# Buckling

Local Buckling vs Buckling, Methods for Solving

Solid Mechanics III - Methods of Solving Problems

Structural Instabilities

Ansys

# What is Buckling?

- Buckling is a structural instability where a structure undergoes rapid sideward deformation.
- Sudden loss of stiffness is the reason for this instability.
- This happens mostly under compressive loads (e.g., crushing soda can) but it may happen under tensile loads too (e.g., rails failure due to sun kink).

# Local Buckling vs Buckling

- Depending on whether a structure is can support load after undergoing buckling, it can be classified as
    1. Local buckling
    2. Buckling
- In case of local buckling, the structure loses a portion of stiffness in a small area but overall it can still support the load.
- In buckling, the structure loses majority of stiffness and can no longer support the load.

# Governing Equations

- Unlike dynamic stabilities, buckling is usually not effected by the inertial effects.
- Generally, there is a sudden transfer of energy from internal energy of the system to its kinetic energy.

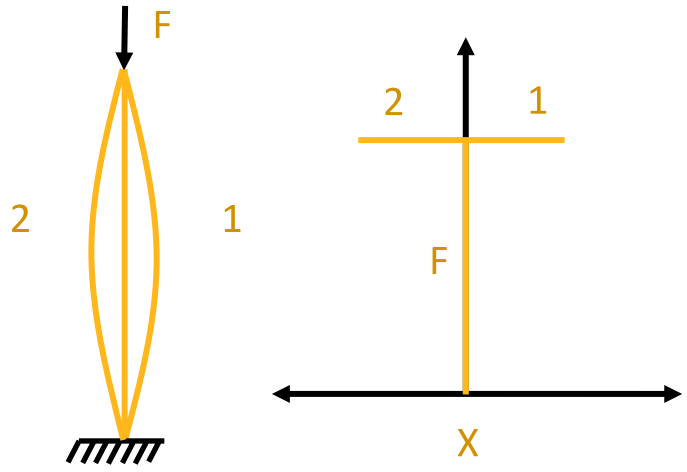
$$\underbrace{[\mathbf{M}]\{\ddot{\mathbf{u}}\}}_{\text{Kinetic energy}} + \underbrace{[\mathbf{K}]\{\mathbf{u}\}}_{\text{Potential energy}} = \{\mathbf{F}(\mathbf{t})\}$$

- Also, in most cases, the point of interest is the load at which the buckling happens rather than how the structure behaves during buckling.
- So, the governing equations for a static analysis are generally the starting point.

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\}$$

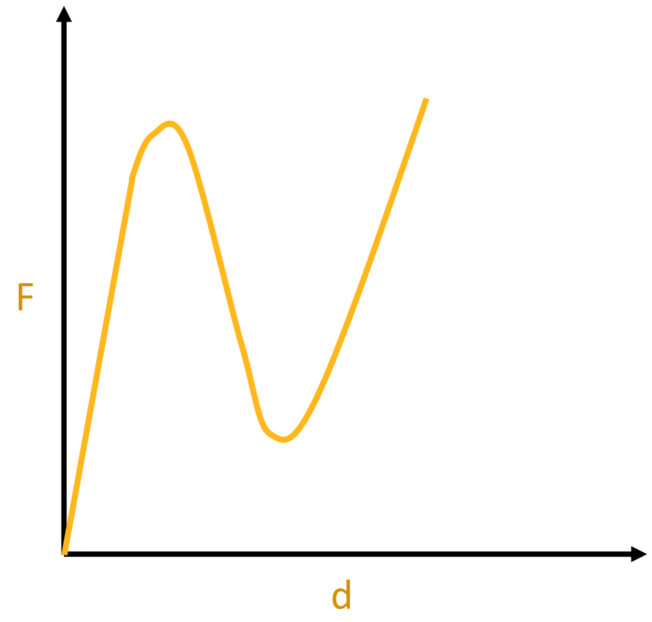
# Buckling of a Column

- Let's consider a column that is fixed in the bottom and subjected to a compressive load.
- The column can support up to a certain load before it buckles.
- The critical load at which the structure buckles is called limiting load.
- At this point, does the column buckle to left or right?
- This point is called bifurcation point on load curve.
- At this point the solution is not unique!
- Requires special methods for solving for critical load.



# Force-Displacement Curve under Buckling

- Let's discuss a typical force-displacement curve.
- Usually a monotonous increase in force with displacement - positive stiffness, K.
- At the onset of buckling there's loss in stiffness followed by a rapid drop to negative stiffness.
- This means there's hardly any resistance offered to deformation.
- After a while, the support can regain stiffness and continue to counter the applied load.




**Insufficient force to counter applied load.**

# Methods for Solving Instability

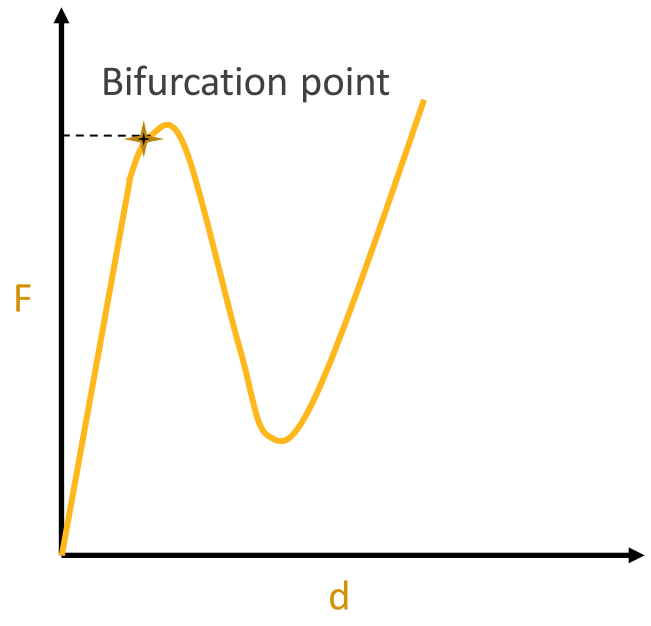
- There are three ways of solving for buckling:
  1. Linear Eigen value analysis
  2. Nonlinear buckling analysis
  3. Transient analysis
- Depending on the application and the objective of the study one of the above methods is chosen.

# Method 1: Eigen Value Analysis

- In most case, the objective of analysis is to calculate load at which structure is expected to buckle.
- In such cases it may be beneficial to solve for the point on load history at which the structure buckles.
- Governing equation at this point is

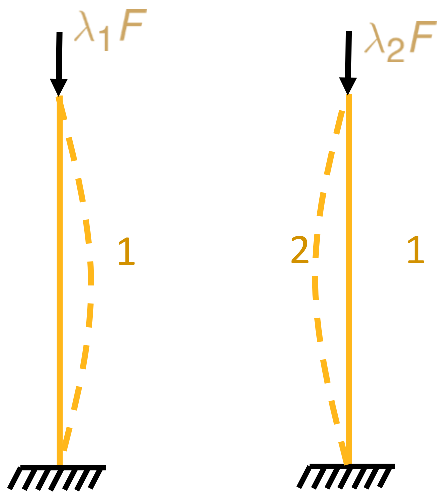
$$[\mathbf{K} + \lambda_i[\mathbf{S}]]\{\Psi_i\} = \{0\}$$

- Outcome of this analysis is the load multiplier ($\lambda_i$) and the mode shape ($\Psi_i$).
- Load multiplier is the multiplying factor to the applied load at which the structure buckles.
- Mode shape is the direction in which the structure buckles at corresponding load multiplier.



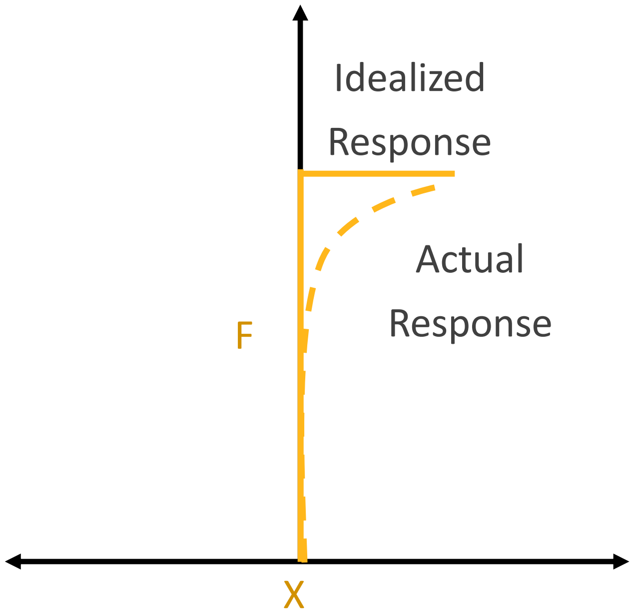
# Cont'd…

- In case of this column example, an eigen value analysis provides the buckling mode shapes, Psi1 and Psi2 and load multiplying factors lambda1 and lambda2.
- These results can be interpreted as:
  - At a load of λ1xF, structure buckles in direction 1.
  - At a load of λ2xF, structure buckles in direction 2.
- Due to symmetry the values of λ1 & λ2 are equal.
- However, eigen value analysis predicts all the possible solutions for the problem.
- This is how non-unique solution can be resolved.



# Cont'd...

- Note that eigen value analysis is a linear analysis.
- It idealizes the force-displacement curve up to bifurcation point.
- It uses
  1. Linear elastic behavior
  2. Small deformation theory is used
  3. Other nonlinearities are ignored
- Load predicted is usually more than actual load.
- But it gives a useful estimate for very little computational cost.



# Method 2: Nonlinear Buckling Static Analysis

- Let's re-visit the governing equations of static analysis.

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{F}\}$$

- In the event of buckling, the reaction force term drops nearly to zero and force balance is not achieved.
- As a result, the structure is not longer in a static state so solving the problem as static analysis becomes difficult.
- So, at the onset of buckling, the solution fails to converge.
- This can be used as a way of gauging the max. load at which the structure buckles.

# Cont'd…

- This method uses all the nonlinearities.
- Unlike linear buckling analysis, this is more accurate.
- This method can be used for predicting both the local buckling and buckling.
- Typically, starting with a linear buckling analysis can provide an upper limit for the buckling load.
- A following nonlinear buckling analysis with the predicted upper load limit will help in getting a more accurate estimate.

Ansys