Flows with Moving and Rotating Objects

Beyond Viscosity – Lesson 4

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Intro

- In many fluid dynamics problems, objects of interest are moving with respect to the quiescent freestream flow.
- If the motion of an object can be assumed to be translational with constant velocity, then the problem can be simply recast into the object's frame of reference according to Newton's 1st law, so the object can be considered to be stationary, with the fluid flowing over the object at a constant freestream velocity.
  - An airplane flying at a constant speed is analyzed using this approach.
- If the object is accelerating with respect to the ambient fluid, or if there are multiple objects moving at different speeds, then such a transformation of the frame of reference becomes more complicated, and, in the case of multiple moving objects, may be impossible.
  - Rotating turbomachinery devices (fan, turbines, etc.) are inherently under rotational acceleration.
  - A stone dropped from an airplane and moving under gravitational acceleration has multiple moving frames of reference.

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Examples of Flows Over Moving and Rotating Objects

Airflow around the blades of a wind turbine

Fluid flow through motors and pumps

Airflow over the rotors of a drone

Airflow around the compressor and turbine blades in an aircraft engine

Airflow over the rotors of a drone

Methodologies to Describe Moving Objects

- Two basic methodologies to describe moving objects are:
  - Solve the governing equations of fluid motion in the reference frame of the ambient fluid and include the actual motion of the object in the analysis.
  - Re-derive governing equations in the reference frame of the accelerating object, provided such derivation is possible and practical.
- In this lesson we will discuss these two methodologies and comment on their advantages and disadvantages.

Analysis in the Stationary Ambient Frame

- The motion of an object in the stationary reference frame of the ambient fluid, as described by the unsteady form of Navier-Stokes equations in the presence of gravitational acceleration, are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0$$

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}\right] = \rho \vec{g} - \nabla p + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu \delta_{ij} \nabla \cdot \vec{V}\right]$$

$$\vec{V}_{fluid} = \vec{V}_{solid}(\vec{x}, t)$$

No-slip condition at the fluid-solid interface

- $\vec{V}_{solid}(\vec{x}, t)$ is the velocity of the object's wetted walls.

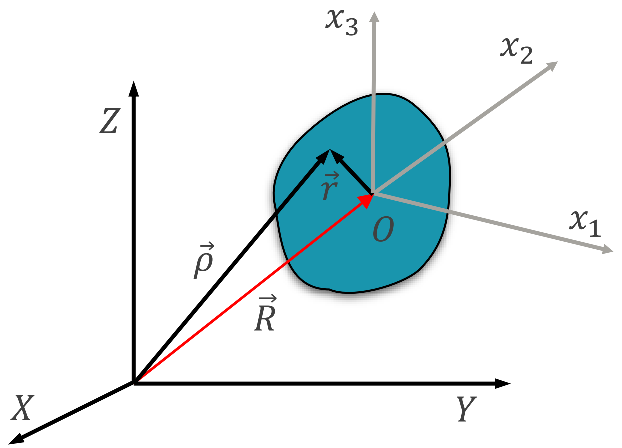
Analysis in the Stationary Ambient Frame

- The governing equations are inherently unsteady when the object's velocity is time-dependent and the transient terms in the equations cannot be dropped.
- In general, there are no analytical approaches available, and the analysis must be carried out using a computational fluid dynamics (CFD) model.
- Assuming the object is a rigid body, its own motion can be described by the laws of the rigid body motion.

$$\vec{V}_{solid}(\vec{x},t) = \underbrace{\vec{V}_t(\vec{x},t)}_{\text{Translational}} + \underbrace{\vec{\Omega}(\vec{x},t)}_{\text{Angular}} \times \vec{r}$$

Center of mass (gravity) velocity

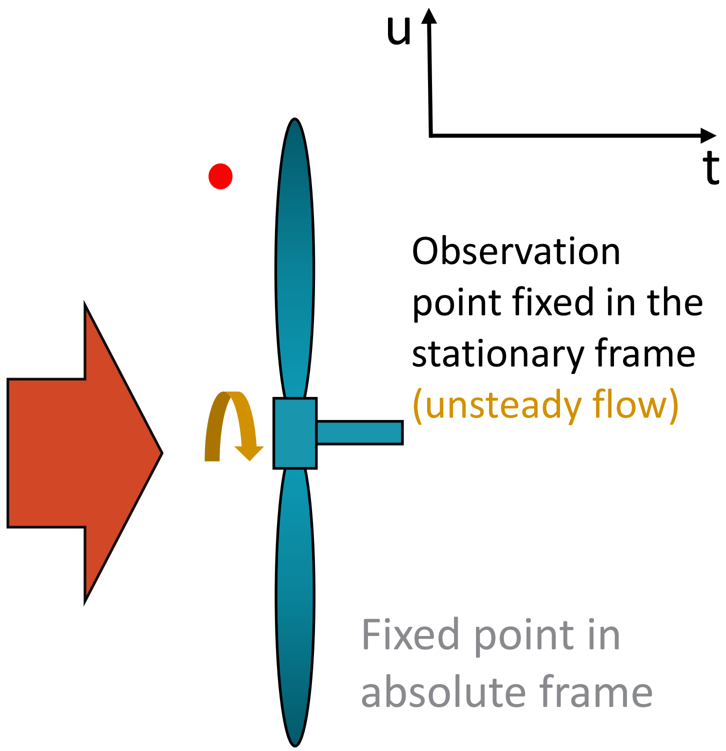
- $\vec{V}_{solid}$ can be:
  - Externally prescribed and unaffected by the fluid forces
    - Maneuvering vehicle
  - Entirely defined by fluid and body forces (gravity)
    - Free-falling object
  - Defined by fluid and external forces
    - Spring-loaded valves

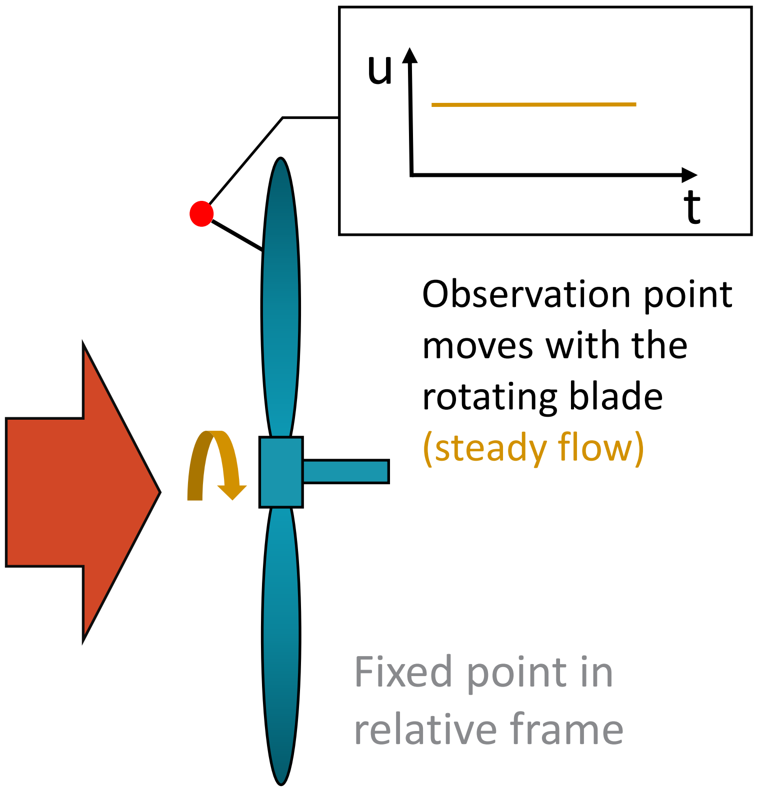


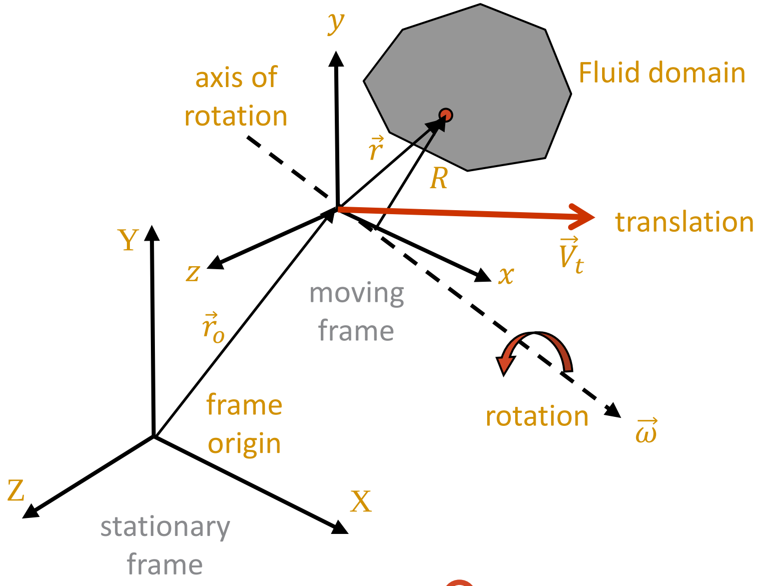
Reference Frames

- Many problems which involve moving components can be modeled using a moving reference frame (MRF). A moving reference frame is defined as a reference frame which is in motion with a prescribed orientation and speed with respect to a stationary (or inertial) reference frame, as shown on the next slide.
- Why use an MRF?
  - A flow field which is unsteady with respect to the stationary frame becomes steady with respect to the MRF
    - Steady-state problems in the moving frame are easier to solve...
      - Simpler boundary conditions
      - Easier to analyze
- Reference frames can be applied to describe translational and rotational motion, but they are more commonly used for rotational motion problems in turbomachinery applications.

Flow Viewed in Fixed and Moving Frames



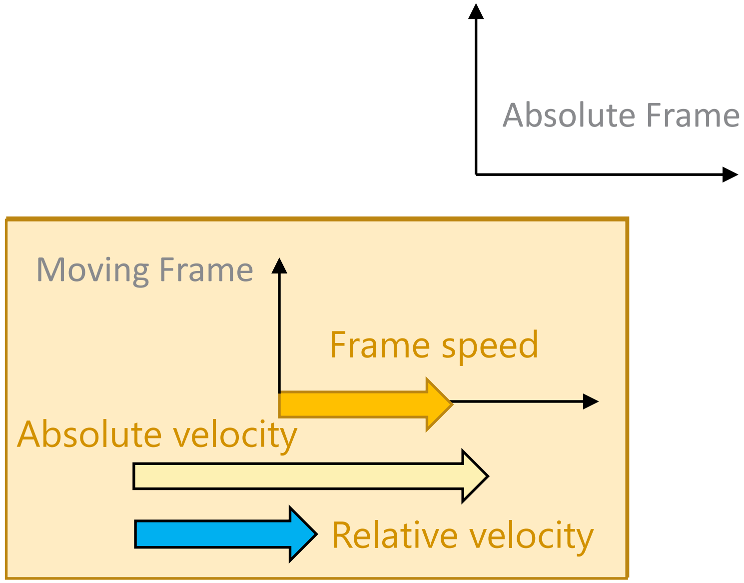




R is perpendicular to axis of rotation

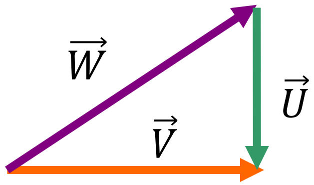
Absolute and Relative Velocities

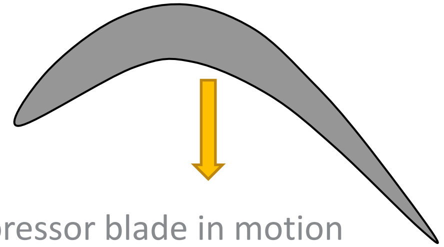
- When the fluid flow is defined with respect to the moving frame, additional acceleration terms will appear in the equations.
  - As the reference frame is accelerating, the fluid will "feel" those accelerations, much like an additional "gravity" force.
  - This will be reflected in Newton's Second law, which is modified to account for the acceleration of the frame.
  - Note that even simple, constant speed rotation of the reference frame results in additional accelerations.
- The velocity of the fluid can be defined with respect to both the stationary and moving frames:
  - Absolute velocity ($\vec{V}$) - Fluid velocity measured with respect to the stationary (absolute) reference frame
  - Relative velocity ($\vec{W}$) - Fluid velocity measured with respect to the moving reference frame
- These are two different velocity fields with different magnitudes and directions – but they are related by the frame speed using vector addition rules called the velocity triangle.



$$\vec{V} = \vec{W} + \vec{U}$$
$$\vec{U} \equiv \vec{\omega} \times \vec{r} + \vec{V}_t$$

$\vec{V} =$ Absolute velocity
$\vec{W} =$ Relative velocity
$\vec{U} =$ Frame velocity





Compressor blade in motion

Governing Equations for MRF - Formulations

- MRF equations are derived by starting with the governing equations in the stationary frame and using the relationship between stationary frame and moving frame velocities to recast the questions in the moving frame, which will give rise to additional acceleration terms.
- Two different governing equation formulations are possible:
  - Relative Velocity Formulation (RVF)
    - Obtained by transforming the stationary frame Navier-Stokes equations to a rotating reference frame
    - Uses the relative velocity as the dependent variable in the momentum equations
    - Uses the relative total internal energy as the dependent variable in the energy equation
  - Absolute Velocity Formulation (AVF)
    - Derived from the relative velocity formulation
    - Uses the absolute velocity as the dependent variable in the momentum equations
    - Uses the absolute total internal energy as the dependent variable in the energy equation

Note: RVF and AVF are equivalent forms of the Navier-Stokes equations!

- ❖ Identical solutions should be obtained from either formulation with equivalent boundary conditions.
- ❖ Scalar transport equations have the same form, independent of the velocity formulation (which affects momentum and energy equations only).

Relative and Absolute Velocity Formulations

Relative velocity formulation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{W} = 0$$

$$\frac{\partial \rho \vec{W}}{\partial t} + \nabla \cdot (\rho \vec{W} \otimes \vec{W}) + \rho(\underbrace{2\vec{\omega} \times \vec{W}}_{\text{Coriolis}} + \underbrace{\vec{\omega} \times \vec{\omega} \times \vec{r}}_{\text{Centrifugal}})$$
$$+\rho(\underbrace{\vec{a}_\omega \times \vec{r}}_{\text{Rotational}} + \underbrace{\vec{a}_t}_{\text{Translational}}) = -\nabla p + \nabla \cdot \bar{\tau}_r + \vec{F}_b$$

$$\frac{\partial \rho e_{tr}}{\partial t} + \nabla \cdot (\rho \vec{W} h_{tr}) = \nabla \cdot (k \nabla T + \bar{\tau}_r \cdot \vec{W})$$
$$+\vec{F}_\mathrm{b} \cdot \vec{W} + \dot{\mathrm{Q}}$$

$e_{tr} = e + 1/2\,(W^2 - U^2)$ - relative total internal energy

$\bar{\tau}_r = \mu \left[\nabla\vec{W} + (\nabla\vec{W})^T - 2/3\,(\nabla \cdot \vec{W})\,\bar{\mathrm{I}}\right]$ - viscous stress

$h_{tr} = e + {}^{p}/_{\rho} + 1/2\,(W^2 - U^2)$ - rothalpy

Note the appearance of additional acceleration terms in the momentum equations

Continuity

Momentum

Energy

$\vec{F}_b$ - body forces

$\dot{Q}$ - heat source

Absolute velocity formulation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{W} = 0$$

$$\frac{\partial \rho \vec{V}}{\partial t} + \nabla \cdot (\rho \vec{W} \otimes \vec{V}) + \rho(\vec{\omega} \times \vec{V} - \vec{\omega} \times \vec{V}_t)$$
$$= -\nabla p + \nabla \cdot \bar{\tau} + \vec{F}_b$$

$$\frac{\partial \rho e_t}{\partial t} + \nabla \cdot (\rho \vec{W} h_t) = \nabla \cdot (k \nabla T - p\vec{U} + \bar{\tau} \cdot \vec{V})$$
$$+\vec{F}_\mathrm{b} \cdot \vec{V} + \dot{\mathrm{Q}}$$

$e_t = e + 1/2\,V^2$ - internal energy

$\bar{\tau}_r = \mu \left[\nabla\vec{V} + (\nabla\vec{V})^T - 2/3\,(\nabla \cdot \vec{V})\,\bar{\mathrm{I}}\right]$ - viscous stress

$h_t = e + {}^{p}/_{\rho} + 1/2\,V^2$- total enthalpy

Note that the frame acceleration terms ($\vec{a}_\omega$ and $\vec{a}_t$) do not appear in the AVF

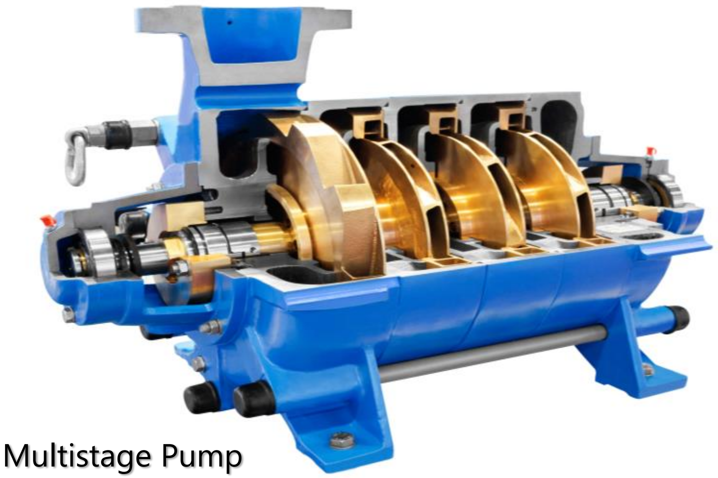
Applications of Rotating Frames of Reference

- Flows within rotating frames of reference occur frequently in science and engineering applications:
  - Compressors and turbines
  - Fans and pumps
  - Rotating cavities, seals and bearings
  - Mixing equipment
  - Fluid coupling devices and torque converters
  - Air motors
  - Marine and aircraft propellers
  - And many more…

Types of Rotating Machines

- Turbomachinery
  - Machines that add work to or extract work from a fluid
    - Compressors, fans, pumps - add work to achieve a pressure rise in the fluid
    - Turbines, windmills - extract work from a fluid to produce power or drive other machines
- Mixing equipment
  - Machines that are designed to mix fluid (and possibly solid) materials for use in chemical processing applications
    - Industrial mixing tanks
- Other Devices
  - Disk cavities and labyrinth seals in gas turbine engines
  - Electric motors
  - Disk drives
  - Rotating tires on automotive vehicles

Examples of Rotating Machinery



Classification of Turbomachinery by Flow Direction

- Axial machines
  - Flow through the machine is (in general) aligned with the axis of rotation
    - Propellers, axial fans/compressors/turbines, swirlers
- Centrifugal machines
  - Flow through the machine is (in general) perpendicular to the axis of rotation
    - Liquid pumps, centrifugal fans/compressors, radial turbines
- Mixed flow machines
  - Flow through the machine is somewhere between axial and centrifugal
    - Mixed flow compressor or pump

Summary

- We discussed two approaches to fluid flows with moving objects: (1) the intrinsically unsteady method of analyzing an object's motion in the stationary frame of the ambient fluid and (2) solving for the flow in the reference frame of the moving objects.
- The moving reference frame approach is widely used in rotating machinery applications, and we briefly discussed some of them.

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