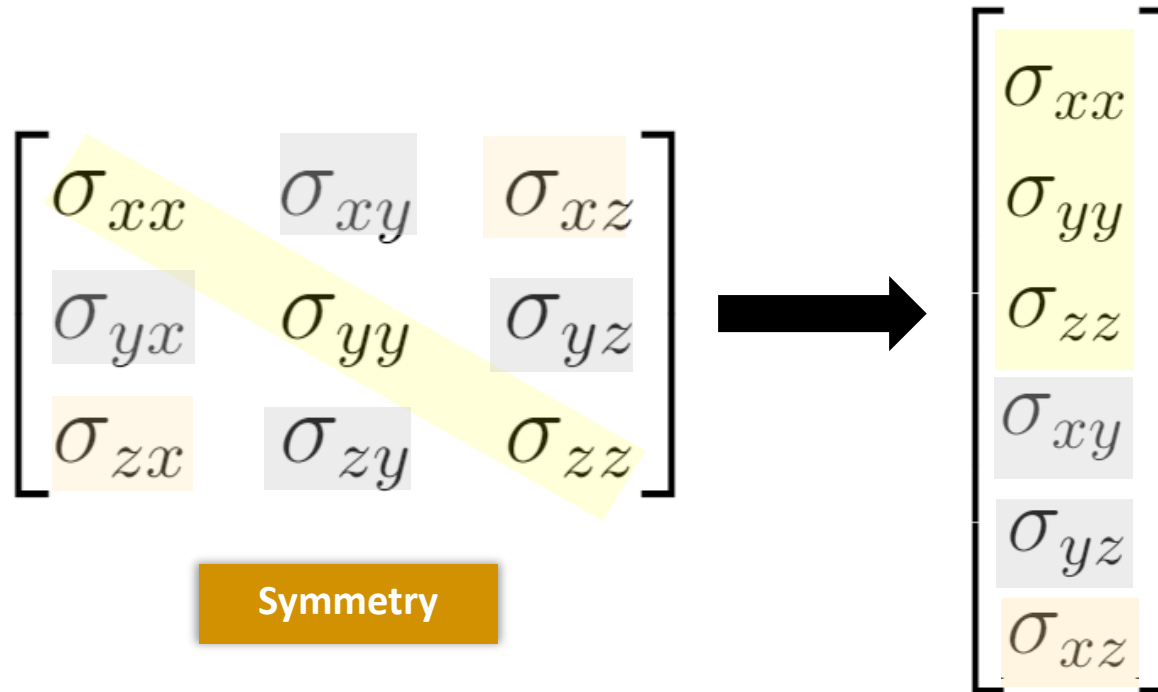


Shear Stress

Stress Tensor: Voigt Notation

For easier notation, the stress tensor are often written in a vector format in derivation.

- The first three components of stress vector are normal stress
- Because of symmetry, we can choose one of the three off-diagonal stress pairs to the vector



 **Ansys**

