Major Losses in Pipes and Ducts

Real Internal Flows – Lesson 3

Ansys

Losses in Pipe Flows

- In this lesson we will discuss some practical aspects of internal flows related to hydraulic losses and describe methodologies for estimating losses in pipes with different surface roughness and cross section shape.

- Energy losses of a fluid flow through a pipe or an internal system are normally described in terms of the hydraulic head.

- Recall that Bernoulli's equation represents the conservation of energy in the fluid. The total energy head in incompressible flow in a gravitational field is given by:

$$H = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

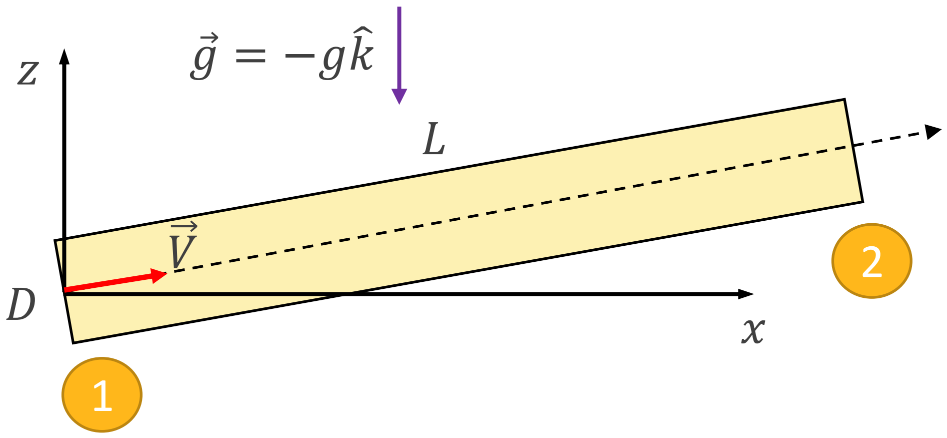
- The hydraulic head represents the energy of the flow as the equivalent height of a static column of the fluid and has the units of length.

- In an inviscid flow H is constant as previously discussed.

Losses in Pipe Flows (cont.)

- In a viscous flow H is no longer constant along the pipe and the Bernoulli's equation at stations 1 and 2 becomes:

$$H = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_l$$



- The head loss h_l is usually used as a measure of the energy loss in an internal fluid system.

- For the fully developed flow in a level pipe ($z_1 = z_2$), the head loss is given by the scaled pressure drop:

$$h_l = \frac{p_1 - p_2}{\rho g}$$

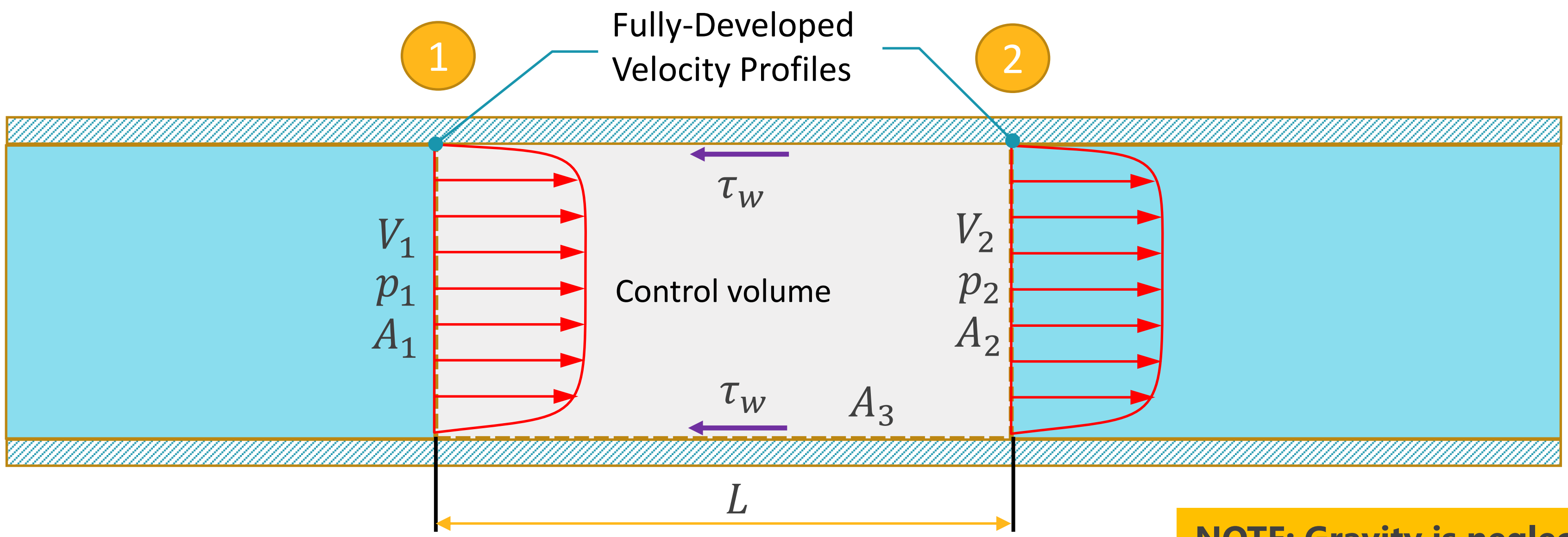
- Energy or head losses are generally divided into two main components:
  - Major losses due to friction in straight pipes
  - Minor losses due to components as valves, bends, etc.

Integral Analysis of Momentum Conservation in a Pipe

- Consider a steady, fully developed, incompressible turbulent flow in a pipe of circular cross section with diameter D.
- We will apply the integral form of the linear momentum equation to a control volume with upstream/downstream stations denoted as 1 and 2, as depicted below. Let's assume:
  1. Velocity profiles at stations 1 and 2 are the same in a fully developed flow, and
  2. Horizontal pipe - so that the gravity effects can be ignored.

$$A_1 = A_2 = \frac{\pi D^2}{4}$$

$$A_3 = \pi D L$$



NOTE: Gravity is neglected

Integral Analysis of Momentum Conservation in a Pipe

- Applying the integral form of the momentum equation gives (noting $\hat{n}$ is an outward pointing unit normal vector):

$$\int_{A_1} \vec{V}(\rho\vec{V}\cdot\hat{n})\,dA + \int_{A_2} \vec{V}(\rho\vec{V}\cdot\hat{n})\,dA = \int_{A_3} -\tau_w dA + \int_{A_1} p\hat{n}\,dA + \int_{A_2} p\hat{n}\,dA$$

- Resolving the integrals using averages at stations 1 and 2, noting the direction of $\hat{n}$ on the boundaries and the fact that due to the fully developed assumption the left-hand side is zero, yields an expression for the mean pressure drop over length L:

$$\Delta p = p_2 - p_1 = \left(\frac{4L}{D}\right)\tau_w$$

- This gives a simple relation between pressure drop and wall shear stress.

- Note that the pressures and shear stress in the above expression are surface area averaged quantities. For pipe flow analysis, it is sufficient for engineering purposes to utilize averaged values for pressure, velocity and other flow properties at specific stations along the pipe.

Head Loss in Level Pipe Flows

- The head loss can now be written as:

$$h_l = \frac{4L\tau_w}{Rg}$$

- This highlights that it is the shear stress that is solely responsible for the head loss in a level straight pipe.

- The expression for head loss is valid for both laminar and turbulent flows.

Darcy Friction Factor

- Recalling definitions of the skin friction coefficient C_f and the Darcy friction factor which is four times C_f:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho V^2}$$

$$f = 4C_f$$

- An equation for pipe pressure drop can be expressed in terms of f as:

$$\Delta p = f\frac{L}{D}\frac{\rho V^2}{2}$$

- Henry Darcy, a French Engineer, developed this relation in the mid-19th century. He did not use the integral analysis; instead he arrived at the pressure drop expression through dimensional analysis arguments.

- For engineering use, the correlation equations are convenient. One can define a pipe flow based on the pipe length, diameter, fluid density, and volume flowrate and then, after determining an appropriate friction factor based on Reynolds number, calculate the mean pressure drop directly. The equation can also be cast in terms of volume flowrate, so that the flowrate can be directly calculated for a given pressure drop.

- Note that f is a function of the Reynolds number for smooth-wall pipes.

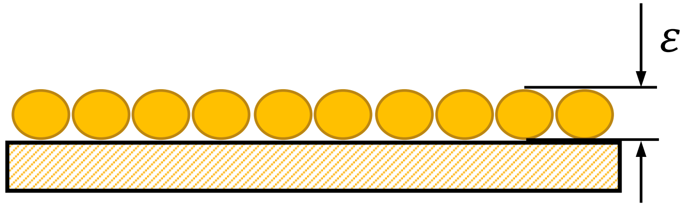
Rough Wall Pipes

- In the real world, pipes are often NOT smooth, but in fact have surface textures which increase friction and hence the turbulence and the pressure drop in the pipe. The texture is seen as a sand roughness height (k_s), or its equivalent, as depicted below.
- The roughness can be characterized by the equivalent sand roughness ε.
- The Darcy friction factor for rough wall pipes is not a function of Reynolds number alone anymore, as it also depends on the surface roughness: $f = f(Re, \varepsilon/D)$.
- The head loss can be expressed as:

$$h_l = f(Re, \varepsilon/D)\frac{L}{D}\frac{V^2}{2g}$$

❖ This is known as the Darcy-Weisbach equation valid for any fully developed steady incompressible pipe flow (level or inclined).

Rough walls of a corroded tube



Uniform Sand Grain

$\varepsilon = grain\ roughness\ height$

L. F. Moody and the Moody Diagram

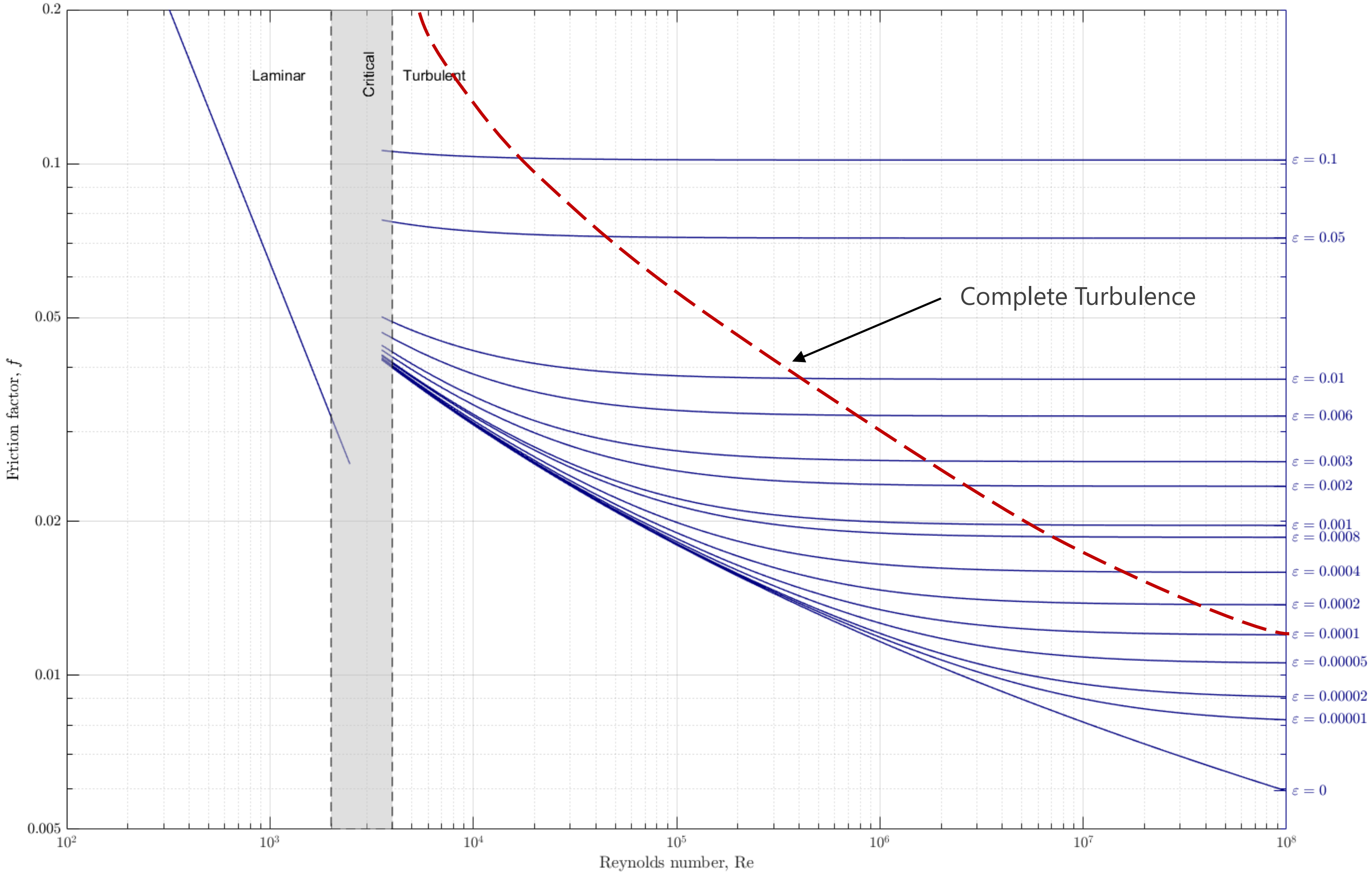
- All of the foregoing equations for pressure drop versus Reynolds number and relative roughness can be summarized in a single plot called the Moody Diagram, which was developed by Lewis Ferry Moody (1880 – 1953), a professor at Princeton University, and published in 1944.

- The Moody diagram combines all the available friction factor data into a single chart and has since become a standard tool for fluids engineers to estimate pipe loss over a range of Reynolds numbers and surface roughnesses.

- A typical, modern version of the Moody diagram is presented on the next slide. Note that this employs the Darcy friction factor definition (some similar charts may actually use the Fanning friction factor).

- Most of correlation data presented in the Moody Diagram was derived from experiments of J. Nikuradse (1933) who used artificially roughened pipes by adhering sand grains of known sizes, then measured pressure drop required to generate certain flow rate, and then converted data into friction factor for a given Re and ε.

The Moody Diagram



The Moody Diagram (cont.)

- As can be seen from the Moody diagram, the laminar regime is unaffected by the wall roughness and described by the single curve $f = 64/Re$. This is not surprising if we recall the parabolic shape of the velocity profile in a laminar pipe flow.

- The friction factor is nearly independent of the Reynolds number for very large values of Re. This range of Re is separated by the dashed *Complete Turbulence* line in the diagram. This independence of Re can be explained by the fact that at larger Re the laminar layer is very thin, and the surface roughness completely dominates near-wall flow characteristics.

- In turns out the entire non-laminar range of the Moody chart can be described by a single correlation called the Colebrook formula:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

- This formula is implicit for f and has to be solved numerically (iteratively). An explicit approximation was made by Haaland (1983):

$$\frac{1}{\sqrt{f}} = -1.8 \log\left(\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$$

The Moody Diagram (cont.)

- The Moody diagram and Colebrook formula are based on experimental data which, unavoidably, carries a certain degree of uncertainty, and the expected accuracy of predictions is usually ~10% which, depending on design objectives, can be acceptable.

- The Moody diagram can also be used for estimating friction factors in non-circular pipes by replacing D with the hydraulic diameter D_h. The accuracy of predictions will be lower because of deviations of circular and non-circular pipe data we discussed for smooth-wall pipes in the previous lesson.

Summary

- A discussion of the pressure losses due to viscous friction (major losses) for internal flows was introduced here.
- Losses in rough wall pipes were discussed, and the Moody diagram and related correlation formulas were also introduced as a tool for estimating pipe losses over a range of Reynolds numbers and wall roughnesses.
- Next, we will consider minor losses due to additional piping components (valves, bends elbows, tees, etc.)

Ansys