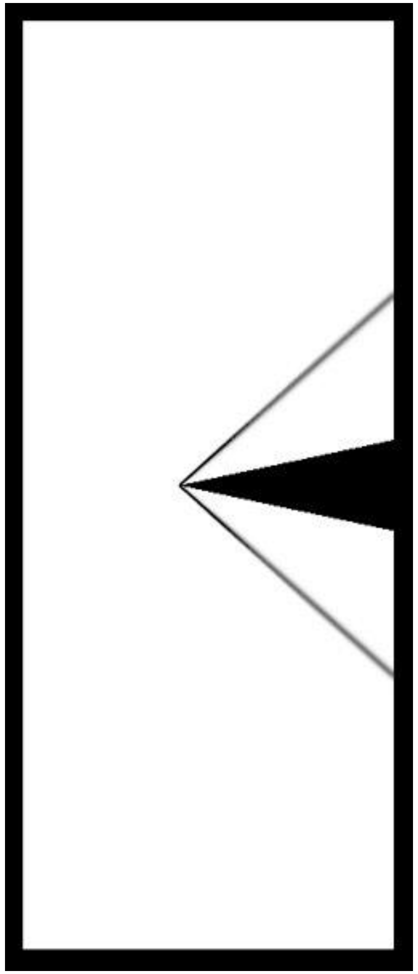
Oblique Shock Waves

Shock-Expansion Theory – Lesson 3

Oblique Shock Waves

- In the previous lesson we covered normal shocks, and now we will expand our discussion to oblique shock waves with wave fronts inclined at an angle to the upstream flow.
- Oblique shocks commonly occur when a supersonic flow is deflected by a concave corner and is forced to turn onto itself.
- A normal shock is a special case of an oblique shock which we will see in our further discussion.
- To analyze oblique shocks we can apply the governing physical laws to the flow passing through the shock and develop relations to describe property changes across the shock wave.
- We will make our considerations simpler by taking advantage of the normal shock results.



Oblique shocks generated at the tip of a wedge placed in supersonic flow (M=2)

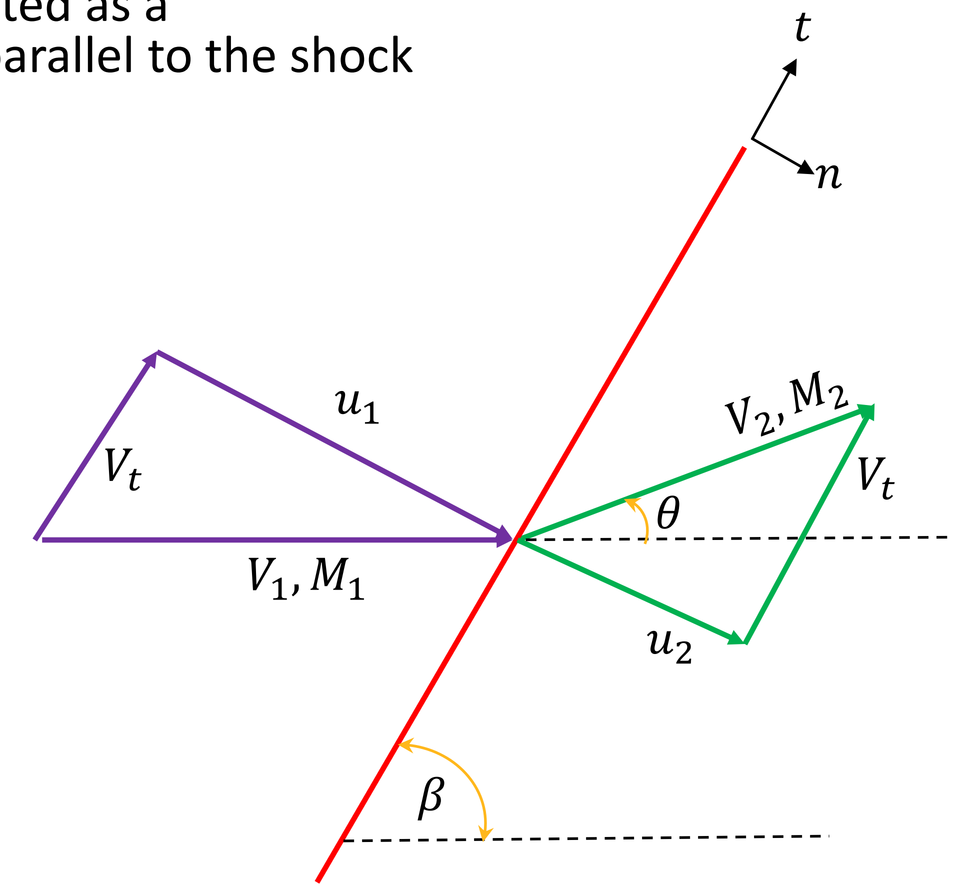
Physical Model of an Oblique Shock Wave

- Let's consider a shock wave oblique to the flow direction.
- The velocity upstream of an oblique shock, V_1, can be represented as a superposition of the normal shock velocity, u_1, and a velocity parallel to the shock front, V_t:

$$V_1 = \sqrt{u_1^2 + V_t^2}$$

$$\beta = \tan^{-1}(u_1/V_t)$$

- Since the normal velocity downstream of the shock, u_2, is less than u_1, the flow always turns towards the shock, and the angle of deflection θ is positive.
- The relations between upstream and downstream conditions can be easily defined from the normal shock relations since the superposition of a uniform velocity V_t does not affect static pressure or any other static parameters.
- Noting that the upstream normal velocity is given by $u_1/a_1 = M_1 \sin\beta$, relations for an oblique shock can be obtained simply by replacing u_1/a_1 with $M_1 \sin\beta$ in the normal shock expressions.



Oblique Shock Relations

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2\sin^2\beta}{(\gamma-1)M_1^2\sin^2\beta + 2}$$

$$\frac{p_2 - p_1}{p_1} = \frac{2\gamma(M_1^2\sin^2\beta - 1)}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = 1 + \frac{2(\gamma-1)}{(\gamma+1)^2}\frac{(\gamma M_1^2\sin^2\beta + 1)}{M_1^2\sin^2\beta}(M_1^2\sin^2\beta - 1)$$

$$\frac{s_2 - s_1}{R} = \ln\frac{p_{01}}{p_{02}} = \left[1 + \frac{2\gamma}{\gamma+1}(M_1^2\sin^2\beta - 1)\right]^{1/(\gamma-1)}\left[\frac{(\gamma+1)M_1^2\sin^2\beta}{(\gamma-1)M_1^2\sin^2\beta + 2}\right]^{-\gamma/(\gamma-1)}$$

- The ratios of static thermodynamic quantities depend only on the normal to the shock velocity component which must be supersonic upstream of the shock, $M_1 \sin\beta \geq 1.0$, which defines the minimum inclination angle. The maximum angle is the normal shock, $\beta = \pi/2$:

$$\sin^{-1}(1/M_1) \leq \beta \leq \pi/2$$

- The Mach number downstream of the shock can be obtained by noting $u_2/a_2 = M_2\sin(\beta - \theta)$:

$$M_2^2\sin^2(\beta-\theta) = \frac{1 + \frac{\gamma-1}{2}M_1^2\sin^2\beta}{\gamma M_1^2\sin^2\beta - \frac{\gamma-1}{2}}$$

Relations Between Wave Angle β and Deflection Angle θ

- Noting from the velocity triangles, $\tan\beta = u_1/V_t$ and $\tan(\beta - \theta) = u_2/V_t$, and utilizing the continuity equation and the density ratio relation from the previous slide,

$$\frac{\tan(\beta - \theta)}{\tan\beta} = \frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} = \frac{(\gamma - 1)M_1^2 \sin^2\beta + 2}{(\gamma + 1)M_1^2 \sin^2\beta} \quad \Rightarrow \quad \tan\theta = 2\cot\beta \frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}$$

- This expression can be re-arranged by dividing the numerator and denominator of the left formula above by $1/2\, M_1^2 \sin^2\beta$:

$$M_1^2 \sin^2\beta - 1 = \frac{\gamma + 1}{2} M_1^2 \frac{\sin\beta \sin\theta}{\cos(\beta - \theta)}$$

which can be approximated for small deflection angles as:

$$M_1^2 \sin^2\beta - 1 \approx \left[\frac{\gamma + 1}{2} M_1^2 \tan\beta\right] \cdot \theta$$

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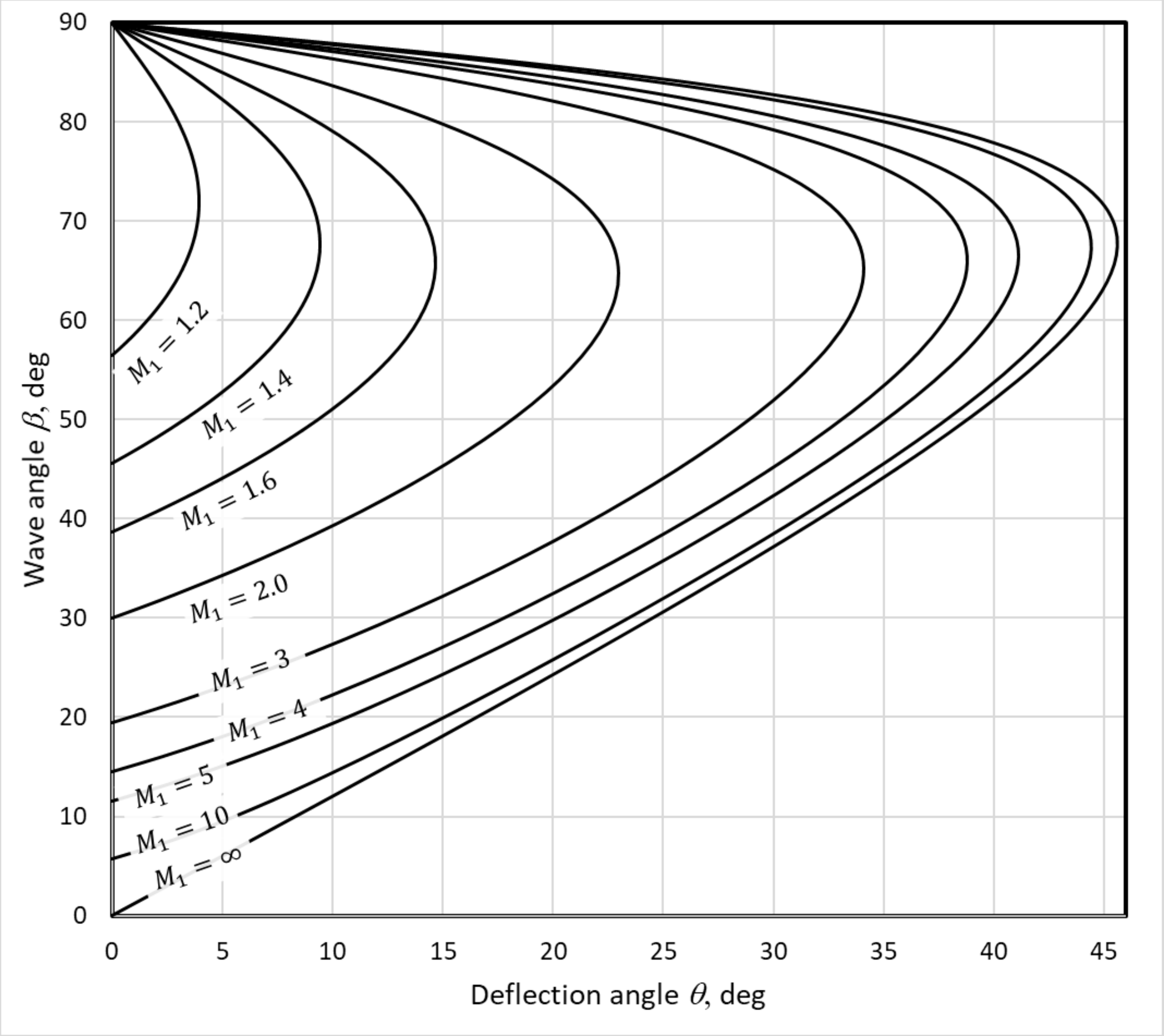
Relations Between Wave Angle β and Deflection Angle θ (cont.)

- The plot on the next slide shows β-θ curves for different Mach numbers. Each curve has a maximum, θ_{max}, and yields two roots for β when $\theta < \theta_{max}$:
  - The larger value of β gives the strong shock solution. The Mach number behind a strong shock is subsonic.
  - The smaller value of β gives the weak shock solution, which more commonly occurs in nature than the strong shock. The Mach number behind a weak shock is supersonic .
  - A strong shock can be generated if the backpressure behind the shock is increased by an external mechanism.
  - M_2 is subsonic in the strong solution and supersonic in the weak solution.
  - The deflection angle θ is zero at $\beta = \pi/2$ (normal shock) and $\beta = \sin^{-1}(1/M_1)$ (Mach wave which we will discuss later on in this lesson).
  - For a fixed deflection angle, M_1 decreases from high to low supersonic values for weak shocks until it reaches a point at $\theta = \theta_{max}$ where weak solutions are no longer possible. For lower values of M_1, there is not a solution for a straight oblique shock, and the shock becomes detached.

Relations Between Wave Angle β and Deflection Angle θ (cont.)

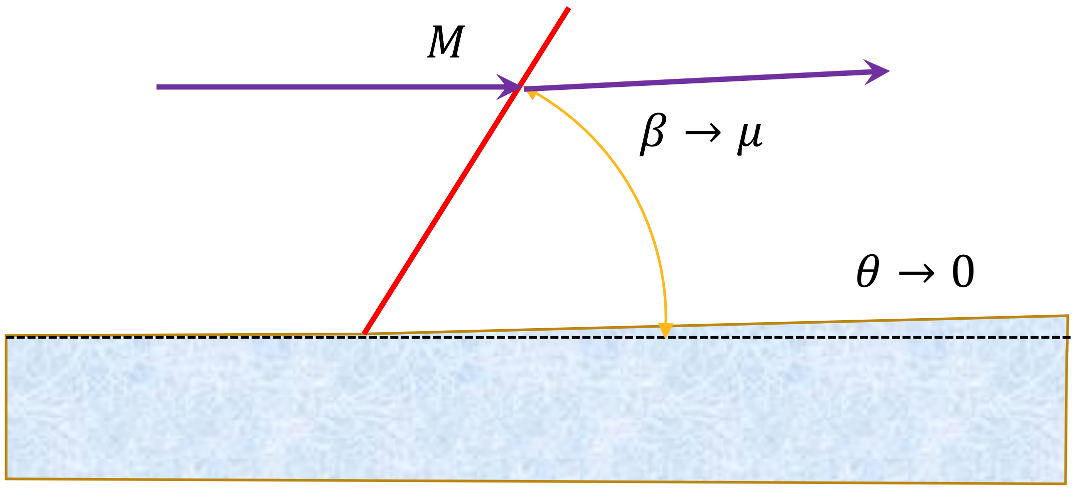


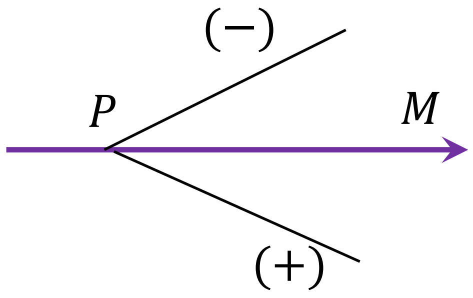
Mach Waves and Mach Lines

- Let's consider the limit as the deflection angle θ goes to zero.
- From the pressure ratio solution, it follows that the angle this wave makes with the surface is a function of Mach number only. This "wave" is also called a Mach wave and the angle μ is the Mach angle given by:

$$\mu = \sin^{-1}(1/M)$$

- The pressure jump also goes to zero, and, strictly speaking, this is not a wave, but simply a characteristic angle associated with the Mach number.
- The name Mach wave can be misleading as it is commonly used for weak but finite waves produced by small disturbances.
- At any point P in the flow field, there are two Mach lines, (+) and (-), intersecting a streamline at the angle μ in 2D. In 3D, Mach lines form a conical surface.





- The Mach lines are also called characteristics, as they trace the propagation of one-dimensional waves.

Weak Oblique Shocks

- In the previous slide, we considered the zero-deflection angle limit. Now let's assume that θ is small but finite. The β-θ relation can then be simplified as:

$$M_1^2 \sin^2\beta - 1 \approx \left[\frac{\gamma+1}{2} M_1^2 \tan\beta\right] \cdot \theta$$

$$M_1^2 \sin^2\beta - 1 \approx \frac{\gamma+1}{2} \frac{M_1^2}{\sqrt{M_1^2-1}} \theta$$

$\Longrightarrow$

$$\tan\beta \approx \tan\mu = \frac{1}{\sqrt{M_1^2-1}}$$

$$\frac{p_2-p_1}{p_1} \approx \frac{\gamma+1}{2} \frac{\gamma M_1^2}{\sqrt{M_1^2-1}} \theta$$

$\Longrightarrow$ *wave strength proportional to the deflection angle*

- From the weak normal shock analysis, entropy is proportional to the 3rd power of shock strength, thus $\Delta s \sim \theta^3$.
- The deviation between the wave angle β and Mach angle μ , $\epsilon = \beta - \mu$, can be approximated under the assumption $\epsilon \ll \mu$ as:

$$\epsilon \approx \frac{\gamma+1}{4} \frac{M_1^2}{M_1^2-1} \theta$$

For a finite small deflection angle, the wave direction differs from the Mach direction by an amount on the order of magnitude of θ.

- Finally, the change of flow speed across a weak oblique shock is:

$$\frac{V_2}{V_1} \approx 1 - \frac{\theta}{\sqrt{M_1^2-1}}$$

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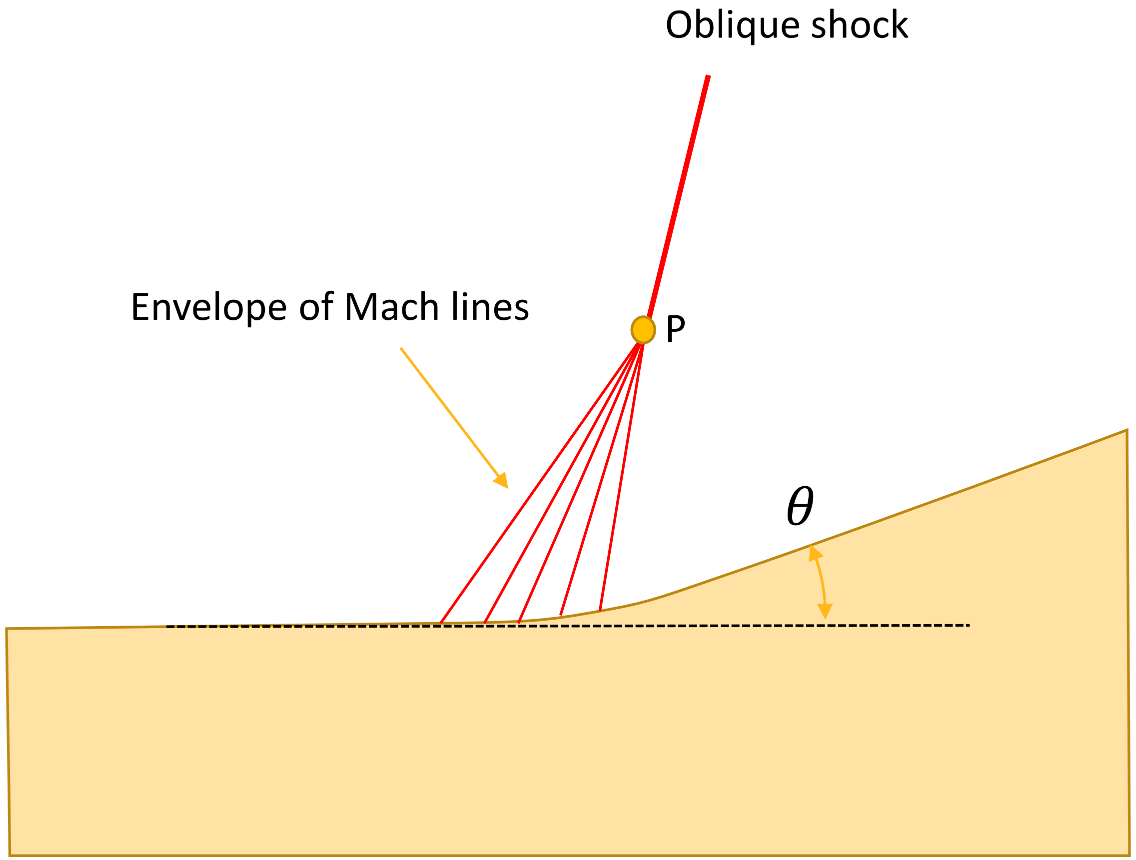
Supersonic Compression by a Curved Wall

- Until now, we have considered a sharp concave corner geometry, which supports a simple weak solution oblique shock wave.
- What about smoothly curved concave walls?
- For this type of geometry, we see that a family of very weak compression waves (Mach waves) are formed.
- These Mach waves coalesce at point P and form an oblique shock wave consistent with the deflection angle θ.
- Note that the radius of curvature must remain fixed for the wall, although for real, 3D geometry, the surface may have a complex shape, in which case compression or expansion waves may form and interact in various ways.

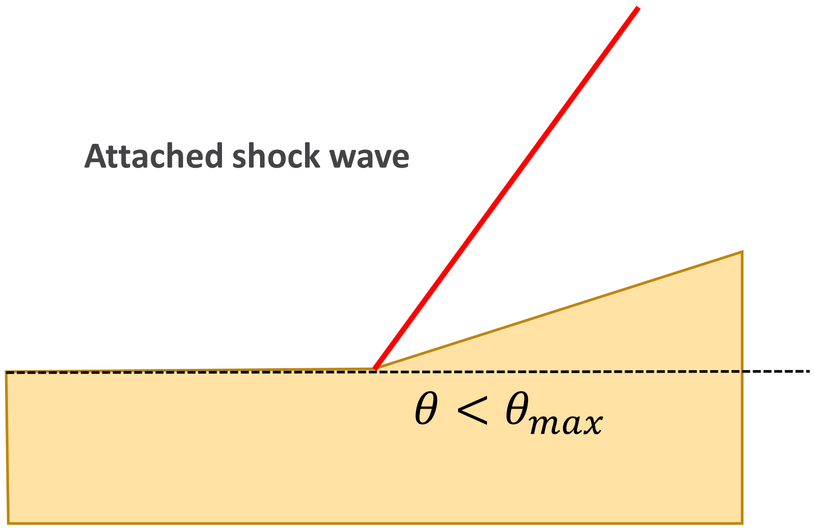


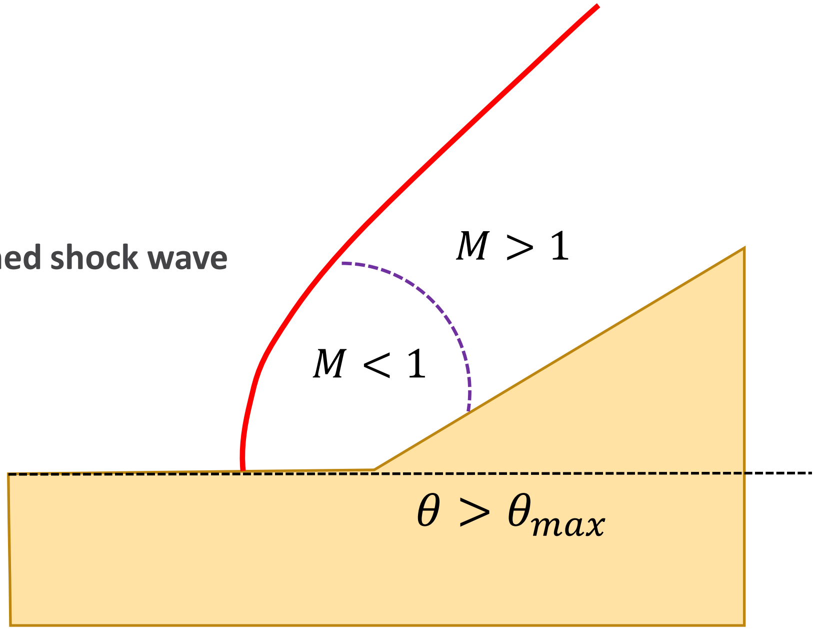
Detached Shock Waves

- As can be seen from the β-θ chart, for a given upstream Mach number, there exists a maximum deflection angle (θ_{max}) that is possible for the flow to turn.

- What if we try to turn the flow more than this angle? The flow will instead create a detached curved shock wave (also called a bow shock) as shown in the illustration. There will be a region of subsonic flow near the corner which eventually accelerates to supersonic downstream.

- Note that θ_{max} increases with Mach number, meaning that a straight shock solution is possible for larger deflection angles at higher Mach numbers.



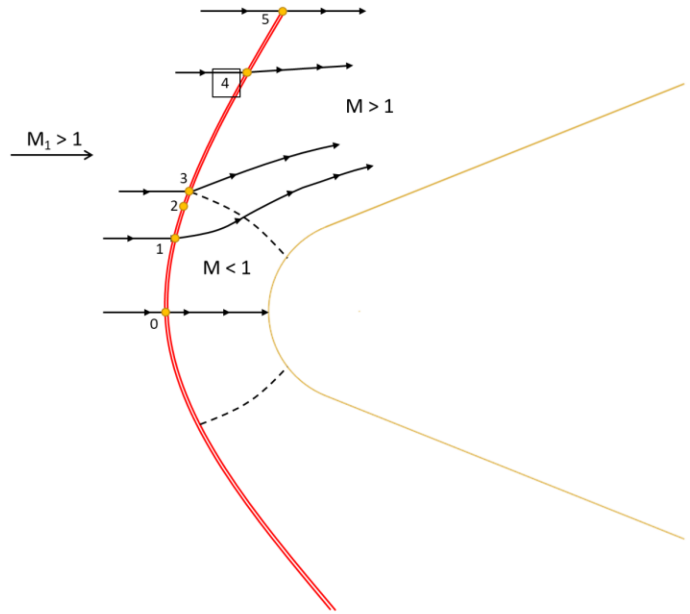


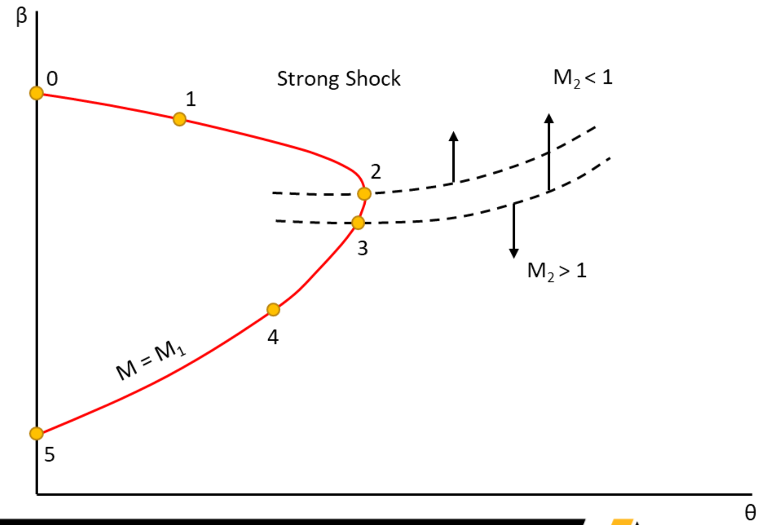
A General Case of Detached Shocks In Front of Blunt Bodies

- A supersonic flow over a blunt body is characterized by a strong curved bow shock generated in front of the body. This shock can be represented by a superposition, or a blend, of a normal shock, strong oblique shocks and weak oblique shocks.

Point	Condition
0	normal shock, subsonic flow behind
1	strong oblique shock, subsonic flow behind the shock
2	divide between strong and weak solutions
3	sonic flow behind the shock
3 - 5	weak oblique shock, supersonic flow behind the shock

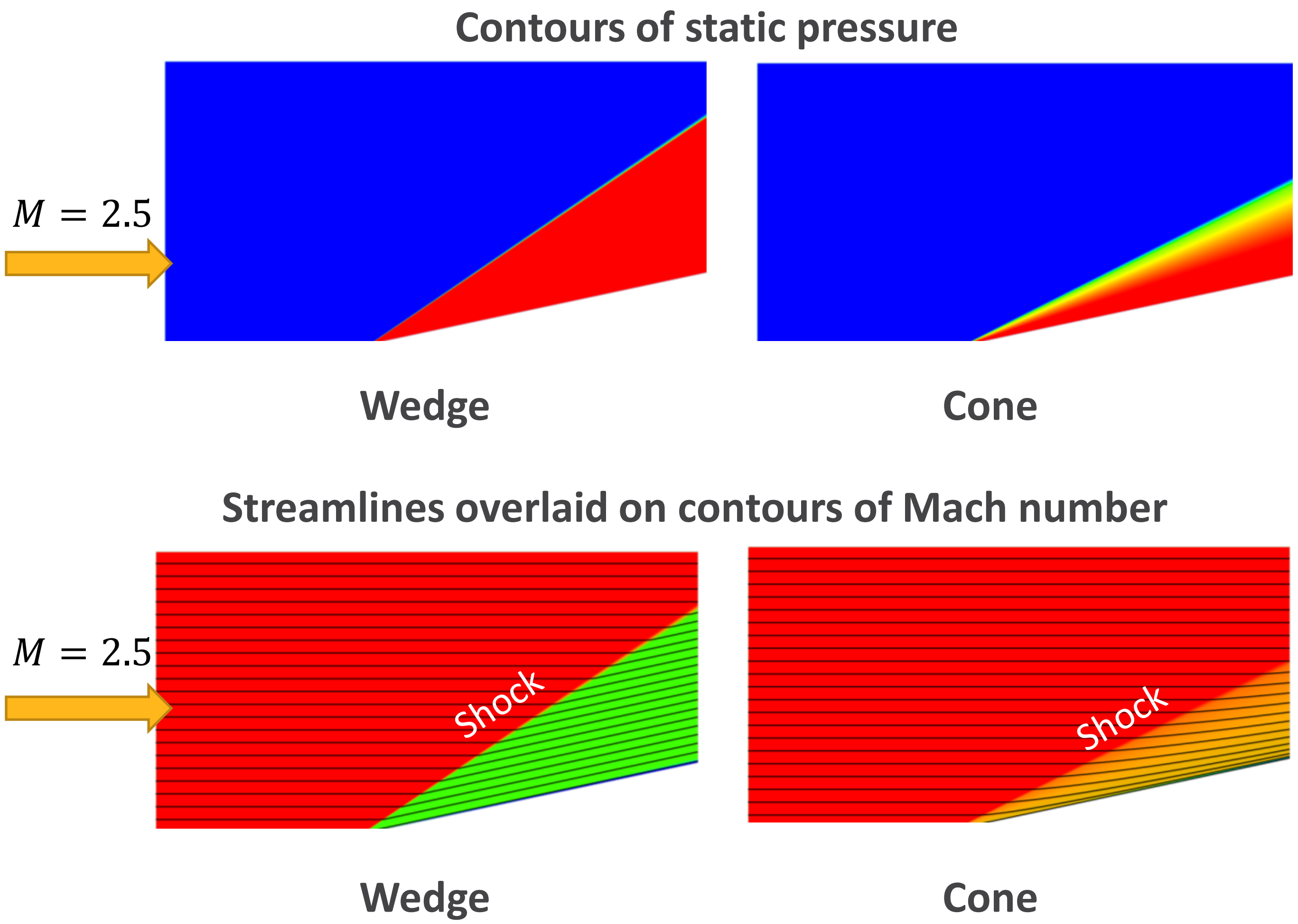
- A detached bow shock is complex and cannot be described analytically. Numerical techniques are required for solving supersonic flows over blunt bodies.





Supersonic Flow Over Wedges and Cones

- The foregoing analysis applies to 2D planar shock waves, and it can be directly applied to flow over two-dimensional wedges.
- A supersonic flow over a conical surface is, however, not as simple as that over a 2D wedge, since a uniform flow downstream of the shock is not possible as it does not satisfy the continuity equation.
- The conical flow problem can be solved using two observations:
  - There is limited upstream influence
  - Absence of characteristic length
- Under these assumptions, properties vary only with the angle , i. e., the conditions are constant along each ray from the cone vertex. Such flows are called conical.
- Unlike the 2D wedge flow, there is additional isentropic compression occurring up to the surface pressure and flow streamlines are curved behind the shock.

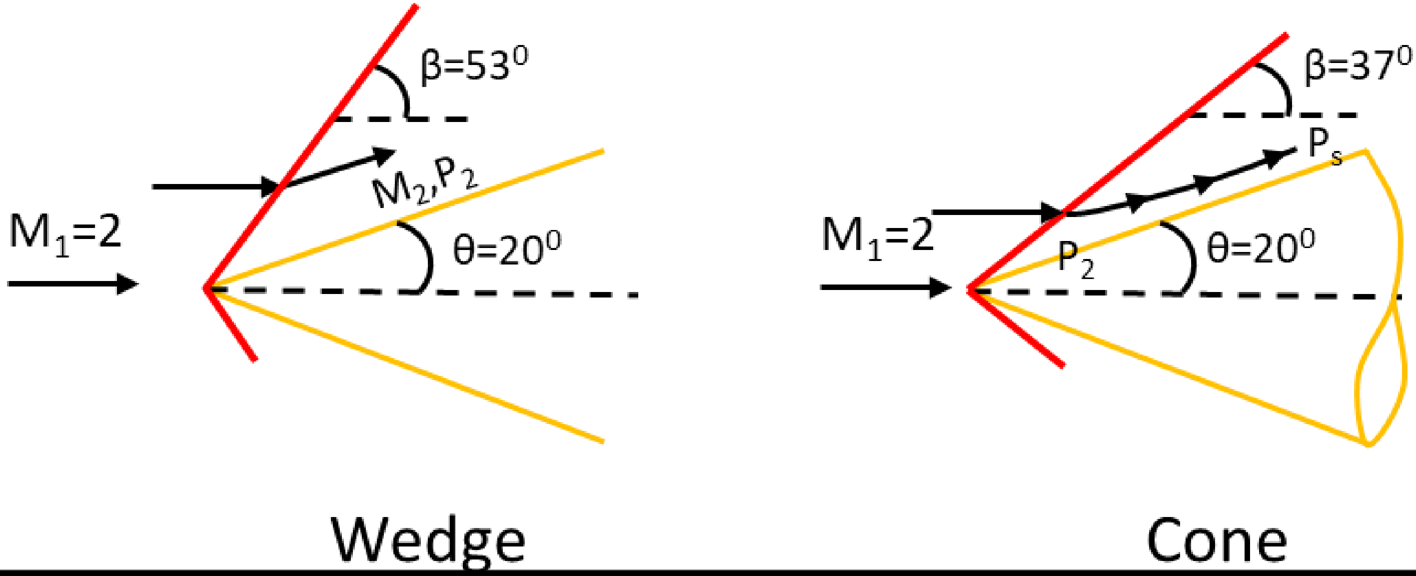


Supersonic Flow Over Wedges and Cones

- The solution is represented by an ordinary differential equation called the Taylor-Maccoll equation, requiring numerical solution:

$$\frac{\gamma-1}{2}\left[1-V_r^2-\left(\frac{dV_r}{d\omega}\right)^2\right]\left[2V_r+\cot\omega\frac{dV_r}{d\omega}+\frac{d^2V_r}{d\omega^2}\right]-\frac{dV_r}{d\omega}\left[V_r\frac{dV_r}{d\omega}+\frac{dV_r}{d\omega}\frac{d^2V_r}{d\omega^2}\right], = 0 \quad V_\omega=\frac{dV_r}{d\omega}$$

- This equation is solved for $V_r(\omega)$ by marching the solution from the initial condition at θ to the cone surface where $V_\omega = dV_r/d\omega = 0$. Isentropic relations are then used to determine flow variables along each ray.



Summary

- We examined oblique shock waves, their properties and relations in this lesson.
- We also discussed detached and bow shocks which can be thought of as generalized combinations of normal and oblique shocks.
- We also examined how the oblique shock theory can be applied to solve supersonic flows over wedges and corners.

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