# Numerical Modeling of Contact

How is contact modeled?

What are advantages and disadvantages different formulation?

Solid Mechanics II – Capturing Complex Response

Contact

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# Overview

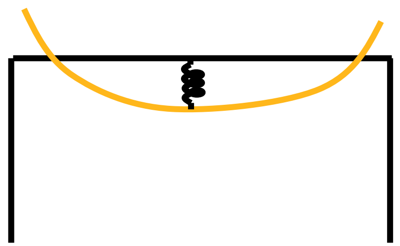
- Requirements of Contact Formulation
- Penalty Formulation
- Lagrange Formulation
- Penalty vs Lagrange Formulation

# Requirements of Contact Formulation

- Numerically, contact is enforced by following three rules:
  - No penetration between the two bodies
  - Balance between the applied and the contact forces in case of static problems.
  - Conservation of momentum in case of dynamic problems (e.g., impact)
- These are the requirements for any contact formulation that is introduced into the model in order to enforce contact between bodies.
- In general, there are two popular contact formulations that can meet these requirements:
  - Penalty formulation
  - Lagrange formulation.

# Penalty Formulation

- Penalty formulation treats contact as stiff spring that resists bodies from penetrating each other.
- This resistance is modeled in the form of **contact stiffness** which is derived as a function of several factors such as
  - Material properties of both the bodies
  - Geometry of the both the surfaces
  - Kinematics of both the bodies
  - Etc.,

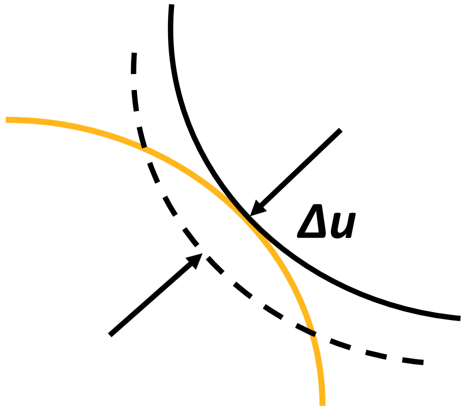


- Contact stiffness is used in both the normal and tangential directions.

# Cont'd…

- Penalty formulation allows for a small amount of penetration, delta u, between the two bodies.
- Contact stiffness, K, is calculated and restoring force, F, is calculated to prevent this penetration.

$$\{F_C\} = [K]\{\Delta u\}$$




- This restoring force is nothing but the contact force.
- Contact stiffness should be large enough to provide restoring force enough to reduce the penetration to close to zero.

# Cont'd…

- Penalty formulation is used for calculating both contact pressure and also frictional stresses.
- If $u_n$ is the contact gap, and $\Delta u_1$ and $\Delta u_2$ are the sliding distances in lateral directions, then the contact traction vector is defined by

$$P = \begin{cases} 0, & \text{if } u_n \geq 0 \\ K_n U_n, & \text{if } u_n < 0 \end{cases}$$

$$\tau_i = \begin{cases} K_t, & \text{if } ||\tau|| = \sqrt{\tau_1^2 + \tau_2^2} < 0 \\ \mu_i P \frac{\Delta u_i}{||\Delta u_i||}, & \text{if } ||\tau|| = \sqrt{\tau_1^2 + \tau_2^2} \geq 0 \end{cases}$$

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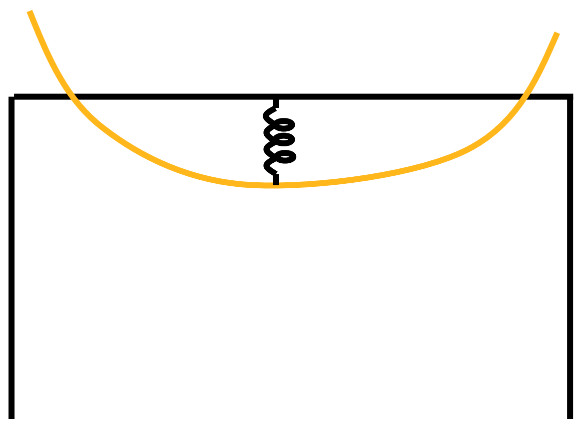
# Lagrange Formulation

- Lagrange formulation treats contact as a constraint.
- Unlike penalty formulation it does not require contact stiffness.
- Instead enforces contact penetration to be zero.
- This is the constraint that it uses in calculations.



# Cont'd…

- Contact traction vector is included as part of the total reaction force vector.
- Contact gaps, and sliding distance are also solved for as additional degrees of freedom.

$$\{F_r\} = [\ P,\ \tau_1,\ \tau_2,\ u_n,\ \Delta u_1,\ \Delta u_2\ ]^T$$

- The equations of motion are then solved for by imposing the constraint on contact gap, $u_n$ and sliding distances $\Delta u_1$ and $\Delta u_2$.

# Contact Penetration

- Due to a strong constraint imposed on the contact gap, the solution results in zero penetration.
- So the contact calculations are most accurate using this method.
- However, the constraint may result in over constraint or very high forces at the contact which may result in force imbalance.

# Penalty vs Lagrange Formulation

- The contact formulations discussed in this lesson have both advantages and disadvantages compared to each other.

| Features | Penalty Formulation | | Lagrange Formulation | |
|---|---|---|---|---|
| | Advantages | Disadvantages | Advantages | Disadvantages |
| Penetration | | Some amount of penetration is allowed. | Zero penetration is allowed. | |
| Contact stiffness | | System response is dependent on contact stiffness. | System response is not dependent on contact stiffness. | |
| Over constraint | Over constraints are not possible in this formulation. | | | Possible over constraints. |
| Computational time | Relatively inexpensive. | | | Relatively expensive. |

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