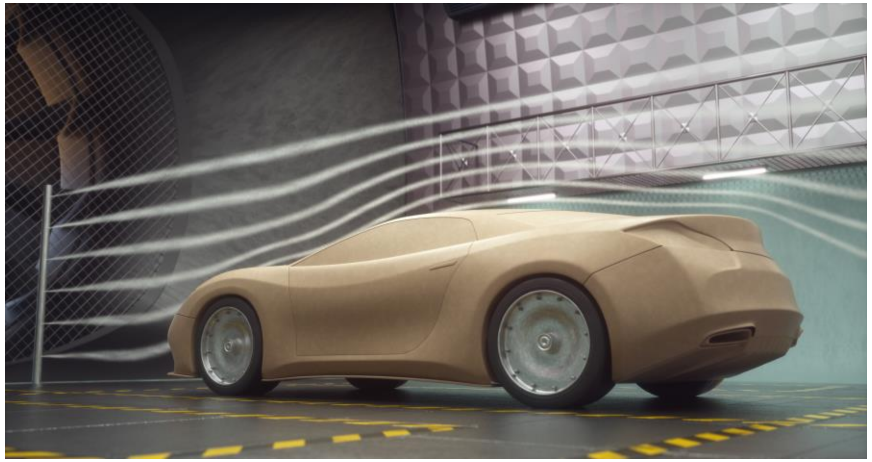
Quasi 1D Flows

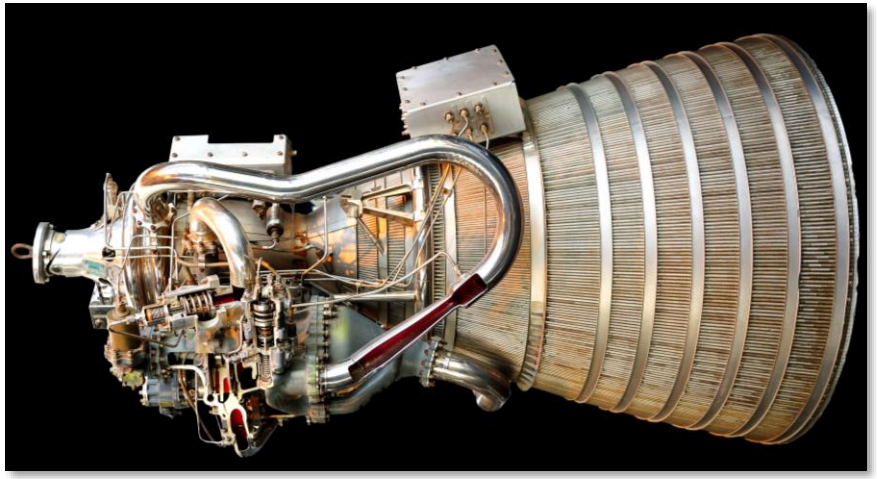
Internal Flows – Lesson 3

Intro

- In this lesson we will analyze quasi one-dimensional flows.
- Unlike a truly one-dimensional flow, the area of the passage varies in such flows. However, the variation $A = A(x)$, is gradual And therefore it is sufficiently accurate to neglect the y and z variations, and to assume that the flow properties are functions of x only.
- In such flows, it is the area change that causes the properties to vary along the x −direction.
- A wide range of engineering applications such as wind tunnels, rocket engines, etc., can be analyzed with a fair degree of accuracy using the methods discussed in this lesson.



Wind Tunnel

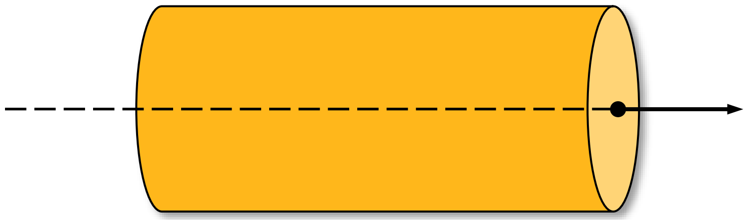


Rocket Engine

Basics of 1D and Quasi 1D Flows

- The flow is considered compressible and has attained steady-state.
- The working fluid is assumed to be an ideal gas with known thermodynamic properties.
- Property variations are assumed to be isentropic (except across shock waves).
- Velocity and thermal property profiles are uniform across the passage.
- No separated or reversed flow,
- No heat transfer or work input to the fluid.
- The duct cross-sectional area:
  - Is constant for 1D flows
  - Is a function of x for quasi 1D flows

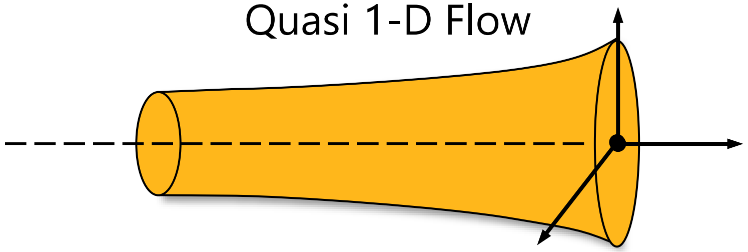
1-D Flow



$$A = constant$$

$$p = p(x), \rho = \rho(x), T = T(x), u = u(x)$$

Quasi 1-D Flow



$$A = A(x)$$

$$p = p(x), \rho = \rho(x), T = T(x), u = u(x)$$

Governing Equations for 1D Flow

$$\dot{m} = \rho A V = \text{constant}$$

Mass

$$dp + \rho V dV = 0$$

Momentum

$$h_0 = h + 1/2\, V^2$$

Energy (First Law)

$$s_0 = s = \text{constant}$$

Entropy (Second Law)

$$p = \rho R T$$

Equation of state (ideal gas)

Stagnation relations:

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

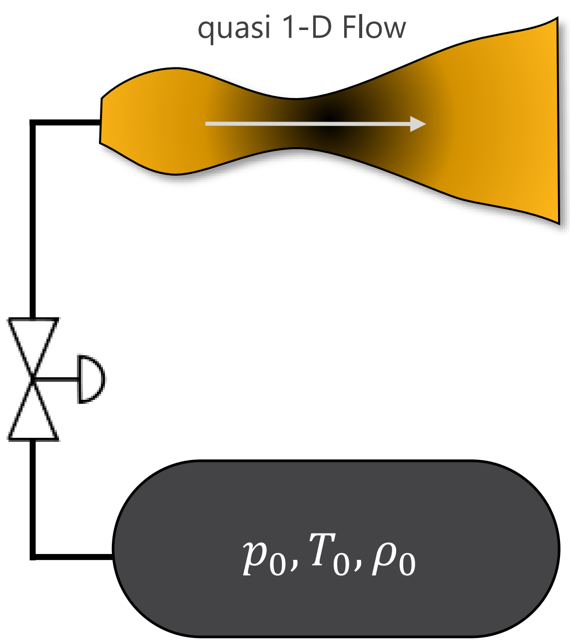
$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(\gamma-1)} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{1/(\gamma-1)}$$

Mass flow rate

- The quasi 1D continuity equation and the isentropic equations can be manipulated to give the following equation for the mass flow rate:

$$\dot{m} = \frac{p_0 A}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{2(\gamma-1)}{\gamma+1}}$$



- This form shows that the mass flow rate is proportional to the fluid total pressure and the duct area, while is inversely proportional to the total temperature of the fluid.
- Typically, the inlet conditions for a quasi 1D model are set assuming that the fluid originates from a tank at the stagnation (zero velocity) pressure and temperature.
- The fluid is assumed to accelerate isentropically to the inlet of the passage, and we can define the inlet condition as known stagnation (total) flow properties (p_0, T_0, ρ_0 etc.).

Area – Velocity Relation

- Governing equations in differential form:

$$d(\rho u A) = 0 \qquad dp = -\rho u du \qquad dh + u du = 0$$

- Using the continuity equation we can get:

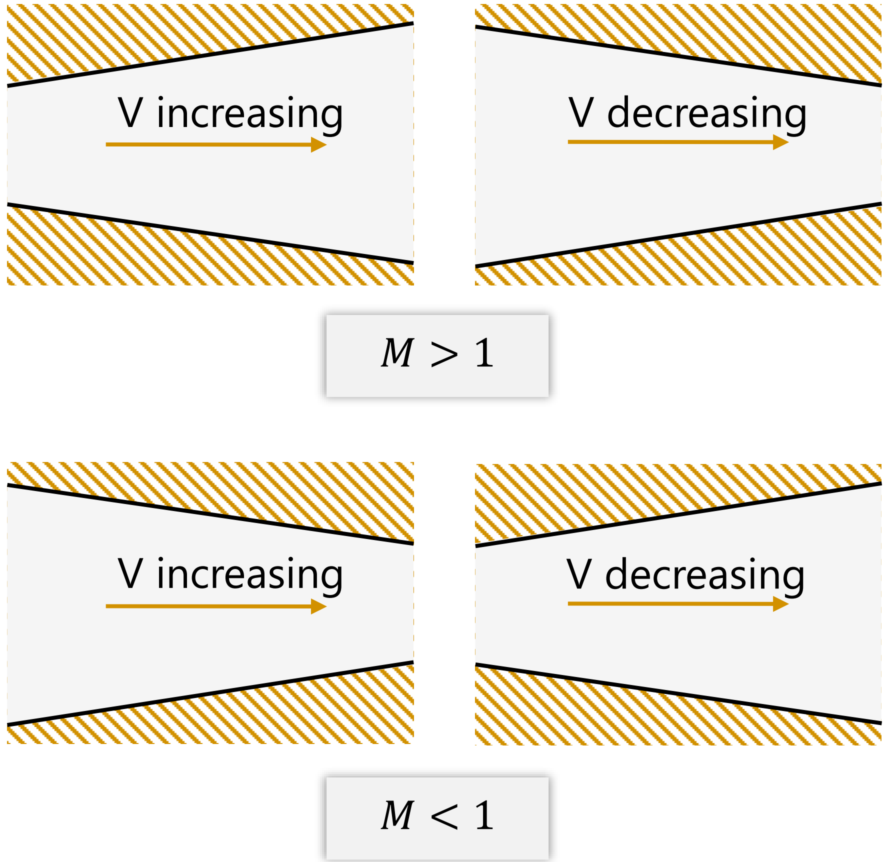
$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

- For an isentropic flow, any change in pressure dp is accompanied by a corresponding isentropic change in density $d\rho$:

$$\frac{dp}{d\rho} = \left(\frac{\partial p}{\partial \rho}\right)_s = a^2$$

- Combining these equations and manipulating them, we get the Area – Velocity Relation:

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$$



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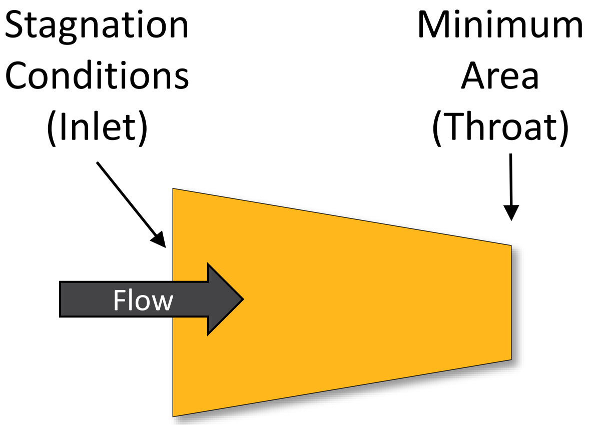
Critical State in Compressible Flow

- As the gas accelerates, its velocity (and Mach number) increases, while the static pressure and temperature decrease. When the fluid velocity reaches Mach number = 1, an important limiting effect is imposed on the passage mass flow rate.
  - This flow state is the critical state and is denoted with the superscript * (e.g. p^*, T^*, V^*, ρ^* etc.).
  - For our 1D passage we denote the area at which sonic conditions are achieved as A^*.
- Let's consider a converging nozzle from the energy equation:

$$V = \sqrt{2(h_0 - h)} = a_0 \sqrt{\frac{2}{\gamma - 1}\left[1 - \left(\frac{p}{p_0}\right)^{\gamma/(\gamma-1)}\right]}, \qquad a_0 = \sqrt{\gamma R T_0}$$

- The passage mass flux (G) can then be defined as: $G = \dot{m}/A = \rho V$
- Let's denote the minimum area of the duct as the throat.
- For a given mass flow rate, the maximum mass flux must occur at the throat, since it is the smallest section of a duct.



Converging Nozzle

Choking

- We denote the area and flow properties there with a subscript t.
- We can now write: $G_{max} = \rho_t V_t \qquad \frac{1}{G_{max}}\frac{dG_{max}}{dx} = \frac{1}{\rho_t}\frac{d\rho_t}{dx} + \frac{1}{V_t}\frac{dV_t}{dx} = 0$

Note that the derivative of the mass flux is zero at the throat.

- From the 1D momentum equation: $dp_t + \rho_t V_t dV_t = 0$
- Under the assumption of isentropic flow, combining the above yields:

$$V_t^2 = \frac{dp_t}{d\rho_t} = a_t^2 \quad or \quad V_t = a_t$$

This shows that when the throat is small enough so that the velocity becomes equal to the speed of sound (or Mach number = 1), then the throat is the point of maximum mass flux. This physical effect is called choking, as it represents the maximum mass flow rate which can be achieved in a converging passage from a given stagnation condition.

Choking and Flow Measurement

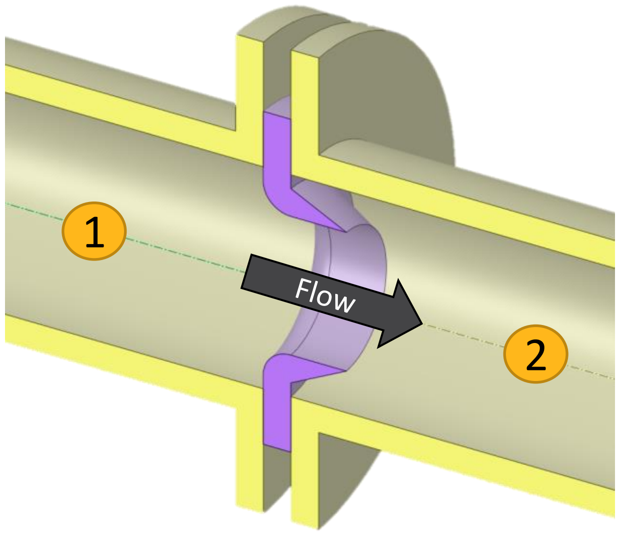
- Since throat conditions are at sonic velocity, they are also critical conditions and can be denoted by the * superscript (e. g. $A_t = A^*, T_t = T^*, p_t = p^*,$ etc.).

$$\frac{T_0}{T^*} = \frac{\gamma+1}{2} \qquad \frac{p_0}{p^*} = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} \qquad \frac{\rho_0}{\rho^*} = \left(\frac{\gamma+1}{2}\right)^{1/(\gamma-1)}$$

Sonic (throat) relations

- The critical (choking) mass flow rate can be calculated substituting M= 1 in the relation presented earlier:

$$\dot{m}^* = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/[2(\gamma-1)]}$$



- This equation provides the basis for flow measurement using choked converging passage nozzles.
  - The choked mass flow equation can be used to determine the mass flow rate $\dot{m}^*$ from the upstream total pressure and temperature, the nozzle throat area A^* and known gas thermodynamic properties (γ, R).
  - The total pressure and temperature can be measured using pitot-static tubes and thermocouple probes.
  - The downstream pressure ratio p_2/p_{0_1} should be less than the choking threshold (for example, 0.528 for air).

Critical Area Ratio and 1D Flow Solution

- Using the continuity equation, we can develop an equation for the passage area to critical (throat) area ratio :

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{(\gamma+1)/[2(\gamma-1)]}$$

- Notice that as $M \rightarrow 1$, $A/A^* \rightarrow 1$ – that is, the passage area approaches the critical (throat) area. We can now formalize the 1D flow calculation procedure:

Specify flow conditions at the inlet (stagnation properties and inlet Mach number). Passage geometry is known, $A(x)$.

→ At the inlet Mach number, calculate the area ratio $A(0)/A^*$ Since $A(x)$ is known, calculate A^*

→ For any downstream station calculate $A(x)/A^*$ and use the area ratio equation to solve for the Mach number at the station. (This can be done numerically, or one can consult tables.)

→ Knowing the Mach number, use the isentropic relations to obtain all flow properties.

This can be easily set up in a spreadsheet or computer program, thus avoiding the use of tables or charts.

Quasi 1D Flow Solution

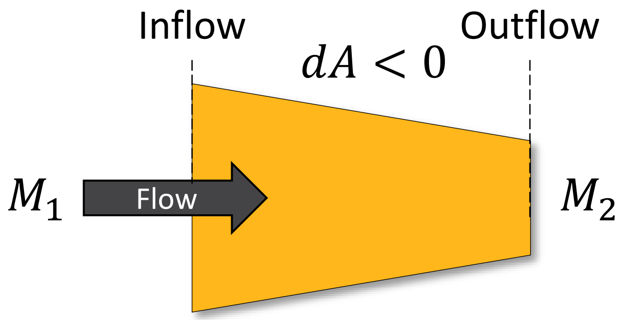
- Note that A^* is calculated based on the inlet conditions. However, in our actual passage geometry, the physical minimum area may be larger than A^*. If this is the case, then the flow will not choke at the throat.
- It is also possible that the minimum area is smaller than A^*. In this case, the prescribed inlet Mach number is not possible! In fact, for the prescribed stagnation pressure and temperature, the inlet Mach number will be lower and the physical throat in the passage will be choked.
- Let's analyze this issue further by looking at the variation in Mach number with passage area. Using the quasi 1D governing equations and isentropic relations, we can develop the following relation:

$$\frac{dM}{M} = K\frac{dA}{A}$$

$$K = -\frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}$$

- These equations lead to four limiting cases analyzed in the next slides.

Converging Passage



- Subsonic Inflow

$$M_1 < 1, \quad K < 0, \quad dA < 0$$
$$dM > 0, \; M_2 > M_1 \quad \rightarrow$$ Mach increases (up to 1)

- Supersonic Inflow

$$M_1 > 1, \quad K > 0, \quad dA < 0$$
$$dM < 0, \; M_2 < M_1 \quad \rightarrow$$ Mach decreases (down to 1)

A converging passage is a nozzle for subsonic inflow and a diffuser for supersonic inflow

- Sonic Inflow, Supersonic Outflow

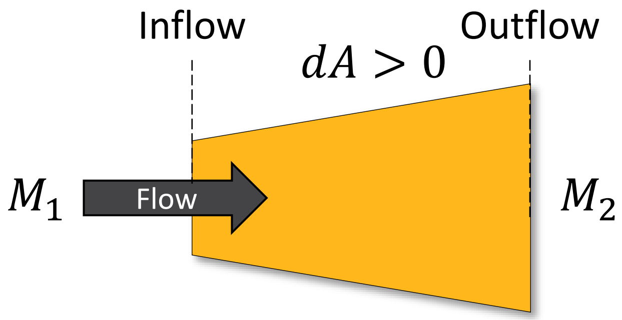
$$M_1 = 1, \quad M_2 > 1, \quad K > 0, \quad dA < 0$$
$$dM < 0, \quad M_2 < M_1 \quad \rightarrow$$ Impossible!

- Sonic Inflow, Subsonic Outflow

$$M_1 = 1, \quad M_2 < 1\,, \quad K < 0, \quad dA < 0$$
$$dM > 0, \quad M_2 > M_1 \quad \rightarrow$$ Impossible!

A sonic flow <u>cannot</u> enter a converging passage!

Diverging Passage



- Subsonic Inflow

$$M_1 < 1, \quad K < 0, \quad dA > 0$$
$$dM < 0, \; M_2 < M_1 \quad \rightarrow \quad \text{Mach decreases}$$

- Supersonic Inflow

$$M_1 > 1, \quad K > 0, \quad dA > 0$$
$$dM > 0, \; M_2 > M_1 \quad \rightarrow \quad \text{Mach increases}$$

A diverging passage is a diffuser for subsonic inflow and a nozzle for supersonic inflow

- Sonic Inflow, Supersonic Outflow

$$M_1 = 1, \quad M_2 > 1, \quad K > 0, \quad dA > 0$$
$$dM > 0, \quad M_2 > M_1 \quad \rightarrow \quad \text{Mach increases}$$

- Sonic Inflow, Subsonic Outflow

$$M_1 = 1, \quad M_2 < 1, \quad K < 0, \quad dA > 0$$
$$dM < 0, \quad M_2 < M_1 \quad \rightarrow \quad \text{Mach decreases}$$

A sonic flow can enter a diverging passage!
Note that the outflow may be subsonic or supersonic.

Summary

- In this lesson we learned how to analyze quasi 1D compressible flow problems having variable area.
- The concepts of choking and critical flow properties were introduced.
- We also saw that there are physical limitations on the type of flow that is permissible in a converging or diverging passage.

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