Discussion of Implicit and Explicit Methods and Time-step Size

Time Integration Method – Lesson 3

Ansys

Discussion of Implicit and Explicit Methods and Time-step Size

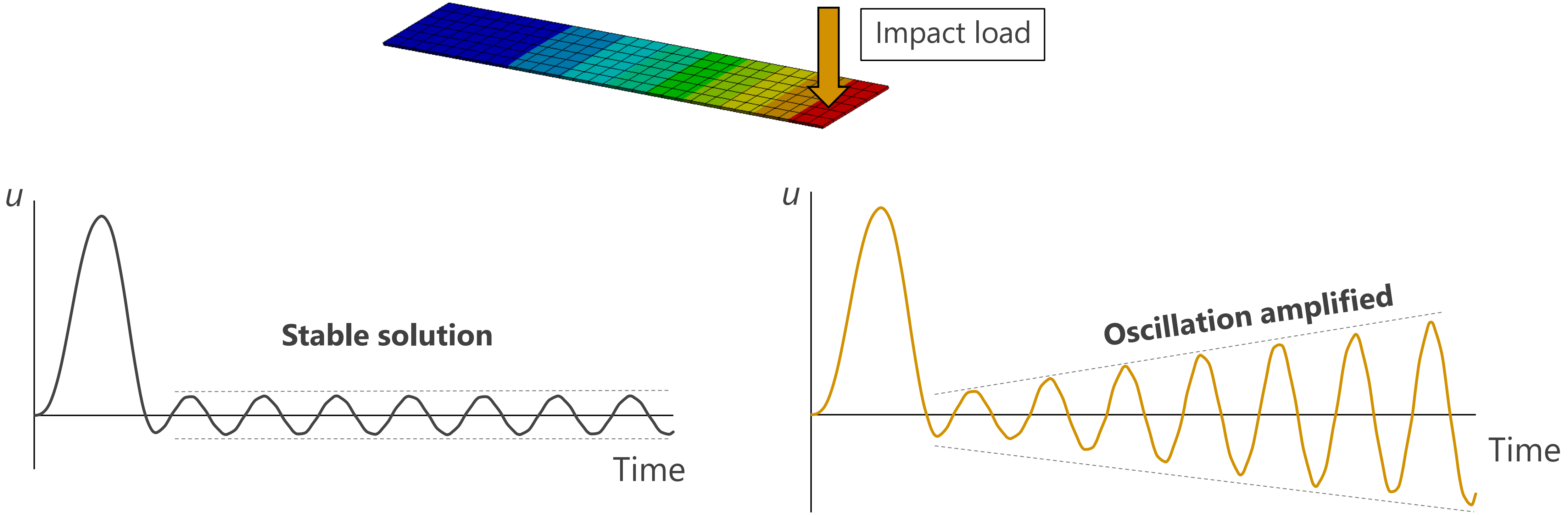
It is clear that implicit methods require extra computation effort to solve an equation for each time-step and could be very hard to solve for practical problems. The explicit method seems to have no such trouble.

- The explicit method is known to have stability issues if the time-step size is not wisely chosen, while the implicit method is proven to be stable for a linear system.
- Because of the limitation of time-step size, explicit methods usually are not used in dynamic problems of long duration.

	Explicit	**Implicit**
Stability	Depends on time-step size	Unconditionally stable
Time-step size	Very small	Large
For nonlinear problem	Does not need iterations	Needs iterations
Solving speed for one step	Fast	Slow

Time-step Size of Explicit Method: Stability

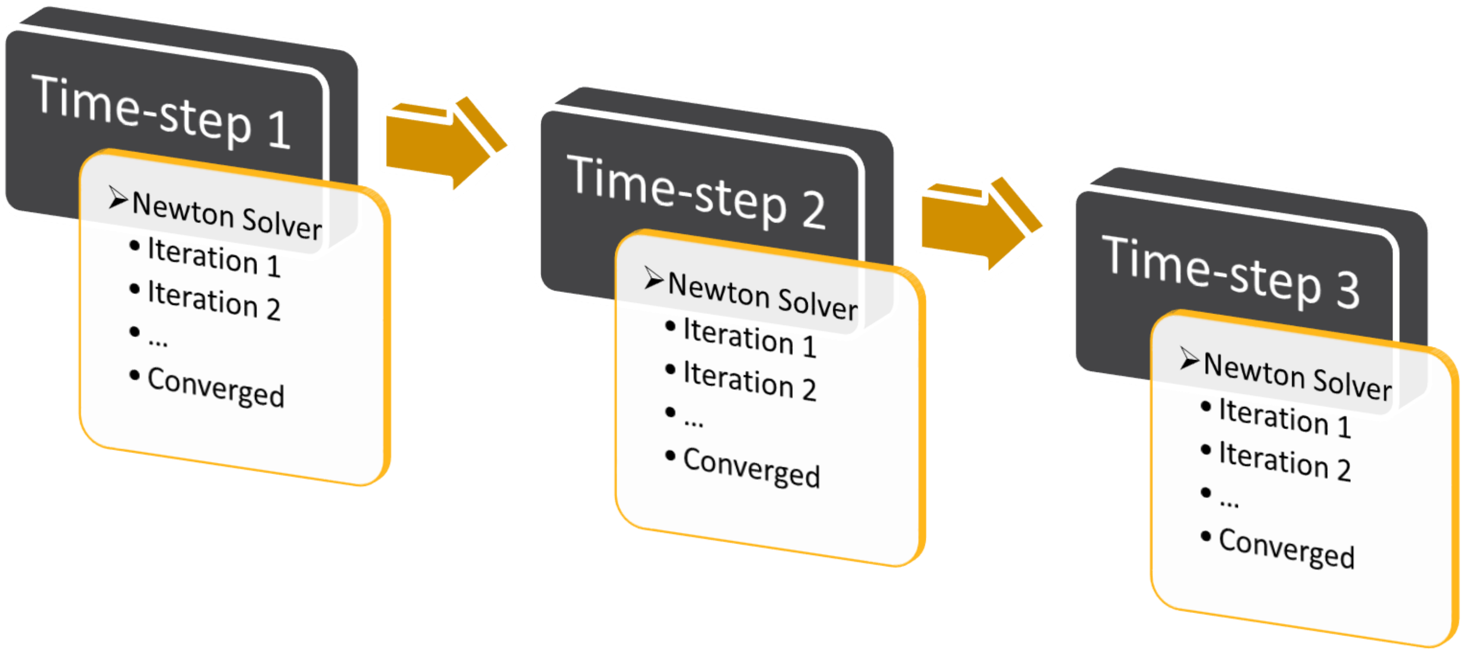
What does an unstable solution look like? The amplitude of the oscillation grows in time without bound, leading to an explosive numerical instability.



Time Integration: Implicit and Explicit

For the implicit method, the solution is implicit in the final equation, which needs to be solved. For the explicit method, the solution is explicit, i.e., is directly obtained.

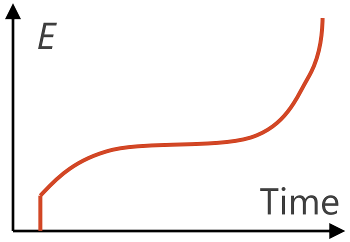
- If it's a nonlinear problem, for the implicit method, Newton iteration is needed to find the solution in each time-step.



Time-step Size of the Explicit Method

Below is a guideline for choice of implicit and explicit time integration methods for dynamic analysis.

Days	*Hours*	*Minutes*	*Seconds*	*Milliseconds*
Creep	**Quasi-Static**	**Static/Dynamic**	**Drop Test**	**Impact**



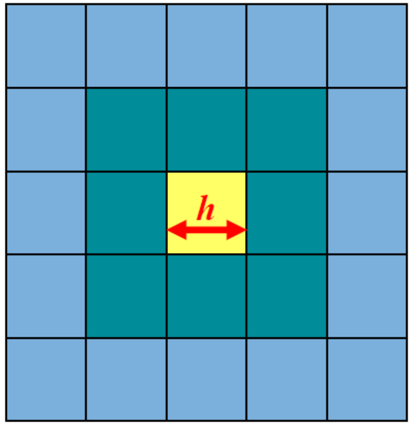
Implicit Methods

Explicit Methods

Time-step Size of the Explicit Method: Stability

To enforce stable results, the time-step size is limited so that a stress wave cannot travel farther than the smallest element characteristic length in a single time-step. This is called the Courant-Friedrichs-Lewy (CFL) condition.

$$\Delta t \leq f * \left[\frac{h}{c}\right]_{\min}$$



- *h* is the characteristic length of a finite element. In explicit analysis, having uniform element size is very important, because the time-step size is controlled by the smallest element.
- *c* is the sound speed in the material.
- *f* is a safety factor, usually equal to or smaller than 1.

Time-step Size of the Explicit Method: Stability

When the material’s sound speed wave is not available, we can use Young’s modulus and density to make an approximation.

- There are transverse waves and longitudinal waves in material.
- Transverse waves travel more slowly than longitudinal waves. Therefore, the time-step size is controlled by longitudinal wave speed.
- A longitudinal elastic wave can be calculated by Young’s modulus and density:

$$c = \sqrt{\frac{E}{\rho}}$$

- The larger the Young’s modulus or the smaller the density is, the larger the sound speed is and the smaller time-step size should be used.

Time-step Size of the Explicit Method: Mass Scaling

Given a material and analysis information, find out how many time-steps are required to solve this problem.

- Young's modulus $E = 2 \times 10^{11} Pa$ *and* density $\rho = 7850\ Kg/m^3$
- Characteristic element size $h = 2 \times 10^{-3}\ m$
- Required analysis time is $2\ s$

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{7850}} = 5048 m/s$$

$$\Delta t = f * \frac{h}{c} = 1 * \frac{2 \times 10^{-3}}{5048} = 3.96 \times 10^{-7} s$$

$$n = \frac{T}{\Delta t} = \frac{2}{3.96 \times 10^{-7}} = 5 \times 10^{6}$$

At least 5×10^6 time-steps are required for this explicit analysis. If the analysis time is 20 *s*, 5×10^7 time-steps are needed. In real analysis, is there a way to increase time-step size so as to expedite the analysis?

Time-step Size of the Explicit Method: Mass Scaling (cont.)

Mass scaling is a simulation method to reduce the run time for large-scale explicit analysis.

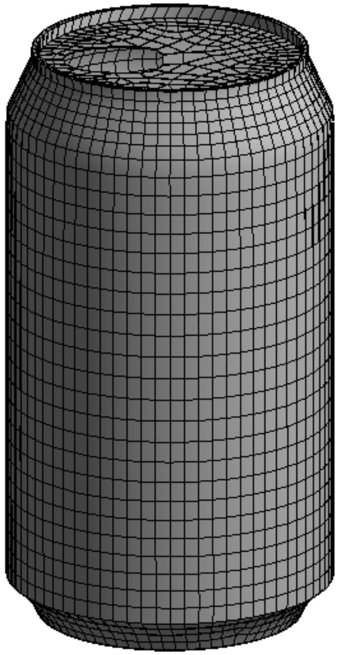
$$c\downarrow = \sqrt{\frac{E}{\rho\uparrow}} \qquad \Delta t\uparrow \leq f * \left[\frac{h}{c\downarrow}\right]_{\min}$$

- Based on the formulation to calculate material sound velocity and time-step requirement, the larger the material's mass is, the larger the required time-step can be.
- By mass scaling, the mass of the *smallest elements* (not all elements) is scaled to be larger, so that the time-step size used for the analysis can be increased.
- Since only a small portion of the elements is subject to mass scaling, the global accuracy of the analysis can be restricted to an acceptable range.

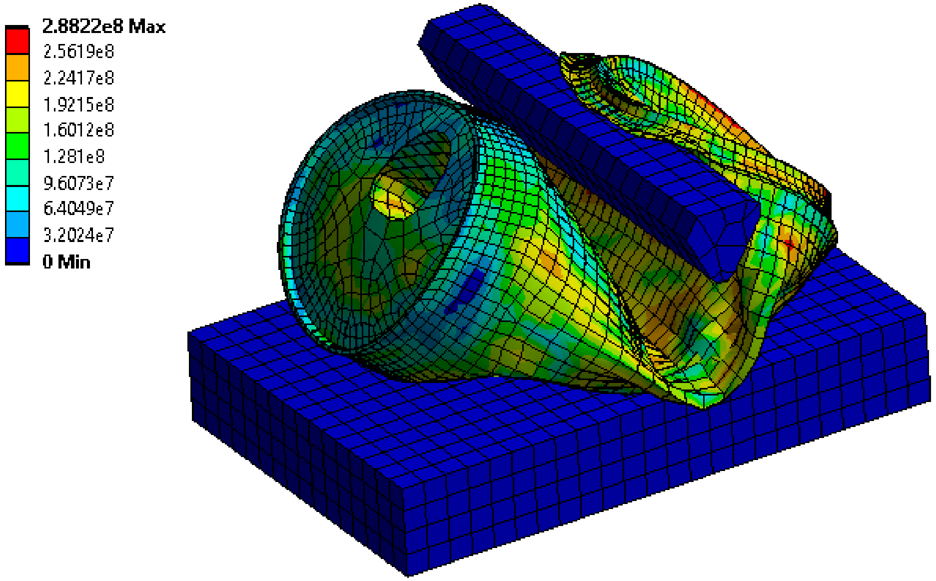
Time-step Size of the Explicit Method: Mass Scaling (cont.)

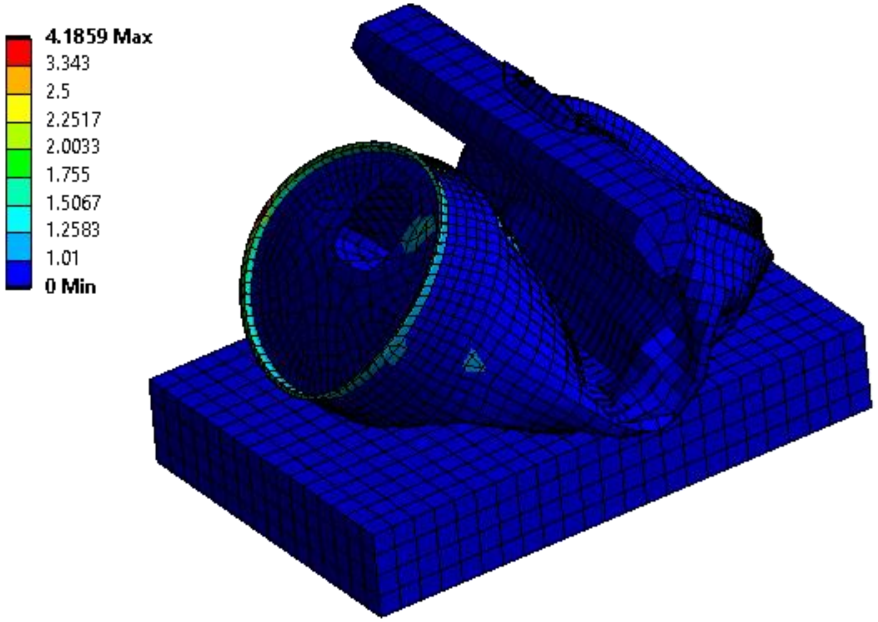
Let's apply mass scaling to a can crash analysis.

1. Originally, the critical time-step for the can crash problem is $\Delta t = 1 \times 10^{-7} s$, which means that if using Δt larger than $1 \times 10^{-7} s$, the solution will be unstable.
2. If we want to solve the analysis by $\Delta t = 2 \times 10^{-7} s$, we can apply mass scaling to the structure.
3. Check how many elements are affected by mass scaling.



Time-step Size of the Explicit Method: Mass Scaling (cont.)





Considerably fewer elements are affected by mass scaling

Time-step Size of the Explicit Method

Mass scaling cannot be abused to drastically increase the time-step size of the explicit method, because global mass change will cause erroneous analysis results. The implicit method is recommended for long-duration analysis.

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